Beyond Planarity: A Spring-Based Approach

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avoid clutter

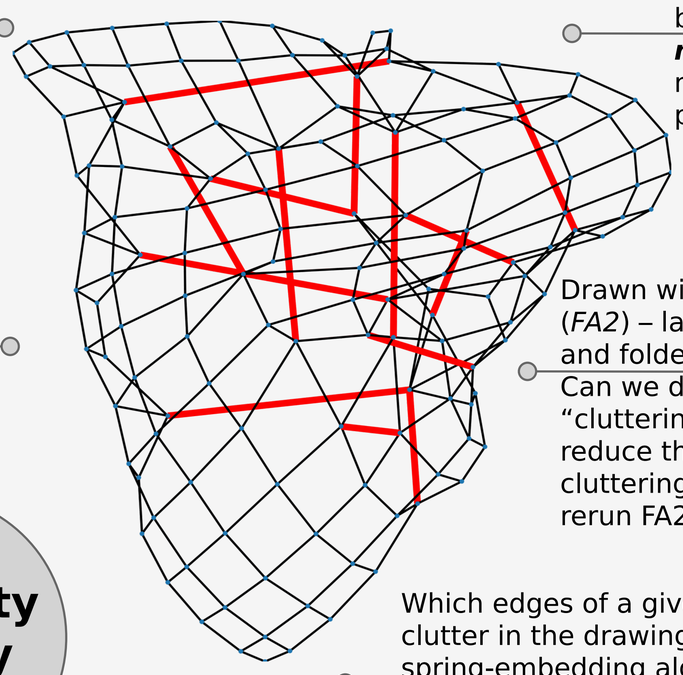
The ultimate goal when constructing a readable graph drawing is to avoid *clutter* that prevents viewers from grasping the structure of the graph

avoid crossings

We know humans perform better on user tasks in drawings with fewer crossings [13] and tend to prefer such drawings [12, 6]

planarity is easy

Testing whether a graph can be drawn without edge crossings [7], i.e. in a *planar* way, as well as constructing its planar drawing [14], can be done in linear time

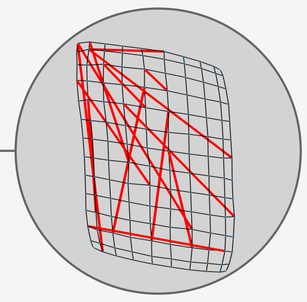


near-planarity is hard

In practice, however, graphs are rarely planar, but can still have a planar substructure and thus be *nearly planar*. **Can we draw graphs in a *near-planar* manner?** Formal definitions of near-planarity lead to hard computational problems [5, 2, 1, 10]

what if we knew bad edges?

Drawn with ForceAtlas2 [8, 3] (*FA2*) – layout appears cluttered and folded inwards.
Can we do better if we know the "cluttering edges"? YES - reduce the weights of the cluttering edges to 0.01 and rerun FA2.



research question

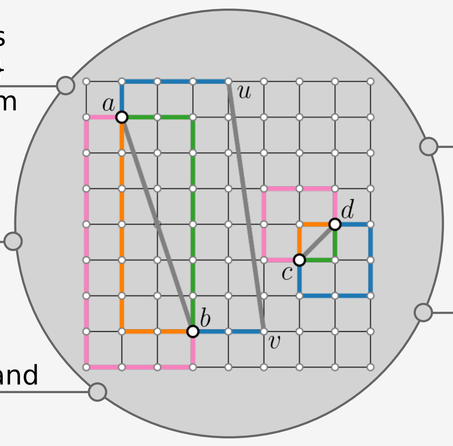
Which edges of a given nearly planar graph create clutter in the drawings generated by a spring-embedding algorithm?
How do we weight these edges to create a drawing where a planar substructure is clearly visible?

Heuristic

1a Find the lengths of the vertex-disjoint paths between end-vertices of an edge $e = \{u, v\}$ using the max-flow Edmonds-Karp algorithm [4] as described in [9].

1b Construct a *footprint* $f(e) = [\ell_1, \ell_2, \dots]$ for each edge e. Here ℓ_i – the number of vertex-disjoint paths of length d between the end-point of e

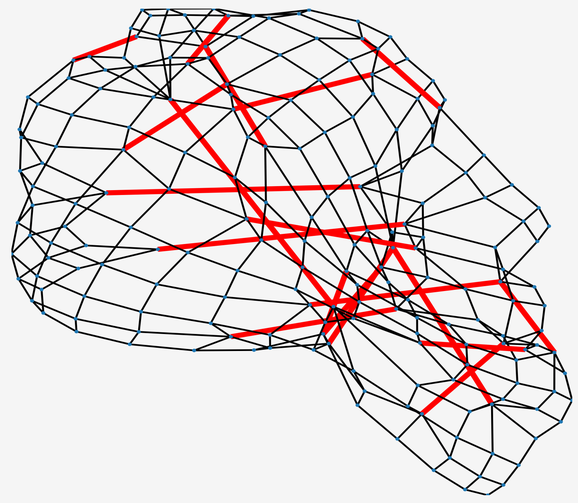
1c Standardize the footprints $f(e)$ to a user-specified number of dimensions k and some function $\mathcal{M}$, where $\mathcal{M}$ can be mean/minimum/maximum.

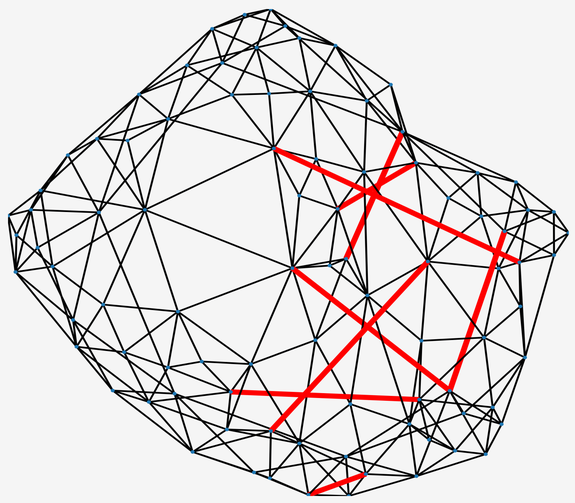


2 Using footprints, find cluttering edges using the *Isolation Forest* technique [11], which is designed to find anomalous data points in a high-dimensional space.

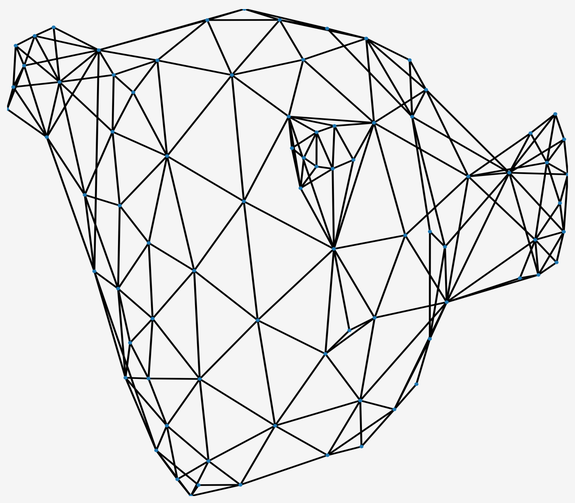
3 Set the weight of each edge e classified as cluttering to $\mathcal{M}(f(e))$

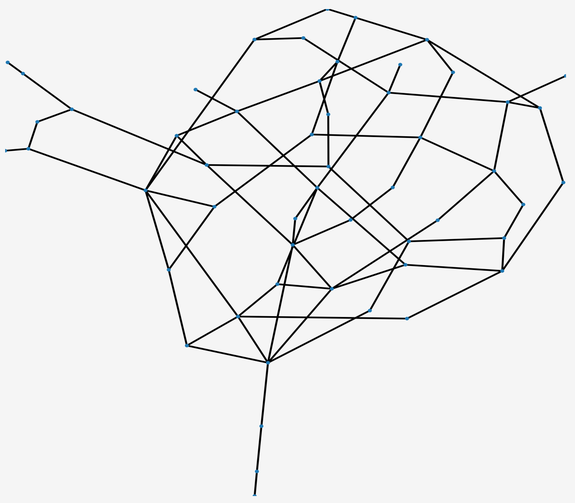
Original layouts



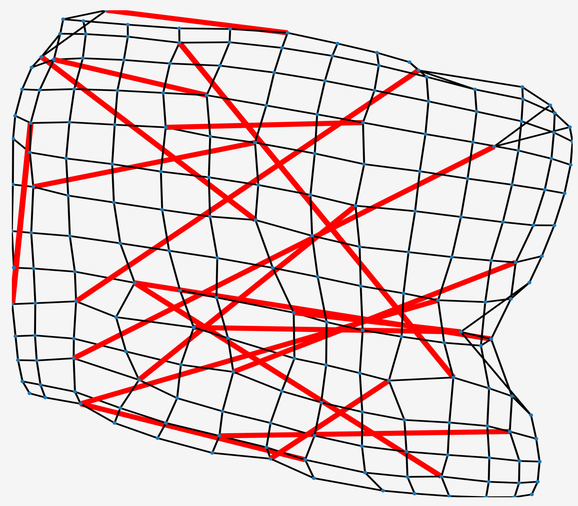


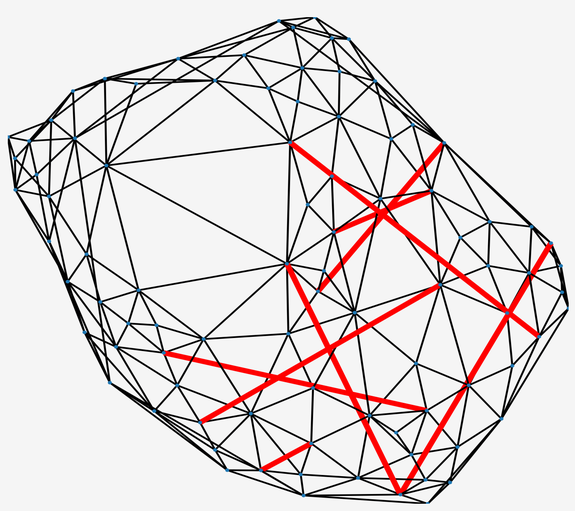
Results

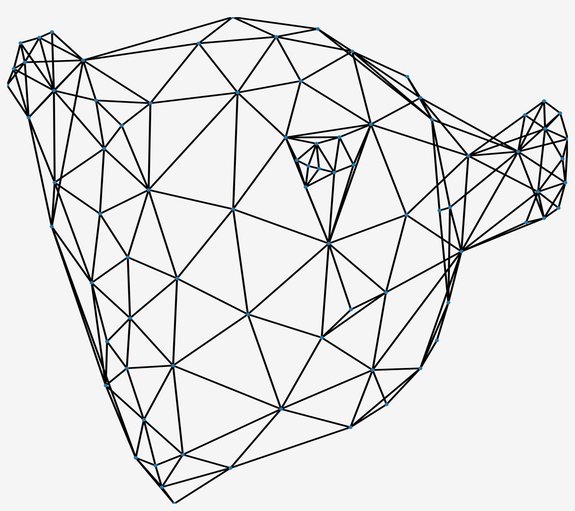


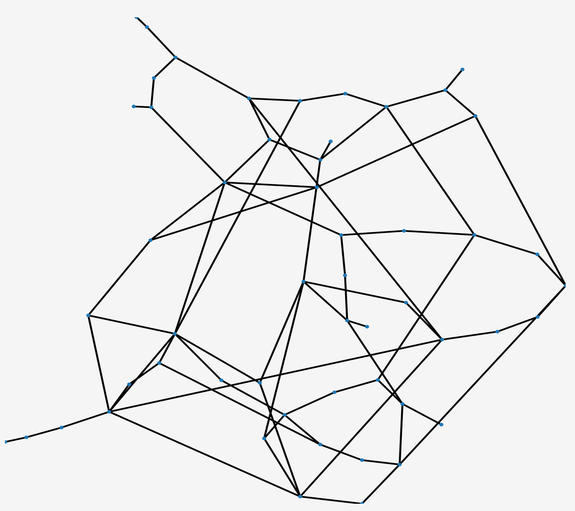


Layouts produced by the *heuristic*









References

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