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A Method for Predicting the First Sighting of the New Crescent Moon

by

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Summary

This note gives an empirical algorithm for predicting first sighting of the new crescent moon, and a neat method for estimating the best time for making the observation.

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Summary

A single parameter method is obtained for predicting first sighting of the new crescent moon, based on the Indian method. Based on the method of Bruin, a simple rule is given that determines the time of best visibility in the twilight sky.

The six ranges of the single test parameter q are calibrated by applying the q-test to a standard set of 295 first sightings of the new crescent moon that cover the period 1859 to 1996. The ranges of the parameter correspond to the following visibility types and visibility codes for the new crescent moon:

(A) easily visible to the unaided eye;
(B) visible under perfect atmospheric conditions;
(C) may need optical aid to find the thin crescent moon before it can be seen with the unaided eye;
(D) can only be seen with binoculars or a telescope;
(E) below the normal limit for detection with a telescope;
(F) not visible, below the Danjon limit.

1. Introduction.

Methods for predicting first sighting of the new crescent moon have been around since the time of the Babylonians and maybe before that. The earliest methods depended upon parameters such as the age of the Moon (*Age*) and the time from sunset to moonset (*Lag*). In Medieval times the methods became slightly more sophisticated, and they included more technical parameters such as ecliptic latitude and longitude.

In the twentieth century empirical methods have been developed based on functional relationships between the arc of light ($ARCL$), arc of vision ($ARCV$) and the relative azimuth (DAZ). In this note I examine three of the twentieth century methods, due to Maunder (1911), the Indians, *The Indian Astronomical Ephemeris*, (1996), and Bruin (1977). My method is an adaptation of these three methods.

2. The basic variables.

The angles $ARCL$, $ARCV$ and DAZ, always in degrees, are defined as follows:

$ARCL$ is the angle subtended at the centre of the Earth by the centre of the Sun and the centre of the Moon.

$ARCV$ is the geocentric difference in altitude between the centre of the Sun and the centre of the Moon for a given latitude and longitude, ignoring the effects of refraction.

DAZ is the difference in azimuth between the Sun and the Moon at a given latitude and longitude, the difference is in the sense azimuth of the Sun minus azimuth of the Moon.

Angles $ARCL$, $ARCV$ and DAZ satisfy the equation

$$\cos ARCL = \cos ARCV \cos DAZ \qquad (2{\cdot}1)$$

so only two of the angles are independent variables.

For angles less than about 22° this approximates to

$$ARCL^2 = ARCV^2 + DAZ^2 \qquad (2{\cdot}2)$$

Although $ARCL$ and $ARCV$ are not directly observable, for historical reasons it is difficult to discontinue using them.

3. The basic data for three twentieth century methods.

This section gives the basic data for the method of (a) Maunder, (b) the Indians and (c) Bruin.

(a) The basic data for the Maunder method are given on page 359 of Maunder (1911), and are reproduced in Table 1:

Table 1: Maunder					
DAZ	0°	5°	10°	15°	20°
ARCV	11·0	10·5	9·5	8·0	6·0

Table 1 gives $ARCV$ as a function of DAZ, i.e. $ARCV = \mathbf{f}(DAZ)$. If $ARCV > \mathbf{f}(DAZ)$ then the crescent is visible. On the other hand if $ARCV < \mathbf{f}(DAZ)$ it is not visible. Thus in principle, the degree of visibility is equivalent to testing the value of a single parameter q, where $q = ARCV - \mathbf{f}(DAZ)$. In section 5 it is shown how q is calibrated for the Indian method using a standard data base of observations of lunar first sightings.

Fitting a quadratic polynomial in DAZ to $ARCV$ using the data in Table 1 by the method of least squares, yields a perfect fit, which indicates that Maunder was using a quadratic to represent his data. The visibility criterion is that the crescent is visible if

$$ARCV > 11 - |DAZ|/20 - DAZ^2/100 \qquad (3{\cdot}1)$$

(b) Since 1966, the basic data for the Indian method have been given in the Explanation to *The Indian Astronomical Ephemeris*, which is based on Schoch (1930). In the 1996 edition, for example, they are found on page 559 under the section"Heliacal rising and setting of planets". They give the data in a similar form to Maunder, i.e. a table of $ARCV$ in terms of DAZ, which is reproduced here in Table 2.

Table 2: Indian					
DAZ	0°	5°	10°	15°	20°
ARCV	10·4	10·0	9·3	8·0	6·2

In this case a quadratic polynomial in DAZ fitted to $ARCV$ by the method of least squares produces the following criterion:

$$ARCV > 10{\cdot}3743 - 0{\cdot}0137\,|DAZ| - 0{\cdot}0097\,DAZ^2 \qquad (3{\cdot}2)$$

(c) The basic data for the Bruin method are contained in figure 9, page 339 of Bruin (1977). This diagram yields $ARCV$ as a function of W the width of the crescent moon, and they are reproduced in Table 3. Note that the entry for $W = 0'\!.3$ has been extrapolated, and that Bruin does not extend his curves beyond $W = 3'$.

Table 3: Bruin						
W	0′.3	0′.5	0′.7	1′	2′	3′
ARCV	10·0	8·4	7·5	6·4	4·7	4·3

In this case a cubic polynomial in W is fitted to $ARCV$ by the method of least squares. A cubic polynomial is required because the curve has an inflexion. Moreover, since the coefficient of W^3 is negative, it guarantees that the test criteria is eventually satisfied, provided that W is large enough. The criterion is that the crescent is visible if

$$ARCV > 12{\cdot}4023 - 9{\cdot}4878\,W + 3{\cdot}9512\,W^2 - 0{\cdot}5632\,W^3 \qquad (3{\cdot}3)$$

where W is the width of the crescent in minutes of arc and is given by

$$W = 15\,(1 - \cos ARCL) = 15\,(1 - \cos ARCV\ \cos DAZ) \qquad (3{\cdot}4)$$

Notice that Bruin took the semi-diameter of the Moon to be a constant 15′, and that W is a function of $ARCV$ and DAZ.

The criterion for Maunder (3.1) and the Indian method (3.2), can also be expressed as a function of $ARCV$ and W, as follows:

$$ARCV > 13{\cdot}1783 - 9{\cdot}0812\, W + 2{\cdot}0709\, W^2 - 0{\cdot}3360\, W^3 \tag{3·5}$$

$$ARCV > 11{\cdot}8371 - 6{\cdot}3226\, W + 0{\cdot}7319\, W^2 - 0{\cdot}1018\, W^3 \tag{3·6}$$

Note that an additional cubic term in W is required to maintain precision.

Finally for comparison I give the expression for Bruin in the alternative form $ARCV$ as a polynomial in DAZ, although the precision is poor when DAZ exceeds about 20°.

$$ARCV > 10{\cdot}136 \ + 0{\cdot}14\,|DAZ| - 0{\cdot}03\, DAZ^2 \tag{3·7}$$

The curves $ARCV = \mathbf{f}(DAZ)$ (i.e. $q = 0$) are drawn in Figure 1, for the three methods. As explained in section 3(a), visibility occurs when $ARCV > \mathbf{f}(DAZ)$ (i.e. $q > 0$). Note that the Indian and Bruin test are very similar between $DAZ = 0°$ and $DAZ = 20°$. For $DAZ > 20°$ the Bruin curve behaves quite differently from the Indian curve and has a strong inflexion. At high latitudes, when the orbit of the Moon is almost parallel with the horizon, this shape of curve produces predictions of first sighting that are far too late. Experimental data are required in this region to improve predictions at high latitudes.

From 1996 March, HM Nautical Almanac Office decided to abandon its test based on the Bruin method for one based on the Indian method, using the expression (3.6) since it produced more sensible results for old moonage sightings at high latitudes, which occur at least once a year for latitudes around 55°. It also uses the topocentric width of the crescent W' in place of W, which is calculated as follows:

$$\mathrm{SD} = 0{\cdot}27245\,\pi \tag{3·8}$$

$$\mathrm{SD}' = \mathrm{SD}\,(1 + \sin h \sin \pi) \tag{3·9}$$

$$W' = \mathrm{SD}'\,(1 - \cos ARCL) \tag{3·10}$$

where a dash indicates that a co-ordinate is topocentric, SD is the semi-diameter of the Moon, π is the parallax of the Moon and h is the geocentric altitude of the Moon.

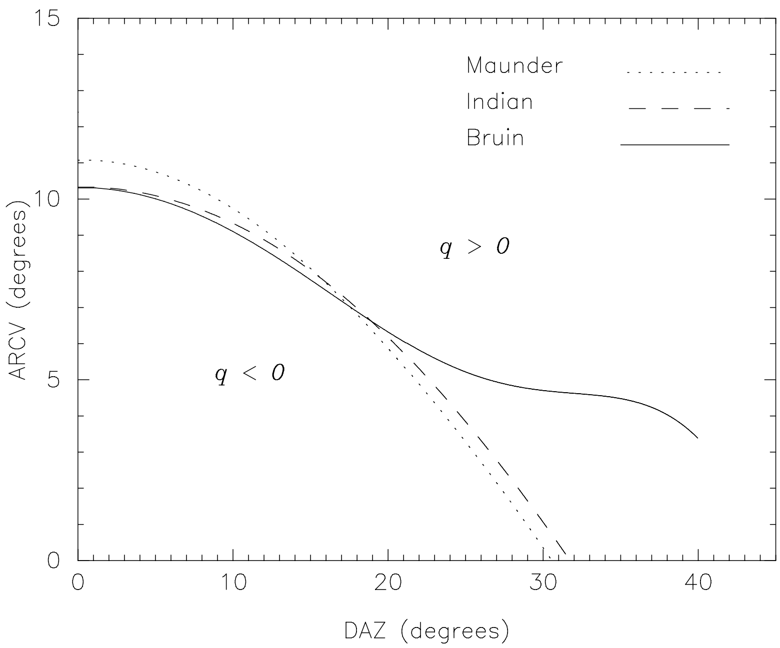

Figure 1: Comparison of Maunder, Indian and Bruin.

4. The concept of best time.

If the crescent moon is to be observed as early as possible it is important to know when is the best time for making the observation. If the observation is made too soon after sunset then the twilight sky may be too bright to pick out the faint crescent moon. The observer has to wait until the contrast between the crescent moon and the twilight sky has increased sufficiently for the Moon to be seen. Although the twilight sky becomes darker, the brightness of the crescent moon also diminishes due to atmospheric extinction as the altitude of the setting Moon decreases, so there is an optimum time for making the observation. In critical cases the observation is only possible within a short period of the best time.

Soon after sunset the observer will be using daylight vision to detect faint objects. Objects like stars are best found by looking straight at them. As the Sun sinks to about 5° below the horizon, night vision takes over. In this case much fainter objects can be seen, but it is necessary to use averted vision. Around this time first magnitude stars begin to pop out of the sky. In critical cases, with an elongation of say 8° and the Sun 5° below the horizon, and in the perfect geometrical situation with $DAZ = 0°$, there is not much time left to observe the Moon before it sets in the murk on the horizon. It becomes increasingly more difficult to observe the crescent as the Danjon limit is approached simply due to these geometrical constraints.

Schaefer (1988), (see pages 519 and 520) calculates the best time from the logarithm of the actual total brightness of the Moon divided by the total brightness of the Moon needed for visibility for the given observing conditions. I have found it difficult to repeat his calculations exactly because the reference he quotes for his theoretical method, Schaefer (1990), is not readily available in most libraries. Furthermore, in his other papers that are relevant to the subject, he gives insufficient or conflicting information.

I have found a simple rule, however, based on Bruin (1977), which is sufficient for most purposes. Bruin (1977) gives a method for calculating the possibility of observing the Moon at any instant after sunset, and the results are given in figure 9 on page 339 of his paper. He plots a series of curves of visibility of $h+s$ (arc of vision) against s for $W = 0'.5$, $0'.7$, $1'$, $2'$ and $3'$, where $90° - s$ is the geocentric altitude of the Sun, and h is the geocentric altitude of the Moon. Hence s is the depression of the Sun below the horizon and $h+s = ARCV$. Each of these curves has a minimum, which Bruin says is when the situation is at an optimum. On the curve for $W = 0'.5$, he marks the minimum as point C. If a straight line is drawn through the origin (at $h+s = 0°$ and $s = 0°$) and through point C (at $h+s = 9°$, and $s = 4°$), it is found that this line passes directly through the minima of the series of curves for different W. Hence at the best time $4h = 5s$. If T_s is the time of sunset and T_m is the time of moonset, then the best time is given by

$$T_b = (5\,T_s + 4\,T_m)/9 = T_s + \frac{4}{9}\,Lag \qquad (4{\cdot}1)$$

Provided the derivation of the Bruin curves is sound, they yield, amongst other things, a very simple rule for determining the best time. It is therefore an important exercise to re-determine Bruin's figure 9 using modern theories for the brightness of the twilight sky as a function of s, (and azimuth of the Sun), the brightness of the Moon as a function of phase, the minimum contrast observable to the human eye for a thin crescent shape, various effects of the atmosphere, such as seeing and extinction, the effects of age of the observer, and other relevant effects.

To this end I first attempted to re-determine figure 9 in Bruin (1977) using his theories, with the aid of the computer package called MathCad. I wanted to extend his curves to a wider range of W, and confirm my result for determining best time. Unfortunately my attempts failed, because scaling factors have to be applied to make the transformations produce sensible results. In his paper Bruin says he has applied a "Gestalt" factor to obtain his results.

From the curves in Figure 1 it appears that for $DAZ < 20°$ Bruin adjusted his results to agree with the Indian method. Doggett and Schaefer (1994) have made some strong comments about Bruin's assumptions for his model, and they point out that some of his quantities are orders of magnitude out.

In spite of these difficulties Bruin's method is a very important approach to the problem because in principle it provides answers to many questions that the Maunder and Indian method cannot address. I am therefore making a fresh attempt to calculate Bruin's curves using the modern approach of Schaefer.

The problem of predicting heliacal rising and setting of stars is similar to the problem of predicting first sighting of the new crescent moon. Three relevant papers have been written by Schaefer (1985), (1986) and (1987) on this topic, and I have managed to reproduce his work, apart from some inconsistencies between the three papers, which still need sorting out. A fourth paper by Schaefer (1993a), repeats all the relevant formulae, but again there are inconsistencies. In this last paper he quotes the expression by Allen (1963) for the apparent magnitude of the Moon as a function of phase. This is the remaining piece of information that is needed in the calculation to predict first sighting of the new crescent moon.

In general, the magnitudes of the stars are fixed, and empirical rules are found for predicting heliacal rising and setting that depend upon magnitude and $ARCV$, Lockyer (1894). There is also some dependence upon DAZ, which is fixed for each star.

Unlike the stars, the Moon is always near to the Sun at first sighting. Moreover the apparent magnitude of the Moon depends upon $ARCL$, which in turn is a function of $ARCV$ and DAZ, and therefore we would expect there to be an empirical rule that is a function of $ARCV$ and DAZ for making the prediction. The problem reduces to finding an expression for that function. There is one difference in the calculation that is often overlooked in the literature, a star is a point source, whilst the Moon is a thin crescent.

I have to agree with Schaefer that his method is very short, taking only a few lines of programming. I can now produce theoretically the curves $ARCV$ as a function of DAZ, and even predict what happens in daylight. I could produce and extend the curves of Bruin to find the best time using modern theories, although there are more logical ways of performing this calculation using a computer. As Schaefer points out, using different extinction coefficients I find an enormous difference between a site at high altitude with a clear dry atmosphere, and one at sea level with a humid or dusty atmosphere, so my empirical approach must be confined to a specific type of site where altitudes above sea level and extinction factors are confined to within narrow limits.

5. The basic data set of observations for calibrating first visibility parameters.

A list of 252 observations of first sighting has been published by Schaefer, (1988) and by Doggett, and Schaefer, (1994). The list was later extended to 295 observations by Schaefer (1996). The list includes cases of both sightings as well as non-sightings. Even non-sightings provide relevant information for calibration purposes.

I have re-calculated these data using my simple rule for determining the best time, and displayed the set of 295 observations in order of decreasing q in Table 4. Columns numbered 1 to 18 have the following meaning:

- 1 Number from original list.
- 2, 3, 4 Date of observation in the form year, month, day.
- 5 Morning (M) or evening (E) observation.
- 6 Julian Date of astronomical new moon minus 2 400 000 days.
- 7, 8 Latitude and longitude of observation.
- 9, 10, 11 Arc of light ($ARCL$), arc of vision ($ARCV$) and relative azimuth (DAZ) at best time.
- 12 Age of the Moon (Age) in hours at best time.
- 13 Time in minutes from sunset to moonset (Lag).
- 14 Parallax of the Moon (π) in minutes of arc. Semi-diameter $= 0{\cdot}27245\pi$.
- 15 Topocentric width of the crescent W' in minutes of arc.
- 16 The test parameter (q), which is derived from the Indian method, and is defined in section 6.
- 17 Schaefer's coded description (BES) of how each observation was made. If the only character is a "V", then the Moon was visible to the unaided eye. An "I" means it was not seen with the unaided eye. If the first character is followed by (F) then optical aid was used to find the Moon, which was then spotted with the unaided eye. If the first character is followed by (B) or (T) it was visible with binoculars or a telescope, respectively. In the second and third papers, the rules were changed as follows: If the first character is followed by (I) it was invisible with either binoculars or a telescope. If the first character is followed by (V) it was visible with either binoculars or a telescope.
- 18 A prediction (BDY) of how the observation would be made, based on Schaefer's coded description

in column 17, and derived from the value of the test parameter q at the best time of observation. The methods for setting the empirical limits on q are described in section 6.

I found it necessary to reproduce most of the information concerning the 295 observations, because the original published tables contained so many errors. For example, in the original list of 252 observations, the column containing the Julian Date of the conjunction has the following errors:

No.	11	*for*	2401081·171	*read*	2401082·171
No.	98	*for*	2422933·485	*read*	2422993·485
No.	115	*for*	2440741·239	*read*	2440741·598
No.	116	*for*	2440741·239	*read*	2440741·598
No.	117	*for*	2441038·308	*read*	2441037·308
No.	208	*for*	2427216·422	*read*	2427216·921
No.	222	*for*	2445702·709	*read*	2445702·719

Schaefer (1996) gives corrections for No. 44 and 117. He agrees with Loewinger (1995) that 44 is unreliable and should be deleted, and gives the correct conjunction time for 117. I still found errors in his list of 43 additional observations. For example the data for No. 285 is for the following day on 1991:05:16, and the following significant discrepancies were noticed in the conjunction times:

No.	256	*for*	2445702·709	*read*	2445702·719
No.	257	*for*	2445702·709	*read*	2445702·719

There are other differences in the columns for *ARCL*, *ARCV*, *DAZ*, and *Age*, some of which can be explained by a different estimate for the best time. Unfortunately Schaefer did not include his estimate for best time in the original list, and there is no simple way of finding out what it should be. There are also many errors in *Lag*, which are too frequent to report. See also the report by Loewinger (1995), for further comments on errors.

Table 4: The 295 observations listed in order of decreasing q

No.	Date				JD − 2 400 000	Lat	Long	*ARCL*	*ARCV*	*DAZ*	*Age*	*Lag*	π	W'	q	Visibility BES	BDY
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
							(A)		$q > +0{\cdot}216$		166 entries						
	year	m	day			°	°	°	°	°	h	m	′	′			
37	1870	7	25	M	4271·971	38·0	23·7	40·3	31·1	−26·1	−81·2	−173·7	55·4	3·6	+3·730	V	V
73	1879	7	22	E	7549·879	38·0	23·7	42·2	19·1	37·8	81·4	99·3	58·2	4·1	+2·804	V	V
48	1872	9	30	M	5069·146	38·0	23·7	29·3	29·2	−2·4	−60·3	−148·4	54·8	1·9	+2·749	V	V
74	1879	12	11	M	7696·963	38·0	23·7	32·5	25·2	−20·6	−54·7	−145·6	60·9	2·6	+2·665	V	V
75	1879	12	15	E	7696·963	38·0	23·7	30·6	24·4	18·6	53·1	139·9	59·9	2·3	+2·433	V	V
274	1984	10	26	E	45998·008	15·6	35·6	29·7	24·7	16·7	51·8	106·5	59·8	2·2	+2·409	V	V
72	1879	6	22	E	7520·347	38·0	23·7	34·2	21·8	26·5	70·4	120·7	56·5	2·7	+2·359	V	V
71	1879	5	23	E	7490·744	38·0	23·7	28·3	24·1	14·9	60·8	138·2	55·0	1·8	+2·182	V	V
270	1984	8	28	E	45939·310	15·6	35·6	27·1	23·5	13·5	45·2	96·0	61·0	1·8	+2·143	V	V
266	1984	6	1	E	45851·200	15·6	35·6	25·0	24·7	3·6	48·1	111·5	58·0	1·5	+2·100	V	V
285	1991	5	16	E	48390·692	39·0	−76·8	26·1	23·1	12·3	44·6	133·6	60·8	1·7	+2·035	V	V
263	1984	4	3	E	45792·007	15·6	35·6	24·5	24·5	0·2	52·4	101·1	55·1	1·4	+2·016	V	V
9	1861	9	7	E	1023·425	38·0	23·7	38·9	13·7	36·5	67·1	68·2	60·4	3·7	+2·014	V	V
70	1878	11	26	E	7312·884	38·0	23·7	30·6	18·7	24·3	54·7	109·3	58·6	2·2	+1·846	V	V
35	1868	6	22	E	3504·115	38·0	23·7	30·2	18·4	24·0	51·8	101·3	60·7	2·3	+1·827	V	V
19	1864	3	9	E	1938·666	38·0	23·7	21·8	21·6	2·3	37·3	107·9	60·1	1·2	+1·637	V	V
284	1991	3	17	E	48331·841	39·0	−76·8	21·7	21·7	0·9	39·9	110·5	58·4	1·1	+1·617	V	V
261	1984	3	4	E	45762·272	15·6	35·6	21·3	21·2	1·8	45·9	86·0	54·1	1·0	+1·511	V	V
26	1865	3	28	E	2322·728	38·0	23·7	21·2	20·4	5·8	36·0	102·7	60·9	1·1	+1·496	V	V
245	1989	10	3	E	47799·407	36·1	−108·8	34·6	13·4	32·0	75·7	67·0	54·0	2·6	+1·485	V	V
65	1878	1	5	E	6988·086	38·0	23·7	23·6	19·7	13·1	50·1	109·8	54·7	1·2	+1·477	V	V
267	1984	6	30	E	45880·638	15·6	35·6	21·0	20·4	5·0	37·6	90·6	59·3	1·1	+1·461	V	V
276	1984	11	24	E	46027·457	15·6	35·6	22·6	19·5	11·6	40·8	86·6	58·8	1·2	+1·450	V	V
21	1864	6	6	E	2026·986	38·0	23·7	27·0	17·3	20·8	54·8	94·9	55·3	1·6	+1·438	V	V
4	1860	2	23	E	462·318	38·0	23·7	20·7	20·6	2·0	45·3	103·0	54·1	1·0	+1·428	V	V

Table 4: The 295 observations listed in order of decreasing q *continued*

No.	Date				JD − 2 400 000	Lat	Long	*ARCL*	*ARCV*	*DAZ*	*Age*	*Lag*	π	W'	q	Visibility BES	BDY
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
						(A)		$q > +0{\cdot}216$		166 entries		*continued*					
	year	m	day			°	°	°	°	°	h	m	′	′			
272	1984	9	26	E	45968·634	15·6	35·6	22·1	18·9	11·5	36·9	76·0	61·0	1·2	+1·385	V	V
254	1983	11	6	E	45643·432	15·6	35·6	21·7	19·1	10·3	41·3	81·5	57·3	1·1	+1·351	V	V
11	1861	11	4	E	1082·172	38·0	23·7	28·4	14·5	24·5	47·9	79·3	60·8	2·0	+1·318	V	V
17	1864	1	10	E	1879·824	38·0	23·7	19·8	19·2	5·0	32·4	102·8	61·2	1·0	+1·295	V	V
32	1866	4	16	E	2706·795	38·0	23·7	20·7	18·2	9·9	34·6	93·0	61·2	1·1	+1·245	V	V
66	1878	6	2	E	7136·575	38·0	23·7	21·0	18·2	10·5	40·7	105·4	57·4	1·0	+1·232	V	V
282	1990	11	19	E	48212·878	39·0	−76·8	27·9	15·0	23·6	61·4	87·0	54·0	1·7	+1·231	V	V
244	1989	7	5	E	47710·708	37·4	−121·6	24·3	16·5	17·9	47·2	89·4	56·6	1·4	+1·221	V	V
63	1877	11	7	E	6928·868	38·0	23·7	29·6	13·0	26·6	55·1	73·0	57·2	2·0	+1·189	V	V
207	1933	3	27	E	27157·639	48·6	7·7	19·2	18·6	4·9	39·4	112·6	55·1	0·84	+1·158	V	V
259	1984	2	3	E	45732·491	15·6	35·6	19·0	18·6	4·0	40·4	77·9	54·0	0·80	+1·138	V	V
25	1865	1	28	E	2263·897	38·0	23·7	18·5	18·3	2·3	30·9	94·3	60·6	0·85	+1·138	V	V
57	1875	7	4	E	6072·727	38·0	23·7	21·6	16·7	13·7	37·1	94·0	59·4	1·1	+1·129	V	V
61	1877	3	16	E	6693·620	38·0	23·7	18·5	18·4	1·8	38·4	91·4	55·8	0·79	+1·120	V	V
76	1880	1	10	M	7726·444	38·0	23·7	23·8	15·2	−18·4	−41·6	−87·0	60·6	1·4	+1·115	V	V
217	1935	4	4	E	27896·009	48·2	5·1	18·2	17·9	3·6	30·8	109·7	59·6	0·82	+1·077	V	V
24	1864	11	1	E	2175·144	38·0	23·7	23·7	15·2	18·2	48·6	79·6	56·4	1·3	+1·050	V	V
264	1984	5	2	E	45821·657	15·6	35·6	17·8	17·8	0·9	36·7	75·2	56·2	0·73	+1·024	V	V
210	1933	9	18	M	27335·264	48·6	7·7	19·3	17·1	−9·1	−37·9	−101·3	56·6	0·87	+1·024	V	V
50	1872	10	4	E	5069·146	38·0	23·7	24·6	14·3	20·1	49·1	70·1	57·0	1·4	+1·016	V	V
8	1861	8	8	E	994·037	38·0	23·7	29·3	10·8	27·3	52·9	52·3	59·4	2·1	+0·984	V	V
168	1986	10	5	E	46707·288	40·8	−73·2	28·5	11·2	26·3	52·0	57·7	59·5	2·0	+0·979	V	V
12	1861	12	3	E	1111·596	38·0	23·7	21·7	15·1	15·6	37·4	85·1	60·3	1·2	+0·976	V	V
206	1932	4	7	E	26803·556	50·0	36·2	18·2	17·2	6·1	39·7	110·0	54·0	0·74	+0·965	V	V
60	1876	7	22	E	6456·704	38·0	23·7	22·0	14·2	16·9	37·4	73·9	60·5	1·2	+0·908	V	V
268	1984	7	29	E	45909·994	15·6	35·6	17·3	16·5	5·1	28·8	69·7	60·4	0·75	+0·905	V	V
287	1992	3	5	E	48686·057	39·0	−76·8	16·9	16·9	−0·5	34·3	84·4	55·2	0·65	+0·894	V	V
58	1876	2	26	E	6309·764	38·0	23·7	17·3	16·6	5·0	34·5	82·0	57·1	0·71	+0·889	V	V
110	1961	1	17	E	37316·396	34·0	−118·3	16·9	16·3	4·4	28·2	81·3	61·3	0·72	+0·866	V	V
5	1860	6	20	E	580·725	38·0	23·7	20·1	14·7	13·7	37·1	82·1	59·1	1·0	+0·851	V	V
68	1878	7	31	E	7195·403	38·0	23·7	25·2	11·8	22·3	44·4	58·5	60·3	1·6	+0·846	V	V
193	1987	5	29	E	46943·134	39·2	−105·5	17·4	16·2	6·4	35·8	96·3	54·5	0·68	+0·839	V	V
14	1862	3	31	E	1229·823	38·0	23·7	16·4	16·4	0·1	33·6	82·4	54·6	0·61	+0·818	V	V
30	1865	10	21	E	2529·185	38·0	23·7	21·7	13·6	16·9	47·7	68·7	54·2	1·0	+0·770	V	V
55	1874	4	17	E	5630·080	38·0	23·7	16·3	15·5	4·8	27·7	79·8	60·8	0·66	+0·762	V	V
51	1872	12	31	E	5157·776	38·0	23·7	19·8	13·6	14·5	33·2	77·3	61·0	1·0	+0·744	V	V
45	1872	6	7	E	4950·641	38·0	23·7	18·0	14·9	10·2	39·0	84·8	54·2	0·73	+0·734	V	V
235	1989	4	4	M	47622·648	41·9	−88·7	24·0	11·3	−21·2	−40·4	−56·3	61·1	1·4	+0·733	V	V
257	1984	1	4	E	45702·719	15·6	35·6	16·6	15·5	5·9	34·6	67·8	54·7	0·62	+0·732	V	V
23	1864	9	3	E	2115·756	38·0	23·7	27·0	10·3	24·9	59·1	49·2	54·3	1·6	+0·722	V	V
83	1909	2	21	E	18357·954	51·1	0·0	19·2	13·6	13·6	31·2	86·2	61·3	0·93	+0·713	V	V
53	1873	5	27	E	5304·890	38·0	23·7	16·9	15·1	7·7	32·9	85·3	56·5	0·66	+0·713	V	V
211	1934	3	16	E	27512·006	55·6	33·9	15·9	15·1	4·7	28·4	104·6	58·2	0·61	+0·690	V	V
62	1877	6	12	E	6782·106	38·0	23·7	16·4	14·5	7·7	27·9	83·5	60·1	0·67	+0·658	V	V
283	1990	12	18	E	48242·682	39·0	−76·8	19·0	13·6	13·3	42·0	78·3	54·3	0·81	+0·646	V	V
47	1872	9	4	E	5039·537	38·0	23·7	19·7	13·1	14·7	40·4	62·9	55·3	0·88	+0·635	V	V
42	1871	6	19	E	4596·604	38·0	23·7	18·2	13·8	11·9	39·9	77·3	54·4	0·74	+0·629	V	V
34	1867	11	27	E	3296·716	38·0	23·7	16·8	14·4	8·7	34·5	77·0	54·4	0·63	+0·628	V	V
84	1911	6	27	E	19214·057	49·9	2·3	19·2	12·7	14·5	31·4	97·3	60·8	0·93	+0·616	V	V
82	1908	2	3	E	17973·859	56·0	−3·2	19·8	12·3	15·6	32·9	95·1	61·0	1·0	+0·603	V	V
64	1877	12	6	E	6958·420	38·0	23·7	21·2	12·0	17·5	41·5	68·5	56·1	1·0	+0·598	V	V
104	1922	4	28	E	23171·710	−33·9	18·5	17·6	13·6	−11·2	35·6	65·3	56·1	0·72	+0·596	V	V
218	1982	4	24	E	45083·354	37·2	−84·1	16·8	13·7	9·7	28·3	68·5	60·4	0·70	+0·595	V	V
248	1990	4	25	E	48006·686	37·7	−121·5	14·6	14·5	0·5	22·9	76·8	61·3	0·54	+0·591	V	V
18	1864	2	8	E	1909·257	53·5	−2·3	14·9	14·4	4·1	23·6	97·4	61·3	0·56	+0·587	V	V
213	1934	5	14	E	27571·021	48·2	5·1	15·8	14·3	6·7	31·4	101·1	55·2	0·57	+0·587	V	V
117	1971	3	27	E	41037·308	51·0	0·0	14·7	14·4	2·9	23·7	90·1	61·0	0·54	+0·578	V	V
27	1865	4	26	E	2352·094	38·0	23·7	16·0	13·9	7·9	27·5	71·0	60·2	0·63	+0·576	V	V
273	1984	10	25	E	45998·008	15·6	35·6	16·0	13·6	8·3	27·4	55·4	60·5	0·64	+0·553	I	V
216	1935	2	4	E	27837·186	48·2	5·1	15·0	13·9	5·5	24·9	85·0	61·3	0·57	+0·547	V	V

Table 4: The 295 observations listed in order of decreasing q *continued*

No.	Date				JD − 2 400 000	Lat	Long	*ARCL*	*ARCV*	*DAZ*	*Age*	*Lag*	π	W'	q	Visibility BES	BDY
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
						(A)	$q > +0{\cdot}216$		166 entries		*continued*						
	year	m	day			°	°	°	°	°	h	m	′	′			
89	1918	3	13	E	21665·328	50·2	5·1	14·3	14·2	1·6	22·4	85·4	61·3	0·52	+0·545	V	V
286	1991	9	7	M	48507·960	50·4	−4·4	17·1	13·0	−11·2	−30·0	−79·6	59·9	0·73	+0·540	V(V)	V
43	1871	8	17	E	4655·793	38·0	23·7	17·9	12·7	12·6	34·7	63·0	56·7	0·75	+0·527	V	V
236	1989	4	6	E	47622·648	34·0	−107·2	14·0	14·0	−1·2	22·5	66·3	61·1	0·50	+0·510	V	V
88	1916	4	3	E	20956·182	49·4	8·7	14·2	14·1	1·9	26·3	86·7	56·5	0·47	+0·507	V	V
69	1878	10	27	E	7283·458	38·0	23·7	24·4	8·8	22·8	40·9	45·0	60·4	1·5	+0·498	V	V
108	1953	4	14	E	34481·341	51·1	5·3	14·1	13·8	2·9	23·0	92·4	59·9	0·50	+0·497	V(F)	V
249	1990	5	23	M	48035·993	31·6	−110·5	15·1	13·2	−7·4	−24·0	−64·5	61·3	0·58	+0·481	V	V
111	1962	4	5	E	37759·324	−25·8	−28·2	15·1	13·2	−7·3	24·4	55·7	60·6	0·57	+0·477	V(F)	V
277	1984	12	23	E	46056·991	15·6	35·6	15·1	13·3	7·2	27·9	58·5	57·5	0·55	+0·472	V	V
29	1865	7	24	E	2440·271	38·0	23·7	23·7	9·4	21·8	47·5	46·3	55·4	1·3	+0·467	V	V
101	1922	2	28	E	23112·283	−33·9	18·5	21·2	10·8	−18·2	47·0	48·7	54·1	1·0	+0·466	V	V
212	1934	5	14	E	27571·021	50·0	36·2	14·9	13·4	6·5	29·4	99·6	55·2	0·51	+0·463	V	V
291	1996	1	19	M	50103·036	32·4	−111·0	14·5	13·3	−5·8	−22·9	−64·4	61·3	0·53	+0·463	V(V)	V
112	1965	9	24	M	39028·638	51·1	5·3	13·7	13·6	−1·6	−22·5	−84·0	60·2	0·47	+0·461	V	V
125	1976	12	21	E	43133·589	37·6	−122·5	14·1	13·4	4·4	23·3	72·0	59·8	0·49	+0·457	V	V
165	1980	7	14	E	44432·782	41·4	−70·7	20·9	10·7	18·0	41·9	58·5	55·9	1·0	+0·453	V	V
1	1859	7	1	E	226·113	38·0	23·7	16·4	12·3	10·9	27·6	67·1	61·1	0·68	+0·442	V	V
121	1973	3	5	E	41746·505	40·0	−85·0	13·6	13·5	−1·0	24·0	67·4	58·3	0·44	+0·436	V(F)	V
13	1862	1	1	E	1141·080	37·9	22·9	14·6	13·1	6·4	25·9	70·9	59·3	0·52	+0·436	V	V
219	1983	7	9	M	45526·014	37·2	−84·1	15·4	12·6	−8·9	−26·4	−68·5	60·5	0·59	+0·426	V	V
79	1899	3	12	E	14725·329	52·5	13·3	13·3	13·2	1·6	21·8	83·3	59·5	0·44	+0·402	V	V
214	1934	5	14	E	27571·021	55·6	33·9	15·2	12·7	8·4	30·1	117·4	55·2	0·53	+0·399	V	V
109	1954	3	5	E	34806·633	44·5	−88·0	13·2	13·2	0·2	21·1	70·4	61·0	0·44	+0·396	V	V
118	1971	4	25	E	41066·669	39·5	−88·2	13·2	13·2	0·3	21·1	71·0	60·1	0·43	+0·396	V(F)	V
262	1984	4	2	E	45792·007	15·6	35·6	13·4	13·3	1·9	28·1	52·5	54·7	0·41	+0·390	I	V
6	1861	3	12	E	846·067	38·0	23·7	13·4	13·3	−1·6	27·4	64·3	54·0	0·40	+0·386	V	V
90	1919	4	1	E	22049·378	53·9	−1·6	13·5	12·9	4·1	22·2	86·2	61·1	0·46	+0·380	V	V
114	1970	4	6	E	40682·674	48·0	−122·0	13·3	13·0	2·4	23·2	77·8	58·7	0·43	+0·378	V	V
85	1911	8	25	E	19272·677	49·9	2·3	21·5	9·3	19·4	39·0	53·4	58·2	1·1	+0·366	V	V
269	1984	8	27	E	45939·310	15·6	35·6	13·3	12·7	3·8	20·8	50·1	61·2	0·45	+0·354	I	V
208	1933	5	25	E	27216·921	55·6	33·9	15·7	12·1	10·0	32·9	121·0	53·9	0·55	+0·354	V	V
192	1987	4	28	E	46913·566	37·0	−122·0	13·1	13·0	1·6	25·8	67·0	55·7	0·39	+0·353	V	V
146	1978	1	9	E	43517·667	29·7	−98·1	12·8	12·8	0·0	20·3	60·1	61·0	0·42	+0·352	V	V
97	1921	8	4	E	22905·346	−33·9	18·5	12·8	12·8	0·4	20·3	60·9	61·3	0·42	+0·351	V	V
67	1878	7	1	E	7166·022	38·0	23·7	16·1	11·6	11·1	29·8	63·1	58·8	0·63	+0·350	V	V
247	1990	4	25	E	48006·686	41·6	−73·7	12·8	12·8	0·7	19·8	71·5	61·3	0·42	+0·350	I(V)	V
22	1864	8	4	E	2086·106	38·0	23·7	23·8	8·3	22·3	51·3	39·7	54·0	1·3	+0·347	I	V
124	1976	12	21	E	43133·589	29·9	−81·3	12·8	12·7	1·7	20·8	60·5	59·9	0·41	+0·330	V	V
167	1985	1	21	E	46086·603	19·0	−155·0	13·8	12·4	6·1	26·0	53·3	56·0	0·44	+0·320	V	V
80	1900	5	29	E	15168·119	38·7	−0·7	15·3	11·7	10·0	28·9	63·8	57·7	0·56	+0·316	V	V
145	1978	1	9	E	43517·667	30·0	−90·2	12·6	12·6	0·0	19·7	58·8	61·0	0·40	+0·312	V	V
144	1978	1	9	E	43517·667	30·0	−90·2	12·6	12·6	0·0	19·7	58·8	61·0	0·40	+0·312	V(F)	V
221	1983	12	5	E	45673·018	37·2	−84·1	17·0	11·0	12·9	34·3	60·0	55·9	0·66	+0·310	I(V)	V
143	1978	1	9	E	43517·667	30·9	−90·2	12·5	12·5	0·2	19·7	59·4	61·0	0·40	+0·310	V(F)	V
191	1987	4	28	E	46913·566	40·7	−111·9	12·8	12·6	2·4	25·3	69·0	55·8	0·38	+0·305	V	V
215	1934	6	13	E	27600·591	55·6	33·9	19·0	10·1	16·1	41·0	97·1	54·2	0·81	+0·299	V	V
38	1871	2	20	E	4478·076	38·0	23·7	14·5	11·8	8·5	26·7	57·9	56·8	0·49	+0·289	V	V
139	1978	1	9	E	43517·667	27·7	−82·7	12·3	12·3	−0·7	19·3	56·1	61·0	0·38	+0·278	V(F)	V
255	1983	12	5	E	45673·018	15·6	35·6	13·4	12·1	5·8	27·0	51·8	56·1	0·42	+0·276	I	V
16	1862	7	28	E	1348·378	38·0	23·7	22·3	8·2	20·8	44·9	39·0	56·2	1·1	+0·275	I	V
115	1970	6	4	E	40741·598	26·3	−98·2	12·4	12·4	0·5	23·4	59·6	55·6	0·36	+0·273	V	V
189	1987	4	28	E	46913·566	30·6	−104·0	12·4	12·4	−0·1	24·4	58·3	55·8	0·35	+0·272	V(F)	V
190	1987	4	28	E	46913·566	30·6	−104·0	12·4	12·4	−0·1	24·4	58·3	55·8	0·35	+0·272	V	V
129	1977	12	11	E	43488·233	47·8	20·0	13·9	11·6	7·7	21·8	76·2	61·4	0·49	+0·272	V	V
137	1978	1	9	E	43517·667	33·9	−84·3	12·3	12·2	0·8	19·2	60·2	61·0	0·38	+0·271	V(F)	V
56	1875	6	4	E	6043·432	51·5	−2·6	14·2	11·5	8·4	22·7	96·3	60·8	0·51	+0·270	V	V
116	1970	6	4	E	40741·598	28·0	−97·0	12·4	12·4	0·9	23·4	60·6	55·6	0·35	+0·270	V	V
243	1989	6	4	E	47681·330	50·8	−1·0	14·5	11·5	8·8	25·0	92·6	58·8	0·51	+0·270	V	V
130	1978	1	9	E	43517·667	41·6	−93·6	12·4	12·2	2·7	19·5	67·8	61·0	0·39	+0·269	V	V

Table 4: The 295 observations listed in order of decreasing q *continued*

No.	Date				JD − 2 400 000	Lat	Long	*ARCL*	*ARCV*	*DAZ*	*Age*	*Lag*	π	W'	q	Visibility BES	BDY
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
						(A)	$q > +0{\cdot}216$		166 entries		*continued*						
	year	m	day			°	°	°	°	°	h	m	′	′			
141	1978	1	9	E	43517·667	29·9	−81·3	12·2	12·2	−0·2	19·1	57·1	61·0	0·38	+0·268	V	V
140	1978	1	9	E	43517·667	29·9	−81·3	12·2	12·2	−0·2	19·1	57·1	61·0	0·38	+0·268	V	V
123	1976	12	21	E	43133·589	42·0	−91·6	12·9	12·0	4·7	21·0	69·2	59·9	0·41	+0·267	V(F)	V
94	1921	2	8	E	22728·526	34·2	−118·0	12·4	12·3	−1·0	25·3	57·9	54·5	0·35	+0·262	V(F)	V
223	1984	3	3	E	45762·272	37·2	−84·1	14·2	11·7	8·0	29·4	55·9	54·0	0·45	+0·258	I(V)	V
135	1978	1	9	E	43517·667	34·0	−81·1	12·1	12·1	0·7	19·0	59·7	61·0	0·37	+0·255	V(F)	V
136	1978	1	9	E	43517·667	34·0	−81·1	12·1	12·1	0·7	19·0	59·7	61·0	0·37	+0·255	V(F)	V
265	1984	5	31	E	45851·200	15·6	35·6	12·2	12·1	0·9	23·7	52·2	57·5	0·35	+0·246	I	V
188	1987	4	28	E	46913·566	30·3	−97·7	12·2	12·2	−0·2	23·9	57·0	55·8	0·34	+0·244	V	V
134	1978	1	9	E	43517·667	36·0	−79·8	12·1	12·0	1·1	18·8	60·8	61·0	0·37	+0·240	V(F)	V
142	1978	1	9	E	43517·667	43·0	−89·8	12·3	11·9	2·9	19·2	68·3	61·0	0·38	+0·240	V(F)	V
132	1978	1	9	E	43517·667	36·0	−79·8	12·1	12·0	1·1	18·8	60·8	61·0	0·37	+0·240	V(F)	V
133	1978	1	9	E	43517·667	36·0	−79·8	12·1	12·0	1·1	18·8	60·8	61·0	0·37	+0·240	V	V
59	1876	6	22	E	6427·429	38·0	23·7	12·9	11·7	5·4	20·1	66·2	61·3	0·42	+0·239	I	V
138	1978	1	9	E	43517·667	41·9	−87·6	12·2	11·9	2·6	19·1	66·9	61·0	0·38	+0·239	V	V
92	1921	2	8	E	22728·526	35·2	−111·7	12·2	12·2	−0·9	24·8	57·8	54·5	0·34	+0·238	V	V
187	1987	4	28	E	46913·566	41·6	−93·7	12·2	12·0	2·4	24·1	66·6	55·8	0·35	+0·228	I	V
93	1921	2	8	E	22728·526	32·2	−110·0	12·2	12·1	−1·5	24·8	55·2	54·5	0·33	+0·228	V	V
81	1901	4	19	E	15493·401	50·7	−2·8	13·1	11·5	6·4	22·1	73·7	61·3	0·44	+0·227	V	V
126	1976	12	21	E	43133·589	43·8	−87·7	12·7	11·7	5·0	20·7	69·8	59·9	0·40	+0·227	V	V
113	1970	4	6	E	40682·674	44·5	−88·0	12·0	11·9	1·0	20·7	65·6	58·8	0·35	+0·222	V(F)	V
						(B)	$+0{\cdot}216 \geq q > -0{\cdot}014$			68 entries							
131	1978	1	9	E	43517·667	38·9	−76·9	11·9	11·8	1·7	18·5	62·4	61·0	0·36	+0·211	V(B)	V(V)
233	1988	6	13	M	47326·885	37·2	−84·1	12·8	11·6	−5·4	−23·5	−64·5	56·8	0·39	+0·211	I(V)	V(V)
40	1871	5	20	E	4566·948	38·0	23·7	14·2	11·2	8·7	31·2	59·7	54·0	0·45	+0·211	I	V(V)
186	1987	4	28	E	46913·566	30·0	−90·1	11·9	11·9	−0·4	23·4	55·5	55·8	0·33	+0·210	V	V(V)
185	1987	4	28	E	46913·566	40·8	−87·7	12·0	11·8	2·1	23·6	64·6	55·8	0·33	+0·205	I	V(V)
184	1987	4	28	E	46913·566	33·0	−87·4	11·9	11·9	0·3	23·3	57·4	55·8	0·33	+0·204	I	V(V)
164	1979	1	28	E	43901·764	37·8	−122·4	11·9	11·7	2·3	19·6	58·8	61·4	0·36	+0·201	V(F)	V(V)
155	1978	3	9	E	43576·609	50·3	−119·3	12·7	11·5	5·4	23·8	67·6	58·1	0·39	+0·198	V	V(V)
203	1908	5	1	E	18062·149	44·1	3·1	14·7	10·7	10·2	27·7	60·1	57·1	0·51	+0·193	V	V(V)
181	1987	4	28	E	46913·566	33·7	−84·4	11·8	11·8	0·4	23·1	57·5	55·8	0·32	+0·192	I	V(V)
100	1922	1	29	E	23082·492	−33·9	18·5	19·5	8·8	−17·4	42·4	40·5	54·0	0·85	+0·191	I	V(V)
77	1881	3	30	E	8169·440	51·5	−2·6	11·8	11·7	1·6	20·7	73·1	57·8	0·33	+0·189	V	V(V)
183	1987	4	28	E	46913·566	42·7	−84·5	12·0	11·7	2·5	23·5	66·1	55·8	0·33	+0·188	V	V(V)
182	1987	4	28	E	46913·566	42·7	−84·5	12·0	11·7	2·5	23·5	66·1	55·8	0·33	+0·188	V	V(V)
177	1987	4	28	E	46913·566	36·2	−81·7	11·8	11·7	1·0	23·0	59·1	55·8	0·32	+0·183	V(F)	V(V)
178	1987	4	28	E	46913·566	36·2	−81·7	11·8	11·7	1·0	23·0	59·1	55·8	0·32	+0·183	V	V(V)
176	1987	4	28	E	46913·566	36·2	−81·7	11·8	11·7	1·0	23·0	59·1	55·8	0·32	+0·183	I	V(V)
105	1922	5	27	E	23201·253	−33·9	18·5	12·3	11·4	−4·5	22·1	56·4	57·3	0·36	+0·179	V	V(V)
107	1942	12	8	E	30701·584	40·7	−74·0	12·6	11·1	5·8	19·9	61·9	61·2	0·40	+0·172	V(F)	V(V)
180	1987	4	28	E	46913·566	28·0	−82·5	11·7	11·6	−0·8	22·8	52·8	55·8	0·31	+0·171	I	V(V)
179	1987	4	28	E	46913·566	26·7	−81·8	11·6	11·6	−1·1	22·8	51·8	55·8	0·31	+0·164	V(F)	V(V)
173	1987	4	28	E	46913·566	38·9	−77·1	11·7	11·6	1·5	22·8	60·8	55·8	0·31	+0·163	I	V(V)
174	1987	4	28	E	46913·566	38·9	−77·1	11·7	11·6	1·5	22·8	60·8	55·8	0·31	+0·163	V(F)	V(V)
175	1987	4	28	E	46913·566	38·9	−77·1	11·7	11·6	1·5	22·8	60·8	55·8	0·31	+0·163	V(F)	V(V)
170	1987	4	28	E	46913·566	38·9	−77·0	11·6	11·5	1·5	22·8	60·8	55·8	0·31	+0·163	V(F)	V(V)
171	1987	4	28	E	46913·566	38·9	−77·1	11·7	11·6	1·5	22·8	60·8	55·8	0·31	+0·163	I	V(V)
172	1987	4	28	E	46913·566	38·9	−77·1	11·7	11·6	1·5	22·8	60·8	55·8	0·31	+0·163	I(B)	V(V)
209	1933	8	20	M	27305·741	48·6	7·7	12·6	11·2	−5·6	−25·8	−70·0	55·5	0·36	+0·161	V	V(V)
28	1865	6	24	E	2410·832	38·0	23·7	18·6	8·7	16·4	34·2	44·8	57·1	0·81	+0·161	I	V(V)
99	1921	12	30	E	23052·735	−33·9	18·5	18·0	9·1	−15·5	36·7	44·1	54·7	0·73	+0·155	I	V(V)
204	1911	1	31	E	19066·906	51·0	−0·9	16·4	9·7	13·3	31·6	65·2	56·6	0·63	+0·154	V	V(V)
152	1978	3	9	E	43576·609	41·6	−93·6	11·8	11·3	3·3	22·1	56·9	58·1	0·33	+0·153	V	V(V)
153	1978	3	9	E	43576·609	41·6	−93·6	11·8	11·3	3·3	22·1	56·9	58·1	0·33	+0·153	V	V(V)
151	1978	3	9	E	43576·609	40·5	−89·0	11·6	11·2	3·0	21·7	55·4	58·2	0·33	+0·139	V	V(V)
154	1978	3	9	E	43576·609	40·5	−89·0	11·6	11·2	3·0	21·7	55·4	58·2	0·33	+0·139	V	V(V)

Table 4: The 295 observations listed in order of decreasing q *continued*

No.	Date				JD − 2 400 000	Lat	Long	*ARCL*	*ARCV*	*DAZ*	*Age*	*Lag*	π	W'	q	Visibility BES	BDY
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
					(B) $+0{\cdot}216 \geq q > -0{\cdot}014$ 68 entries *continued*												
	year	m	day			°	°	°	°	°	h	m	′	′			
202	1885	12	7	E	9882·053	50·6	5·7	13·5	10·7	8·3	26·8	75·3	54·6	0·41	+0·136	V	V(V)
205	1920	4	19	E	22433·405	43·5	7·0	11·9	10·9	4·8	21·0	59·5	60·2	0·36	+0·126	V	V(V)
163	1979	1	28	E	43901·764	47·6	−122·3	11·7	10·9	4·2	19·2	65·5	61·4	0·35	+0·116	V(F)	V(V)
2	1859	10	27	E	343·523	38·0	23·7	21·4	6·8	20·3	39·2	33·3	57·5	1·1	+0·109	V	V(V)
87	1915	3	16	E	20572·321	49·4	8·7	11·2	11·0	1·7	22·3	63·9	55·2	0·29	+0·094	V	V(V)
31	1866	1	17	E	2618·359	38·0	23·7	11·0	11·0	0·5	19·3	56·3	57·6	0·29	+0·093	I	V(V)
91	1921	2	8	E	22728·526	42·3	−71·1	11·0	11·0	0·1	21·9	57·6	54·5	0·27	+0·081	V	V(V)
78	1889	11	22	M	11329·573	50·9	4·2	11·5	10·6	−4·3	−19·1	−71·3	60·0	0·33	+0·079	V	V(V)
290	1995	6	28	E	49896·534	−30·1	−71·0	10·9	10·9	−0·2	21·5	51·5	54·2	0·27	+0·075	V	V(V)
33	1867	2	5	E	3002·261	38·0	23·7	11·0	10·8	1·6	22·0	53·6	56·1	0·28	+0·071	I	V(V)
128	1977	11	10	M	43458·800	30·7	−104·0	10·7	10·7	−0·7	−18·3	−48·4	60·4	0·29	+0·062	V(F)	V(V)
156	1978	3	9	E	43576·609	41·3	−72·9	11·0	10·7	2·9	20·6	52·9	58·2	0·29	+0·062	V(F)	V(V)
149	1978	3	9	E	43576·609	41·3	−72·9	11·0	10·7	2·9	20·6	52·9	58·2	0·29	+0·062	V	V(V)
150	1978	3	9	E	43576·609	42·7	−73·8	11·1	10·6	3·2	20·7	53·9	58·2	0·30	+0·058	V	V(V)
148	1978	3	9	E	43576·609	42·7	−73·8	11·1	10·6	3·2	20·7	53·9	58·2	0·30	+0·058	V(F)	V(V)
127	1977	2	18	E	43192·651	42·7	−83·6	10·7	10·7	0·4	20·0	55·5	56·7	0·27	+0·057	V(F)	V(V)
201	1987	6	26	E	46972·733	37·0	−122·0	11·3	10·6	4·0	22·4	58·8	54·0	0·29	+0·055	V	V(V)
200	1987	6	26	E	46972·733	33·5	−112·1	11·0	10·6	2·9	21·5	55·3	54·0	0·27	+0·043	V	V(V)
161	1979	1	28	E	43901·764	38·7	−90·3	10·6	10·5	1·9	17·3	52·9	61·4	0·29	+0·038	V(B)	V(V)
159	1979	1	28	E	43901·764	29·7	−82·4	10·5	10·5	0·1	17·1	46·9	61·4	0·28	+0·034	V(B)	V(V)
162	1979	1	28	E	43901·764	42·0	−93·6	10·7	10·4	2·6	17·5	55·5	61·4	0·29	+0·033	V	V(V)
158	1979	1	28	E	43901·764	29·9	−81·3	10·4	10·4	0·1	17·0	46·8	61·4	0·28	+0·028	V	V(V)
160	1979	1	28	E	43901·764	42·0	−91·7	10·6	10·3	2·5	17·3	55·2	61·4	0·29	+0·024	V(B)	V(V)
197	1987	6	26	E	46972·733	30·0	−100·0	10·6	10·4	1·8	20·5	51·6	54·0	0·25	+0·014	I	V(V)
106	1931	8	13	M	26567·352	48·6	7·7	10·5	10·3	−2·0	−16·6	−66·4	60·1	0·27	+0·010	V(T)	V(V)
230	1988	1	19	E	47179·727	32·2	−111·0	12·2	9·7	7·4	19·6	46·3	61·3	0·38	+0·010	I(V)	V(V)
260	1984	3	3	E	45762·272	15·6	35·6	10·9	10·3	3·6	21·5	39·9	54·0	0·27	+0·007	I	V(V)
199	1987	6	26	E	46972·733	40·7	−111·9	11·1	10·2	4·6	21·9	60·3	54·0	0·28	+0·003	V(B)	V(V)
147	1978	3	9	E	43576·609	45·1	−64·2	10·7	10·2	3·5	20·0	53·5	58·2	0·28	+0·002	V	V(V)
86	1913	11	28	E	20099·570	−33·9	18·5	10·3	10·3	0·0	16·4	53·1	58·7	0·26	−0·001	V	V(V)
198	1987	6	26	E	46972·733	39·8	−105·0	10·9	10·1	4·2	21·4	58·8	54·0	0·27	−0·010	I	V(V)
251	1990	5	24	E	48035·993	34·2	−118·1	10·1	10·1	0·7	15·4	51·9	61·2	0·26	−0·014	I(V)	V(V)
252	1990	5	24	E	48035·993	34·2	−118·1	10·1	10·1	0·7	15·4	51·9	61·2	0·26	−0·014	V(V)	V(V)
					(C) $-0{\cdot}014 \geq q > -0{\cdot}160$ 26 entries												
36	1869	5	12	E	3829·172	38·0	23·7	13·5	9·1	10·0	25·6	45·8	56·4	0·42	−0·018	I	V(F)
220	1983	11	5	E	45643·432	37·2	−84·1	13·1	8·9	9·6	24·5	43·6	57·8	0·41	−0·041	I(V)	V(F)
246	1990	4	24	M	48006·686	41·6	−73·7	12·1	9·1	−7·9	−18·8	−46·0	61·1	0·37	−0·048	I(V)	V(F)
10	1861	10	5	E	1052·791	38·0	23·7	20·2	5·5	19·4	33·2	24·4	61·1	1·0	−0·048	I	V(F)
46	1872	7	6	E	4980·267	38·0	23·7	11·4	9·6	6·1	23·8	51·9	53·9	0·29	−0·049	I	V(F)
238	1989	5	5	E	47651·993	39·7	−105·5	9·8	9·8	−0·6	14·6	53·0	60·4	0·24	−0·053	I(V)	V(F)
228	1987	6	26	E	46972·733	37·2	−84·1	10·3	9·8	3·1	19·8	54·3	54·0	0·24	−0·054	I(V)	V(F)
250	1990	5	24	E	48035·993	31·6	−110·5	9·8	9·8	0·0	14·8	48·2	61·2	0·24	−0·054	I(V)	V(F)
49	1872	10	3	E	5069·146	38·0	23·7	12·8	9·0	9·1	24·9	41·5	56·5	0·38	−0·056	I	V(F)
196	1987	6	26	E	46972·733	42·7	−84·5	10·5	9·5	4·4	20·2	58·8	54·0	0·24	−0·083	V(B)	V(F)
157	1979	1	27	M	43901·764	35·2	−111·7	10·4	9·2	−4·8	−16·2	−44·2	61·4	0·27	−0·093	V(B)	V(F)
52	1873	4	27	E	5275·447	38·0	23·7	10·2	9·2	4·4	18·8	45·7	58·5	0·25	−0·106	I	V(F)
120	1972	3	15	E	41391·983	35·5	−117·6	9·6	9·3	−2·6	14·7	42·0	60·8	0·23	−0·110	I(B)	V(F)
119	1972	3	15	E	41391·983	35·5	−117·6	9·6	9·3	−2·6	14·7	42·0	60·8	0·23	−0·110	V(B)	V(F)
225	1984	11	23	E	46027·457	34·0	−84·0	13·3	8·0	10·7	23·8	38·9	59·3	0·44	−0·123	I(V)	V(F)
237	1989	5	5	E	47651·993	43·0	−85·7	9·2	9·2	−0·3	13·4	52·8	60·4	0·21	−0·128	I(V)	V(F)
239	1989	5	5	E	47651·993	42·7	−84·8	9·2	9·2	−0·4	13·3	52·2	60·4	0·21	−0·133	I(V)	V(F)
240	1989	5	5	E	47651·993	42·7	−84·8	9·2	9·2	−0·4	13·3	52·2	60·4	0·21	−0·133	I(I)	V(F)
96	1921	2	8	E	22728·526	38·8	−9·1	9·3	9·2	−1·3	17·8	44·9	54·4	0·20	−0·141	I	V(F)
234	1988	6	14	E	47326·885	37·2	−84·1	9·3	9·2	1·6	16·1	50·6	55·9	0·20	−0·141	I(I)	V(F)
241	1989	5	5	E	47651·993	30·3	−97·0	9·4	9·0	−2·5	13·6	41·7	60·4	0·22	−0·146	I(V)	V(F)
224	1984	5	1	E	45821·657	37·2	−84·1	10·2	8·8	5·1	21·0	43·3	55·8	0·24	−0·153	I(I)	V(F)
289	1995	1	1	E	49718·956	33·0	−106·0	9·1	9·0	−0·3	13·5	43·2	60·2	0·20	−0·153	I(V)	V(F)
95	1921	2	8	E	22728·526	36·5	−6·2	9·2	9·1	−1·7	17·7	42·8	54·4	0·19	−0·155	I	V(F)
122	1973	7	1	E	41863·987	−44·0	170·5	10·6	8·5	−6·3	17·9	50·5	61·1	0·28	−0·156	I(V)	V(F)
102	1922	3	29	E	23142·043	−33·9	18·5	12·9	8·0	−10·2	28·0	34·7	54·7	0·38	−0·157	I	V(F)

No.	Date				JD − 2 400 000	Lat	Long	*ARCL*	*ARCV*	*DAZ*	*Age*	*Lag*	π	W'	q	Visibility BES	BDY
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	(D)					$-0{\cdot}160 \geq q > -0{\cdot}232$			14 entries								
	year	m	day			°	°	°	°	°	h	m	′	′			
44	1871	9	14	M	4685·298	38·0	23·7	9·3	8·9	2·6	−15·4	−42·2	57·1	0·21	−0·163	V	I(V)
281	1990	5	24	E	48035·993	35·6	−83·5	8·9	8·9	0·2	13·2	46·2	61·2	0·20	−0·164	I(V)	I(V)
39	1871	4	20	E	4537·295	38·0	23·7	11·1	8·4	7·2	22·3	40·1	54·4	0·28	−0·175	I	I(V)
253	1983	11	5	E	45643·432	15·6	35·6	9·3	8·8	3·0	17·0	34·4	58·0	0·21	−0·178	I	I(V)
294	1996	1	20	E	50103·036	34·1	−118·3	8·9	8·8	−1·6	12·6	41·0	61·2	0·20	−0·184	I(V)	I(V)
295	1996	1	20	E	50103·036	34·1	−118·3	8·9	8·8	−1·6	12·6	41·0	61·2	0·20	−0·184	I(I)	I(V)
15	1862	4	29	E	1259·477	38·0	23·7	8·9	8·9	0·2	18·1	44·6	54·1	0·18	−0·189	I	I(V)
166	1981	7	30	M	44816·662	42·3	−71·3	10·1	8·3	−5·8	−18·6	−45·2	59·0	0·25	−0·205	I	I(V)
293	1996	1	20	E	50103·036	32·8	−113·2	8·7	8·6	−1·8	12·3	39·2	61·2	0·19	−0·208	I(V)	I(V)
98	1921	10	31	E	22993·485	−33·9	18·5	9·8	8·3	−5·3	17·9	37·8	58·1	0·23	−0·213	I	I(V)
292	1996	1	20	E	50103·036	32·4	−111·0	8·7	8·5	−1·9	12·2	38·6	61·2	0·19	−0·219	I(V)	I(V)
280	1990	2	25	E	47947·872	35·6	−83·5	8·5	8·5	−0·6	14·8	38·3	59·4	0·18	−0·222	I(I)	I(V)
278	1990	2	25	E	47947·872	35·6	−83·5	8·5	8·5	−0·6	14·8	38·3	59·4	0·18	−0·222	V(V)	I(V)
279	1990	2	25	E	47947·872	35·6	−83·5	8·5	8·5	−0·6	14·8	38·3	59·4	0·18	−0·222	I(V)	I(V)
	(E)					$-0{\cdot}232 \geq q > -0{\cdot}293$			4 entries								
271	1984	9	25	E	45968·634	15·6	35·6	8·4	8·2	1·7	12·5	30·5	61·4	0·18	−0·248	I	I(I)
20	1864	5	6	E	1997·510	39·6	26·2	9·1	7·8	4·7	17·3	39·5	57·6	0·20	−0·283	I	I(I)
288	1992	4	2	M	48715·710	19·8	−155·5	8·1	8·1	−0·7	−13·0	−30·9	55·7	0·15	−0·284	I(I)	I(I)
226	1985	4	20	E	46175·724	37·2	−84·1	8·7	7·9	3·6	19·2	37·5	54·0	0·17	−0·287	I(I)	I(I)
	(F)					$-0{\cdot}293 \geq q$			17 entries								
275	1984	11	23	E	46027·457	15·6	35·6	9·2	7·6	5·2	16·3	30·4	59·5	0·21	−0·296	I	I
7	1861	8	7	E	994·037	38·0	23·7	16·0	5·0	15·2	28·7	21·4	59·0	0·63	−0·316	I	I
231	1988	4	16	E	47268·001	37·2	−84·1	7·7	7·6	−1·2	12·4	35·9	59·2	0·15	−0·330	I(I)	I
258	1984	2	2	E	45732·491	15·6	35·6	8·6	7·4	4·5	16·0	29·0	54·1	0·17	−0·340	I	I
169	1986	12	31	E	46795·633	39·0	−77·0	12·4	6·0	10·8	19·0	31·7	61·3	0·39	−0·348	I(B)	I
232	1988	5	15	M	47297·425	37·2	−84·1	7·7	7·1	−2·8	−12·0	−34·8	58·2	0·14	−0·383	I(I)	I
54	1873	12	20	E	5512·285	38·0	23·7	11·6	5·4	10·3	20·5	27·4	58·3	0·33	−0·447	I	I
3	1860	1	23	E	432·512	38·0	23·7	7·1	6·3	3·2	15·6	30·6	54·2	0·11	−0·482	I	I
242	1989	6	3	E	47681·330	19·8	−155·5	7·0	6·3	−3·1	9·2	25·8	59·2	0·12	−0·484	I(I)	I
222	1984	1	3	E	45702·719	37·2	−84·1	8·7	5·1	7·1	17·4	24·9	54·9	0·17	−0·565	I(I)	I
227	1987	4	27	M	46913·566	37·2	−84·1	7·6	5·3	−5·5	−15·0	−22·8	56·6	0·14	−0·573	I(I)	I
41	1871	6	18	E	4596·604	38·0	23·7	7·1	5·4	4·5	15·5	26·9	54·1	0·11	−0·574	I	I
103	1922	4	27	E	23171·710	−33·9	18·5	6·1	5·5	−2·6	11·3	23·4	55·6	0·09	−0·576	I	I
229	1987	9	23	E	47061·630	37·2	−84·1	9·9	4·2	9·0	20·5	16·7	55·9	0·23	−0·620	I(I)	I
194	1987	6	25	M	46972·733	−30·1	−71·0	9·6	4·2	8·6	−18·1	−18·1	54·3	0·21	−0·632	I(B)	I
195	1987	6	26	E	46972·733	−30·1	−71·0	9·0	4·0	−8·0	16·4	16·8	54·0	0·18	−0·671	I(B)	I
256	1984	1	3	E	45702·719	15·6	35·6	5·5	4·2	3·6	10·2	15·3	55·0	0·07	−0·720	I	I

Table 4: The 295 observations listed in order of decreasing q *continued*

6. Derivation of the q-test

The visibility test parameter q is based on the Indian method, which is defined discretely by Table 2 and continuously by inequality (3.6). The parameter q is calculated at the best time from the equation

$$q = (ARCV - (11{\cdot}8371 - 6{\cdot}3226\, W' + 0{\cdot}7319\, W'^2 - 0{\cdot}1018\, W'^3))/10 \qquad (6{\cdot}1)$$

where W' is the topocentric width of the crescent, and q has been scaled by a factor of 10 to confine it roughly to the range -1 to $+1$. (Note the use of the topocentric width of the crescent.) The values of q have been calculated for the 295 observations referred to in section 5, and the results are listed in Table 4 in order of decreasing q. Table 4 has also been partitioned into six ranges of q. These ranges in the q-test have been calibrated empirically by comparing the visibility code Schaefer used for the 295 observational records, with a similar code derived from the calculated value of q. It has also been found necessary to use theoretical arguments to obtain some of the limiting values for q. Table 5 lists the six criteria by type, A to F, by range in q and by visibility code.

Table 5: The q-test criteria			
Criterion	Range	Remarks	Visibility Code
(A)	$q > +0{\cdot}216$	Easily visible, $ARCL \geq 12^\circ$	V
(B)	$+0{\cdot}216 \geq q > -0{\cdot}014$	Visible under perfect conditions	V(V)
(C)	$-0{\cdot}014 \geq q > -0{\cdot}160$	May need optical aid to find crescent	V(F)
(D)	$-0{\cdot}160 \geq q > -0{\cdot}232$	Will need optical aid to find crescent	I(V)
(E)	$-0{\cdot}232 \geq q > -0{\cdot}293$	Not visible with a telescope, $ARCL \leq 8{\cdot}5^\circ$	I(I)
(F)	$-0{\cdot}293 \geq q$	Not visible, below Danjon limit, $ARCL \leq 8^\circ$	I

The limiting values of q were chosen for the six criteria A to F for the following reasons:

(A) A lower limit is required to separate observations that are trivial from those that have some element of difficulty. After some experimentation, it was found that the ideal situation $ARCL = 12^\circ$ and $DAZ = 0^\circ$ produces a sensible cut-off point, for which $q = +0{\cdot}216$. To avoid ambiguities, the constant geocentric quantity W defined by equation (3.4), that was adopted by Bruin for the crescent width, was used to calculate q from equation (6.1) instead of W'.

There are 166 examples in Table 4 when q exceeds this value, and in general it should be very easy to see the Moon in these cases, provided there is no obscuring cloud in the sky.

(B) From observers reports it has been found that, in general, $q = 0$ is close to the lower limit for first visibility under perfect atmospheric conditions at sea level, without requiring optical aid. Table 4 is used to set this lower limit for visibility more precisely. From inspection of Table 4, the significance of $q = 0$ can be seen, but $q = -0{\cdot}014$ is another possible cut-off value. There are 68 cases in Table 4 with q in this range.

(C) Table 4 was used to find the cut-off point when optical aid is always needed to find the crescent moon by matching the q-test visibility code with Schaefer's code. The rounded value of $q = -0{\cdot}160$ was chosen for the cut-off criterion. Entry number 44, the first entry in the next group, was ignored because it is false. In Table 4 there are 26 cases that satisfy this criterion.

(D) In this case Table 4 has too few entries from which to estimate a lower limit for q. The situation is made worse by the fact that where there is an entry, in most cases, the Moon was not seen even with optical aid. In fact it is rare for the crescent to be observed below an apparent elongation of about $7^\circ\!.5$, see Fatoohi *et al.* (1998). Table 4 has 17 cases. This is the current limit below which it is not possible to see the thin crescent moon with a telescope.

Allowing 1° for horizontal parallax of the Moon, and ignoring the effect of refraction, for an apparent elongation of $7^\circ\!.5$, $ARCL = 8^\circ\!.5$. If $DAZ = 0^\circ$ this corresponds to a lower limit of $q = -0{\cdot}232$. Without good finding telescopes and positional information, observers are unlikely to see the crescent below this limit. The only sighting that was seen both optically and visually near this limit was No. 278 for which $q = -0{\cdot}222$. It is an important observation because it is the observation with the smallest elongation, see Table 6. If the Moon were observed near this elongation and it was also at or near perigee, Age would be about 12 hours.

(E) There is a theoretical cut-off point when the apparent elongation of the Moon from the Sun is 7°, known as the Danjon limit. This limit is obtained by extrapolating observations made at larger elongations. Allowing 1° for horizontal parallax of the Moon, and ignoring the effect of refraction, an apparent elongation of 7° is equivalent to $ARCL = 8^\circ$. With $ARCL = 8^\circ$ and $DAZ = 0^\circ$ the corresponding lower limit on q is $-0{\cdot}293$.

(F) In Table 4 there are only 17 cases in this range of q, but three of them (169, 194 and 195) contradict the q-test, in particular, 194 and 195 are anomalous observations. The main reason for the discrepancy must be due to the extremely clear atmosphere experienced on high altitude mountain sites. The elongations, however, are well above the Danjon limit, and since $ARCV$ is about 4°, the observations were probably made using daylight vision. These observations show that the curve $q = 0$ needs modifying for high altitude observations. No 169 was made at $ARCL$ = 12°4, and width W' = 0′39, which are both large. Since $ARCV$ = 6°, it should have been possible to make the observation. This observation shows also that the curve $q = 0$ needs modifying when the atmospheric conditions are so perfect.

Earliest sightings are mentioned in the literature, by Schaefer *et al.* (1993b). For example, No. 252 is type B and Nos. 239, 237 and 241 are type C. None of these observations were made at such a small elongation as No. 278 of type D, for which W' = 0′18. Table 6 lists the first fifty of the 295 observations in order of increasing crescent width, W', which is correlated with increasing elongation. Note that the fifteenth observation No. 278 is the first observation designated type V(V) according to Schaefer's coding. Observation No. 195 is thirteenth on the list, so based on the small crescent width, it was a very sharp observation indeed.

Many people assume that the place of earliest sighting of the new crescent moon, given some criterion such as Lag = 50 minutes, will be the most easterly place that the observation can be made from. This assumption is fallacious. In an ideal situation, earliest sighting occurs at a place where a first sighting parameter such as Lag or q takes a specific value and in addition DAZ = 0°. It is easy to show that in general the place on the parabolic curve on the surface of the Earth where the parameter is a constant will not be the most easterly point on the curve. At the place of earliest sighting the line of constant altitude of the Sun is tangential to the parabolic curve (because DAZ = 0°), which leaves the most easterly point in daylight, except in rare cases when the declination of the Sun is zero and the tangent will then be at the most easterly point.

The list of 295 observations in Table 4 shows bias. For example there is a concentration on early sightings, which occur when $DAZ \approx 0°$ and $ARCL \approx ARCV$. Cases when $ARCL \approx DAZ$ and $ARCV$ is small, which regularly occur at high latitudes, and the Moon is first seen in daylight, are rare. In other words the list of 295 observations concentrates on young moonage sightings at low latitudes, and ignores old moonage sightings at high latitudes.

In fact there are only nine cases for latitude > 50° where $ARCL/ARCV$ > 1·2. Three of these 82, 204 and 215, have ratios larger than 1·6, which makes them more interesting. There are no contradictory cases, but the sample is too small to be helpful for calibration purposes.

Table 6: The first fifty entries of Table 5 listed in order of increasing W'

No.	Date				JD − 2 400 000	Lat	Long	*ARCL*	*ARCV*	*DAZ*	*Age*	*Lag*	π	W'	q	Visibility BES	BDY
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	year	m	day			°	°	°	°	°	h	m	′	′			
256	1984	1	3	E	45702·719	15·6	35·6	5·5	4·2	3·6	10·2	15·3	55·0	0·07	−0·720	I	I
103	1922	4	27	E	23171·710	−33·9	18·5	6·1	5·5	−2·6	11·3	23·4	55·6	0·09	−0·576	I	I
3	1860	1	23	E	432·512	38·0	23·7	7·1	6·3	3·2	15·6	30·6	54·2	0·11	−0·482	I	I
41	1871	6	18	E	4596·604	38·0	23·7	7·1	5·4	4·5	15·5	26·9	54·1	0·11	−0·574	I	I
242	1989	6	3	E	47681·330	19·8	−155·5	7·0	6·3	−3·1	9·2	25·8	59·2	0·12	−0·484	I(I)	I
227	1987	4	27	M	46913·566	37·2	−84·1	7·6	5·3	−5·5	−15·0	−22·8	56·6	0·14	−0·573	I(I)	I
232	1988	5	15	M	47297·425	37·2	−84·1	7·7	7·1	−2·8	−12·0	−34·8	58·2	0·14	−0·383	I(I)	I
231	1988	4	16	E	47268·001	37·2	−84·1	7·7	7·6	−1·2	12·4	35·9	59·2	0·15	−0·330	I(I)	I
288	1992	4	2	M	48715·710	19·8	−155·5	8·1	8·1	−0·7	−13·0	−30·9	55·7	0·15	−0·284	I(I)	I(I)
222	1984	1	3	E	45702·719	37·2	−84·1	8·7	5·1	7·1	17·4	24·9	54·9	0·17	−0·565	I(I)	I
258	1984	2	2	E	45732·491	15·6	35·6	8·6	7·4	4·5	16·0	29·0	54·1	0·17	−0·340	I	I
226	1985	4	20	E	46175·724	37·2	−84·1	8·7	7·9	3·6	19·2	37·5	54·0	0·17	−0·287	I(I)	I(I)
195	1987	6	26	E	46972·733	−30·1	−71·0	9·0	4·0	−8·0	16·4	16·8	54·0	0·18	−0·671	I(B)	I
271	1984	9	25	E	45968·634	15·6	35·6	8·4	8·2	1·7	12·5	30·5	61·4	0·18	−0·248	I	I(I)
278	1990	2	25	E	47947·872	35·6	−83·5	8·5	8·5	−0·6	14·8	38·3	59·4	0·18	−0·222	V(V)	I(V)
279	1990	2	25	E	47947·872	35·6	−83·5	8·5	8·5	−0·6	14·8	38·3	59·4	0·18	−0·222	I(V)	I(V)
280	1990	2	25	E	47947·872	35·6	−83·5	8·5	8·5	−0·6	14·8	38·3	59·4	0·18	−0·222	I(I)	I(V)
15	1862	4	29	E	1259·477	38·0	23·7	8·9	8·9	0·2	18·1	44·6	54·1	0·18	−0·189	I	I(V)
95	1921	2	8	E	22728·526	36·5	−6·2	9·2	9·1	−1·7	17·7	42·8	54·4	0·19	−0·155	I	V(F)
292	1996	1	20	E	50103·036	32·4	−111·0	8·7	8·5	−1·9	12·2	38·6	61·2	0·19	−0·219	I(V)	I(V)
293	1996	1	20	E	50103·036	32·8	−113·2	8·7	8·6	−1·8	12·3	39·2	61·2	0·19	−0·208	I(V)	I(V)
281	1990	5	24	E	48035·993	35·6	−83·5	8·9	8·9	0·2	13·2	46·2	61·2	0·20	−0·164	I(V)	I(V)
20	1864	5	6	E	1997·510	39·6	26·2	9·1	7·8	4·7	17·3	39·5	57·6	0·20	−0·283	I	I(I)
289	1995	1	1	E	49718·956	33·0	−106·0	9·1	9·0	−0·3	13·5	43·2	60·2	0·20	−0·153	I(V)	V(F)
234	1988	6	14	E	47326·885	37·2	−84·1	9·3	9·2	1·6	16·1	50·6	55·9	0·20	−0·141	I(I)	V(F)

Table 6: The first fifty entries of Table 5 listed in order of increasing *W′ continued*

No.	Date				JD − 2 400 000	Lat	Long	*ARCL*	*ARCV*	*DAZ*	*Age*	*Lag*	π	*W′*	*q*	Visibility BES	BDY
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	year	m	day			°	°	°	°	°	h	m	′	′			
96	1921	2	8	E	22728·526	38·8	−9·1	9·3	9·2	−1·3	17·8	44·9	54·4	0·20	−0·141	I	V(F)
294	1996	1	20	E	50103·036	34·1	−118·3	8·9	8·8	−1·6	12·6	41·0	61·2	0·20	−0·184	I(V)	I(V)
295	1996	1	20	E	50103·036	34·1	−118·3	8·9	8·8	−1·6	12·6	41·0	61·2	0·20	−0·184	I(I)	I(V)
44	1871	9	14	M	4685·298	38·0	23·7	9·3	8·9	2·6	−15·4	−42·2	57·1	0·21	−0·163	V	I(V)
237	1989	5	5	E	47651·993	43·0	−85·7	9·2	9·2	−0·3	13·4	52·8	60·4	0·21	−0·128	I(V)	V(F)
239	1989	5	5	E	47651·993	42·7	−84·8	9·2	9·2	−0·4	13·3	52·2	60·4	0·21	−0·133	I(V)	V(F)
275	1984	11	23	E	46027·457	15·6	35·6	9·2	7·6	5·2	16·3	30·4	59·5	0·21	−0·296	I	I
240	1989	5	5	E	47651·993	42·7	−84·8	9·2	9·2	−0·4	13·3	52·2	60·4	0·21	−0·133	I(I)	V(F)
194	1987	6	25	M	46972·733	−30·1	−71·0	9·6	4·2	8·6	−18·1	−18·1	54·3	0·21	−0·632	I(B)	I
253	1983	11	5	E	45643·432	15·6	35·6	9·3	8·8	3·0	17·0	34·4	58·0	0·21	−0·178	I	I(V)
241	1989	5	5	E	47651·993	30·3	−97·0	9·4	9·0	−2·5	13·6	41·7	60·4	0·22	−0·146	I(V)	V(F)
229	1987	9	23	E	47061·630	37·2	−84·1	9·9	4·2	9·0	20·5	16·7	55·9	0·23	−0·620	I(I)	I
120	1972	3	15	E	41391·983	35·5	−117·6	9·6	9·3	−2·6	14·7	42·0	60·8	0·23	−0·110	I(B)	V(F)
98	1921	10	31	E	22993·485	−33·9	18·5	9·8	8·3	−5·3	17·9	37·8	58·1	0·23	−0·213	I	I(V)
119	1972	3	15	E	41391·983	35·5	−117·6	9·6	9·3	−2·6	14·7	42·0	60·8	0·23	−0·110	V(B)	V(F)
228	1987	6	26	E	46972·733	37·2	−84·1	10·3	9·8	3·1	19·8	54·3	54·0	0·24	−0·054	I(V)	V(F)
196	1987	6	26	E	46972·733	42·7	−84·5	10·5	9·5	4·4	20·2	58·8	54·0	0·24	−0·083	V(B)	V(F)
224	1984	5	1	E	45821·657	37·2	−84·1	10·2	8·8	5·1	21·0	43·3	55·8	0·24	−0·153	I(I)	V(F)
238	1989	5	5	E	47651·993	39·7	−105·5	9·8	9·8	−0·6	14·6	53·0	60·4	0·24	−0·053	I(V)	V(F)
250	1990	5	24	E	48035·993	31·6	−110·5	9·8	9·8	0·0	14·8	48·2	61·2	0·24	−0·054	I(V)	V(F)
52	1873	4	27	E	5275·447	38·0	23·7	10·2	9·2	4·4	18·8	45·7	58·5	0·25	−0·106	I	V(F)
197	1987	6	26	E	46972·733	30·0	−100·0	10·6	10·4	1·8	20·5	51·6	54·0	0·25	+0·014	I	V(V)
166	1981	7	30	M	44816·662	42·3	−71·3	10·1	8·3	−5·8	−18·6	−45·2	59·0	0·25	−0·205	I	I(V)
86	1913	11	28	E	20099·570	−33·9	18·5	10·3	10·3	0·0	16·4	53·1	58·7	0·26	−0·001	V	V(V)
251	1990	5	24	E	48035·993	34·2	−118·1	10·1	10·1	0·7	15·4	51·9	61·2	0·26	−0·014	I(V)	V(V)

7. A few conclusions

In this note I have pointed out that there is too much emphasis on making record breaking observations of first sighting of the new crescent moon. The majority of cases are not critical ones and what we need are more observations of first sighting at latitudes above 50° when the Moon is several days old, including observations made in daylight. Northern Europe would be an ideal place to organise another moonwatch program to solve this problem.

I have given a simple method for determining the time of best visibility, which is based on the method of Bruin. His method has many other applications to problems concerning first visibility. It would be useful to update Bruin's calculations using modern theories.

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I would especially like to thank Mr Yaacov Loewinger for encouraging me since 1992 to continue with this project, and for checking my work so thoroughly and making useful suggestions for improving this technical note. It was he who first made me consider expressing the Bruin test as a continuous function in the form of a quadratic polynomial in W. He also helped me to appreciate the value of the original list of 252 observations of first sighting for checking and calibrating other theories.

I am also indebted to Miss Catherine Hohenkerk, who kindly TEXd this technical note for me. She amended the NAO computer program for calculating first sighting to use the method given in this note, and together we updated Astronomical Information Sheet (AIS) No. 6. An example of the computer output is given in Appendix A, and further details concerning the output will be found in AIS No. 6.

I am grateful to Fatoohi *et al.* for producing a paper that gave me fresh insight to the problem.

I would also like to thank Dr Monzur Ahmed for advice on updating my q-test, and broadcasting recent changes on the internet, see http:\\www.starlight.demon.co.uk\ildl\jan98.htm.

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Appendix A

Visibility Predictions for the New Crescent Moon

London
Longitude W000 10
Latitude N51 30

Times are in GMT

New Moon Birth Date year	mth	day	h m	+ d		mth	day	Sun Set h m	Sun Az (°)	Moon at Sunset Age h m	Alt (°)	Az (°)	DAZ (°)	Mag	El (°)	Moon Set h m	Lag Time m	BEST TIME h m	Code
1998	Jan	28	06 01		Wed														
				0	Wed	Jan	28	16 43	241	10 42	4	239	2	-4.1	6	17 21	39	17 00	F
				1	Thur	Jan	29	16 44	242	34 43	14	230	12	-5.5	20	18 39	114	17 35	A
1998	Feb	26	17 26		Thur														
				0	Thur	Feb	26	17 35	257	0 09	-2	257	0	-3.3	0	17 30	-6	17 33	F
				1	Fri	Feb	27	17 37	258	24 11	10	250	8	-4.9	14	18 51	74	18 10	A
1998	Mar	28	03 14		Sat														
				0	Sat	Mar	28	18 27	276	15 13	5	269	7	-4.4	10	19 04	37	18 43	F
				1	Sun	Mar	29	18 29	277	39 15	16	261	15	-5.9	24	20 26	118	19 21	A
1998	Apr	26	11 41		Sun														
				0	Sun	Apr	26	19 15	293	7 34	-1	287	6	-4.0	6	19 16	1	19 16	F
				1	Mon	Apr	27	19 17	294	31 36	10	279	15	-5.5	19	20 36	79	19 52	A
1998	May	25	19 32		Mon														
				0	Mon	May	25	20 00	306	0 28	-5	303	4	-3.9	5	19 25	-35	19 44	F
				1	Tues	May	26	20 01	307	24 29	4	293	14	-5.0	15	20 37	36	20 17	F
				2	Wed	May	27	20 02	307	48 30	13	283	24	-6.3	28	21 40	98	20 46	A
1998	June	24	03 50		Wed														
				0	Wed	June	24	20 22	311	16 32	-1	301	10	-4.4	10	20 22	0	20 22	F
				1	Thur	June	25	20 22	311	40 32	6	290	21	-5.7	22	21 11	49	20 44	B
1998	July	23	13 44		Thur														
				0	Thur	July	23	20 02	305	6 18	-3	301	4	-3.8	4	19 47	-16	19 55	F
				1	Fri	July	24	20 01	304	30 17	2	289	15	-5.1	15	20 23	22	20 11	F
				2	Sat	July	25	20 00	304	54 16	7	278	26	-6.2	27	20 53	54	20 24	A
1998	Aug	22	02 03		Sat														
				0	Sat	Aug	22	19 09	290	17 06	1	282	8	-4.3	8	19 22	13	19 15	F
				1	Sun	Aug	23	19 07	289	41 04	5	271	18	-5.5	20	19 47	40	19 25	B
1998	Sept	20	17 01		Sun														
				0	Sun	Sept	20	18 04	273	1 03	1	273	0	-3.6	2	18 14	10	18 09	F
				1	Mon	Sept	21	18 02	272	25 01	4	262	10	-4.7	12	18 36	34	18 17	F
				2	Tues	Sept	22	18 00	271	48 59	8	251	21	-5.8	23	18 58	59	18 26	A
1998	Oct	20	10 09		Tues														
				0	Tues	Oct	20	16 57	254	6 48	3	252	2	-3.9	5	17 26	28	17 10	F
				1	Wed	Oct	21	16 55	254	30 46	7	242	12	-5.0	15	17 50	54	17 20	B
1998	Nov	19	04 27		Thur														
				0	Thur	Nov	19	16 07	239	11 40	5	236	3	-4.2	7	16 49	43	16 26	F
				1	Fri	Nov	20	16 05	238	35 38	9	225	14	-5.2	17	17 24	78	16 40	A
1998	Dec	18	22 42		Fri														
				1	Sat	Dec	19	15 53	232	17 11	6	227	5	-4.4	9	16 47	55	16 17	F
				2	Sun	Dec	20	15 53	232	41 11	11	216	15	-5.5	20	17 40	107	16 41	A

A Easily visible
B Visible under perfect conditions
C May need optical aid to find the crescent Moon
D Will need optical aid to find the crescent Moon
E Not visible with a telescope
F Not visible, below the Danjon limit

Computed on 7-Apr-98