MECCANO MATH III*

Gerard 't Hooft

Institute for Theoretical Physics
Utrecht University
and

Spinoza Institute
Postbox 80.195
3508 TD Utrecht, the Netherlands

e-mail: `g.thooft@uu.nl`
`internet: http://www.staff.science.uu.nl/~hooft101/`

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*These notes are the sequel of *Meccano Math I* and *II* by the same author.

1. Bi- and trisectors, revisited

To find mathematics literature that is helpful for this subject, one has to go to the 19^{th} century. A solution much better than the one I came up with myself in *Meccano Math II*, was found by Alfred Bray Kempe, an amateur mathematician, known from the Kempe chains, which play a role in the 4-colour problem. His solution for a mechanical trisector was pointed out to me by Luuk Hoevenaars, a former student of ours, and I am thankful for it.

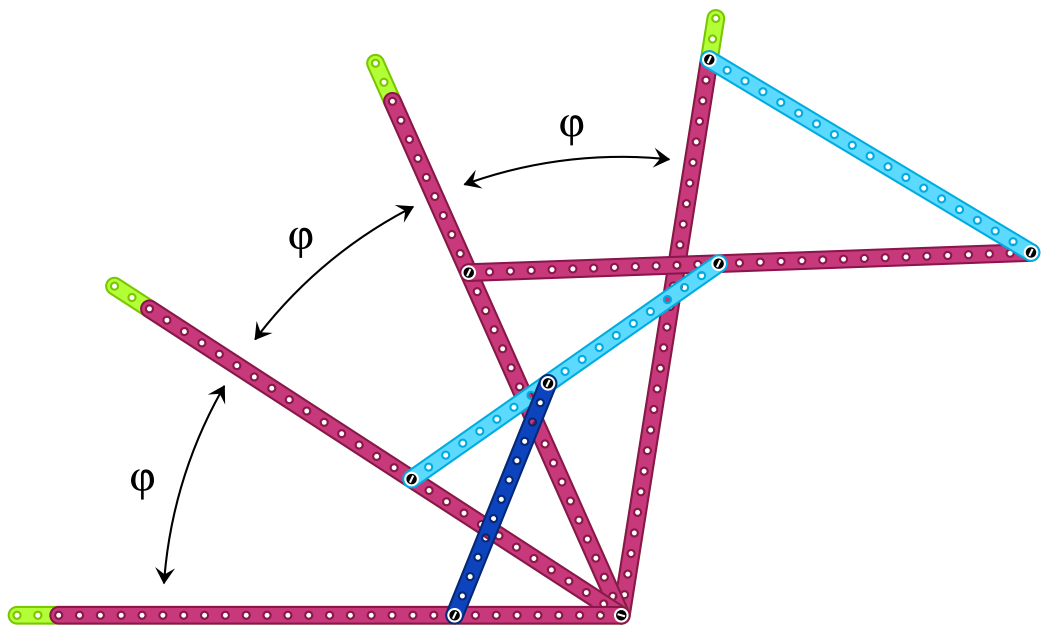

Figure 20: Kempe's trisector. The angles φ are the same. One easily sees how it works: a butterfly structure containing equal triangles is repeated twice, scaled down by a given factor (here $2:3$). The basic strips have lengths 27, 18, 12 and 8.

Figure 20 shows his trisector, which takes no more than 4 auxiliary pieces, as opposed to 10 in *Meccano Math II*. We see that it also contains a bisector, with only 3 auxiliary pieces.

Note that this enables us to make a simpler cube root machine, according to the step described in *Meccano Math II*. Next in line should be simpler straightedges and compasses. Suggestions are welcome.

2. The straightedge, revisited

Again, this is 19^{th} century mathematics. Kempe decribes his construction that leads to the straightedge of Figure 21 The lines $B'AB$ and AC are orthogonal. It is similar to what we had before.

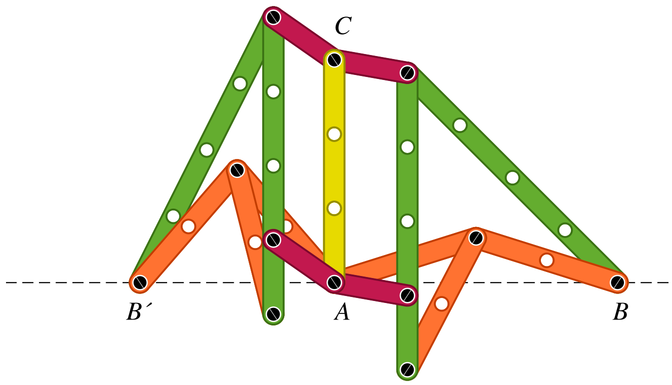

Figure 21: Kempe's construction leading to a straightedge.

A straightedge that is better than this, and Fig. 18 of Meccano II, because it has only 10 moving parts, compared to the 14 in my previous construction, is due to H. Hart[1]. See Fig. 22.

Hart also described the inverter that uses omly 4 moving parts, see Fig. 6 of Meccano I, Chapter 4, where I had started out with 6.

Kempe found various constructions that are actually a lot better than what I could show in Meccano I and Meccano II. Figure 23 shows a construction where two strips are kept neatly on one line: "Kempe's sledge".

References

[1] H. Hart, *A parallel motion*, Proc. London Math. Soc. **6** (1875) p. 137; id., *On some cases of parallel motion*, Proc. London Math. Soc. **8** (1877) p. 286.

[2] A.B. Kempe, *On a general method of producing exact rectilinear motion by linkworks*, Proc. Royal Soc. London, **23** (1875) p. 565.

[3] E.A. Dijksman, *True Straight-line Linkages Having a Rectilinear Translating Bar*

[4] Warren D. Smith, *Plane mechanisms and the "downhill principle"*, Babbage, typeset 765 2, 1998.

[1] I thank Chris Vos for giving me the references.

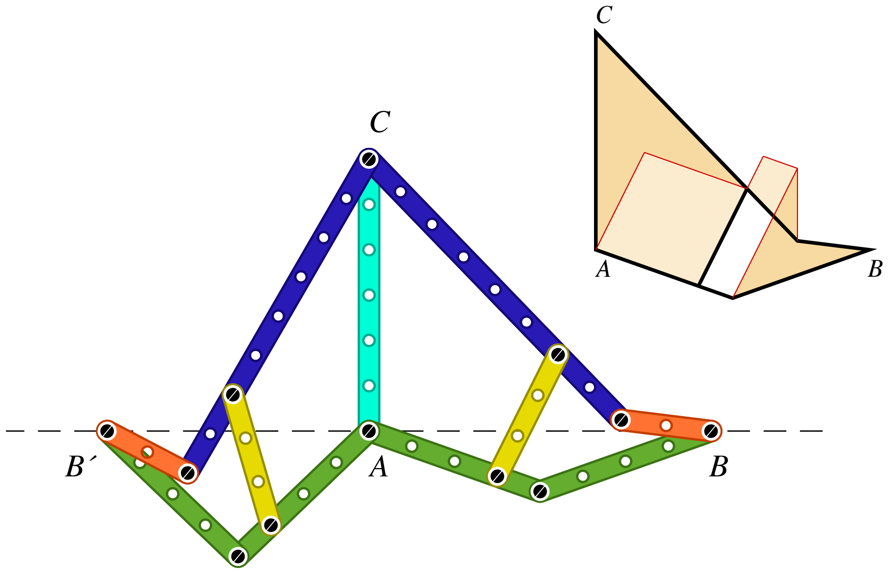

Figure 22: Hart's construction leading to a straightedge. The points A, B and B' are on a straight line but their distances can be varied, within a certain range. The line AC is orthogonal to $B' A B$. Top right: to prove these properties, some auxiliary lines are drawn. The lengths of the lines drawn are all integer in meccano units. The two kite shapes are congruent, and so are the two darts (shaded). By calculating the relations of the angles one finds the orthogonality. The left part of the construction mirrors the right part.

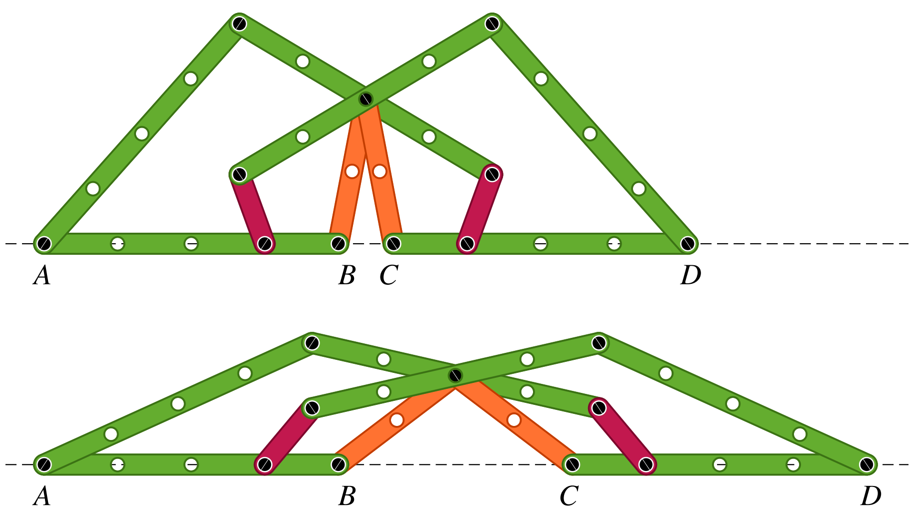

Figure 23: Kempe's sledge. If points A and B are kept fixed, C and D can only move on the dotted line. The four kite figures are all congruent.