Good-for-Game QPTL: An Alternating Hodges Semantics

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The connection between logic and games is a well-established fact. Typically, the satisfiability of a (first-order) logical formula reduces to deciding whether a player has a winning strategy in a zero-sum two-player game. Logic can also be used to reason about coalition-games, by encoding moves of the opposing coalitions by means of existentially and universally quantified variables and by describing the game with a formula over those variables. The quantifier alternation in the formula yields the multiple rounds and the winning condition of the coalition is provided by the underlying unquantified formula.

In some scenarios, infinite-horizon games need being considered. In this case, one cannot rely on a finite alternation of quantifiers. Extending the approach with infinite sequences of quantifiers leads to infinitary logics [12], which depart from the Tarskian point of view because only non-compositional game-theoretic semantics have been provided. A more viable route is to consider first-order extensions of temporal logics that predicate over infinite sequences of temporal points. There, a valuation of the timeline is seen as the result of an infinite sequence of moves, each time point corresponding to a round of the infinite-horizon game. In this setting, however, the satisfiability and the game solution problems do not coincide anymore; indeed, the choices of one player at each round may depend on the future choices of her adversary. In fact, due to the classic interpretation of quantifiers, the choice of a valuation for a quantified variable is made with complete information about the variables quantified before it in the sentence. This limitation of the classic interpretation of quantifiers is well-known and attempts have been made to overcome the *linear dependence* of quantifiers dictated by their relative position in a sentence [8, 9, 2, 17].

Somewhat inspired by the work on dependence logics [18, 14, 1] and by Hodges' Team semantics [10], we propose a novel semantics for Quantified Propositional Temporal Logic (QPTL), an extension of Linear Temporal Logic [16] (LTL) with quantifiers. This new semantics allows for restricting functional dependencies between variables so that their value at each time point is independent of the values of future time points of the other variables. This is achieved by first extending Hodges' teams to sets of sets of assignments[1], called hyperassignments. This two-level structure captures the alternation between the strategic choices of each player: an inner set represents a pruning (by a chosen strategy) of the tree obtained when unfolding the infinite-horizon game. A hyperassignment is not typed: the player choosing first her strategy can be the existential one or the universal one indifferently. Thus, our semantics add the information about who is pruning the tree first through a flag $\exists\forall$ or $\forall\exists$. Intuitively, a hyperassignment $\mathfrak{X}$ is said to $\exists\forall$-model (resp., $\forall\exists$-model) a formula φ *iff* there is some set of assignments $\mathrm{X} \in \mathfrak{X}$ such that for all assignments $\chi \in \mathrm{X}$, it holds that $\chi \models_{\mathrm{LTL}} \varphi$ (resp., for all sets of assignments $\mathrm{X} \in \mathfrak{X}$ there is an assignment $\chi \in \mathrm{X}$ such that $\chi \models_{\mathrm{LTL}} \varphi$). This intuition matches the formalization when φ is quantifier free (i.e. an LTL formula). When the formula contains quantifiers, the evaluation requires two operations on hyperassignments, namely the *extension* and the *dualization*. The extension $\mathrm{ext}(\mathfrak{X}, p)$ of a hyperassignment $\mathfrak{X}$ with regard to a variable p is the hyperassignment defined as follows. $\mathrm{ext}(\mathfrak{X}, p)$ contains exactly the sets X' obtained from a set $\mathrm{X} \in \mathfrak{X}$ where every assignment has been expanded to p in some way. Roughly speaking, the hyperassignment $\mathrm{ext}(\mathfrak{X}, p)$ contains all the possible sets X' corresponding to different ways of expanding every X. Remark that the variable p is supposed to be controlled by the first player to choose her strategy as she chooses the set X' and therefore, the response of p. If we want the second player to control the variable, we dualize the hyperassignment: the dualization swaps the places of the existential and the universal players. We do not formally describe the dualization because its definition is tedious and would not serve the explanation. The dual of the hyperassignment $\mathfrak{X}$ is denoted $\overline{\mathfrak{X}}$. We can completely define our semantics on quantifiers: $\mathfrak{X} \models^{\exists\forall} \exists p.\varphi$ if $\mathrm{ext}(\mathfrak{X}, p) \models^{\exists\forall} \varphi$ and $\mathfrak{X} \models^{\exists\forall} \forall p.\varphi$ if $\overline{\mathfrak{X}} \models^{\forall\exists} \forall p.\varphi$. The semantics for the flag $\forall\exists$ follows the same idea. In contrast to previous work, such as [10, 11], because our formulation allows for a symmetric treatment of the two quantifiers, we obtain a logic that is closed under negation and avoids undetermined

[1]An assignment is a mapping of the variables to valuation on the timeline: $P \to (\mathbb{N} \to \mathbb{B})$

formulae.

The most significant feature of this new approach is the ability to specify various forms of independence constraints among quantified variables. Indeed, during the extension of a hyperassignment, one can restrict the considered strategies with, for example, uniformity conditions. The ability to constraint strategies in different manners provides a powerful tool to fine-tune the semantics of the propositional quantifiers of a formula. In particular, we discuss a specific instantiation of the semantics, the so-called *behavioral semantics*: we restrict all the involved strategies of a given variable to be independent of the future values of the other variables. Formally, when extending a hyperassignment $\mathfrak{X}$, we consider only expansions X' of set $\mathrm{X} \in \mathfrak{X}$ for which if two assignments in X are equal up to some time point t, then the valuations of p expanding those assignment in X' are also equal up to t. This semantics allows one to recover a compositional game-theoretic interpretation of the quantifiers and reconcile the satisfiability and the game solution problems.

On the technical side, we show that the behavioral semantics allows for a reformulation of a prenex formula into a formula where existentially quantified variables are all quantified before the universal ones, but also into a formula where the universally quantified variables are quantified first. Both reformulations use a mixed specification of dependencies on quantifiers to force them not only to be independent of the future but also of the present of some variable. For instance, $\exists p. \forall q. \varphi$ is equivalent to $\forall q. \exists p.$ where the strategy for p does not depend on the present values of q. As a consequence, the novel semantics enjoys 2ExpTime decision procedures for both the satisfiability and the model-checking problem for prenex formulae, because we effectively limit to one alternation of quantifiers. So, we show that the high (non-elementary) complexity of QPTL when interpreted with the standard semantics stems from the fact that unrestricted dependencies among the quantified variables are allowed. On the other hand, the new semantics does not affect expressiveness, although the proposed logic may be non-elementarily less succinct. We believe the generality and flexibility of the semantic settings for quantifiers opens up systematic investigations on the impact of such semantics in quantified temporal logics, such as QCTL [7, 13], HyperLTL/CTL* [4, 5], Coordination Logic [6], and Strategy Logic [3, 15].

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