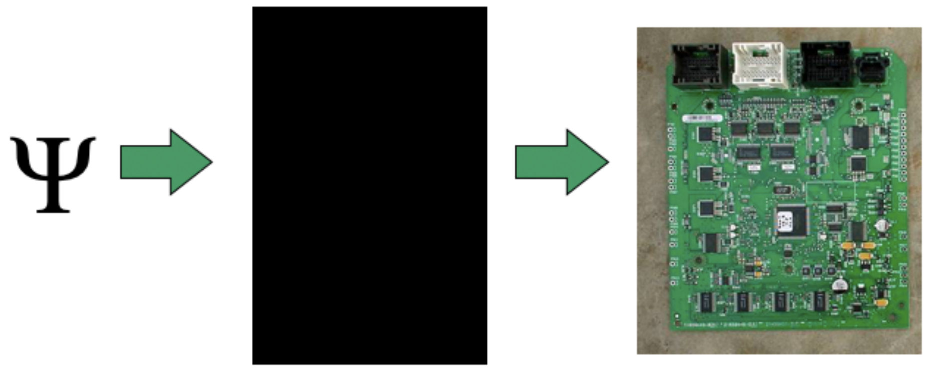
Rational Synthesis



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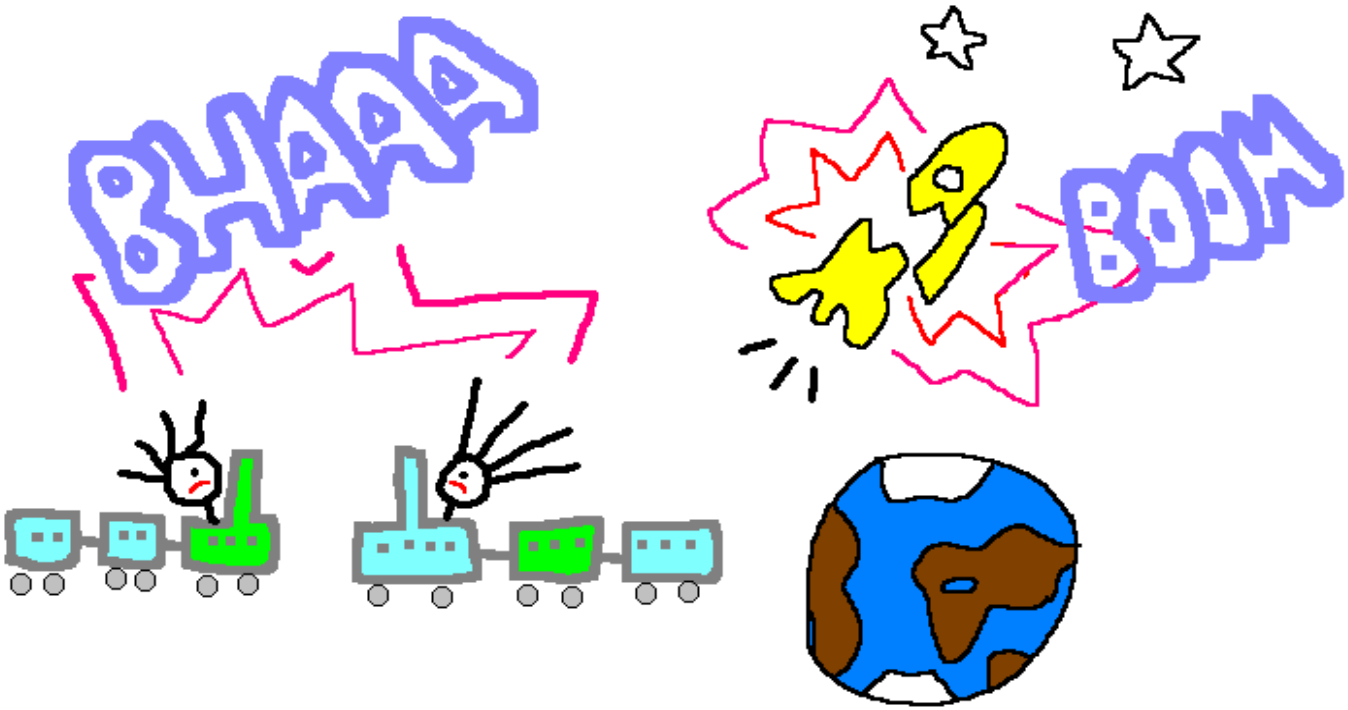
Hebrew University

Joint work with Shaull Almagor, Dana Fisman, Yoad Lustig, Giuseppe Perelli, and Moshe Y. Vardi

Is the system correct?



Synthesis:

Input: a specification ψ.

Output: a system satisfying ψ.

Is the system correct?

Yes! it satisfies its specification.

An open system: Interacts with its environment.

A game:

- A set I of input signals.
- A set O of output signals.
- In each round of the game:
  - the system assigns values to the signals in O.
  - the environment assigns values to the signals in I.
- Together, the system and the environment generate a computation: an infinite word over the alphabet $2^{I \cup O}$.

An open system: $f:(2^I)^* \rightarrow 2^O$

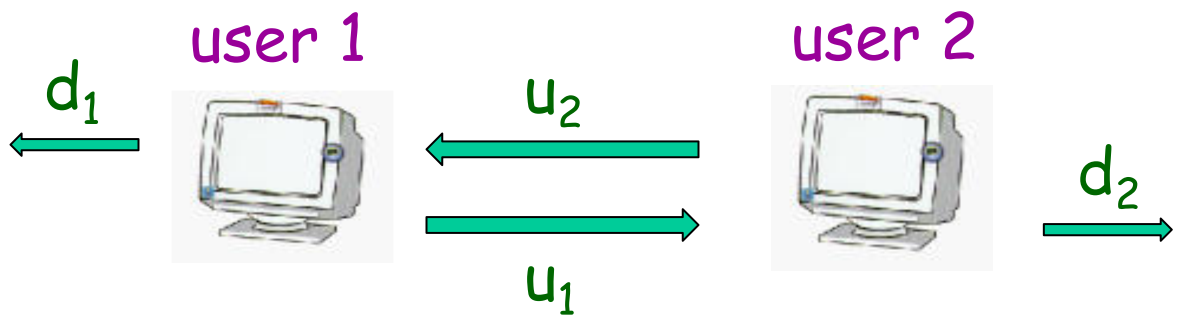
A correct system: a winning strategy.

An open system is correct if it satisfies its specification in all environments.

Too strong: Add assumptions on the environment (behavioral or structural).

Rational synthesis: the components that compose the environment have their own objectives and are rational. [Fisman, Lustig, Kupferman 2010]

An example:



User 1 can download only when User 2 uploads.

User 2 can download only when User 1 uploads.

Both users want to download infinitely often.

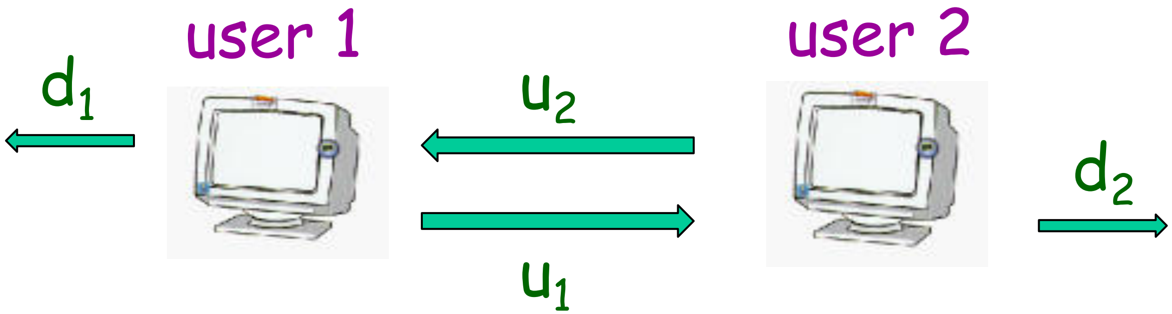
$\varphi_1 = GF(d_1 \wedge u_2)$

$\varphi_2 = GF(d_2 \wedge u_1)$

φ_1 is not realizable:

- fails when User 2 eventually never uploads.

An example:



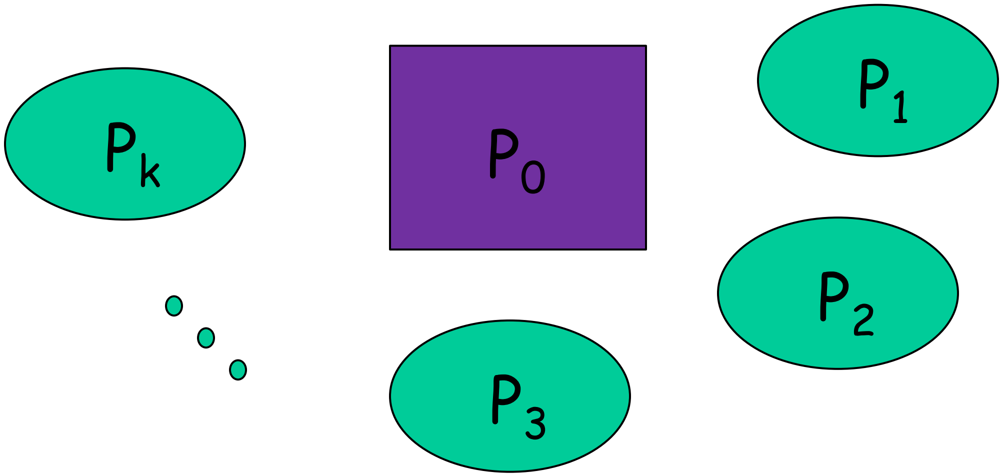
$\varphi_1 = GF(d_1 \wedge u_2)$

$\varphi_2 = GF(d_2 \wedge u_1)$

User 1 to User 2: I will upload, and will continue to upload as long as you upload.

A rational User 2 will upload forever, enabling User 1 to satisfy φ_1.

Rational Synthesis [FKL10]



$X = X_0 \cup \ldots \cup X_k$

P_i assigns values to X_i

Input: objectives ψ and $\varphi_1, \ldots, \varphi_k$.

Output: a $\boxed{\text{stable}}$ profile $\langle f_0, \ldots, f_k \rangle$ that satisfies ψ.

$P_1 \ldots P_k$ have no incentive to deviate

Cooperative Rational Synthesis [FKL10]

Input: objectives ψ and $\varphi_1,\ldots,\varphi_k$.

Output: a stable profile $\langle f_0,\ldots,f_k \rangle$ that satisfies ψ.

We can suggest a strategy to the enviroment...

Cooperative Rational Synthesis [FKL10]

Input: objectives ψ and $\varphi_1,\ldots,\varphi_k$.

Output: a stable profile $\langle f_0,\ldots,f_k\rangle$ that satisfies ψ.

Non-Cooperative Rational Synthesis [KPV13]

Input: objectives ψ and $\varphi_1,\ldots,\varphi_k$.

Output: a strategy f_0 such that every stable profile $\langle f_0,\ldots,f_k\rangle$ satisfies ψ.

How different they are?

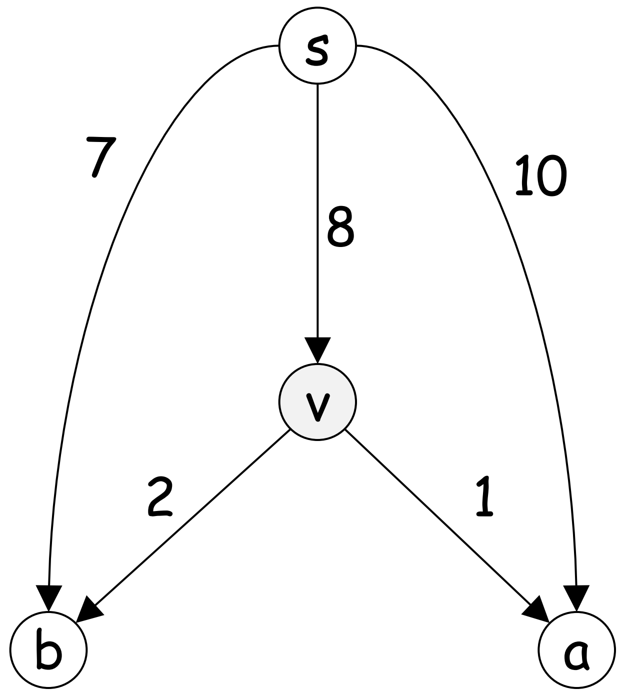
Algorithmic Game Theory

A network

b locations.

communication channels.

6 cost of creating the channel.



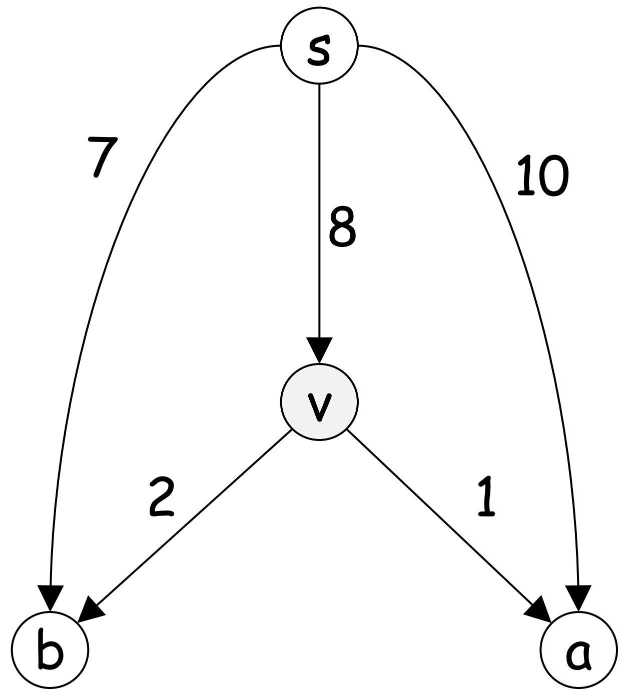
A network formation game

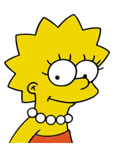
[Anshelevich, Dasgupta, Kleinberg, Tardos, Wexler, Roughgarden 2004]

b locations.

communication channels.

6 cost of creating the channel.





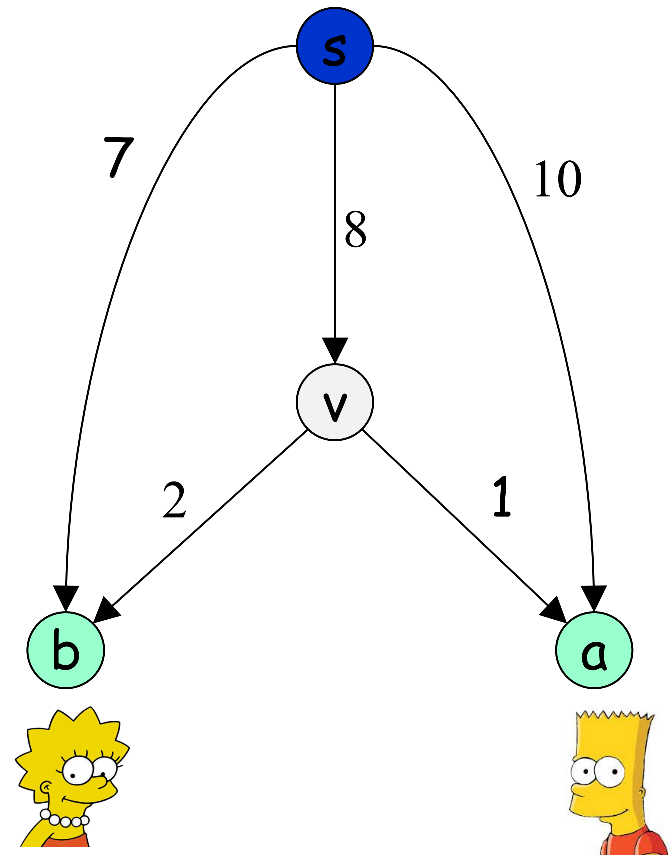
Players that need to transmit messages between locations in the network.

A network formation game: example

Two players need to transmit messages from s

Player 1  needs to reach a

Player 2  needs to reach b



A network formation game: example

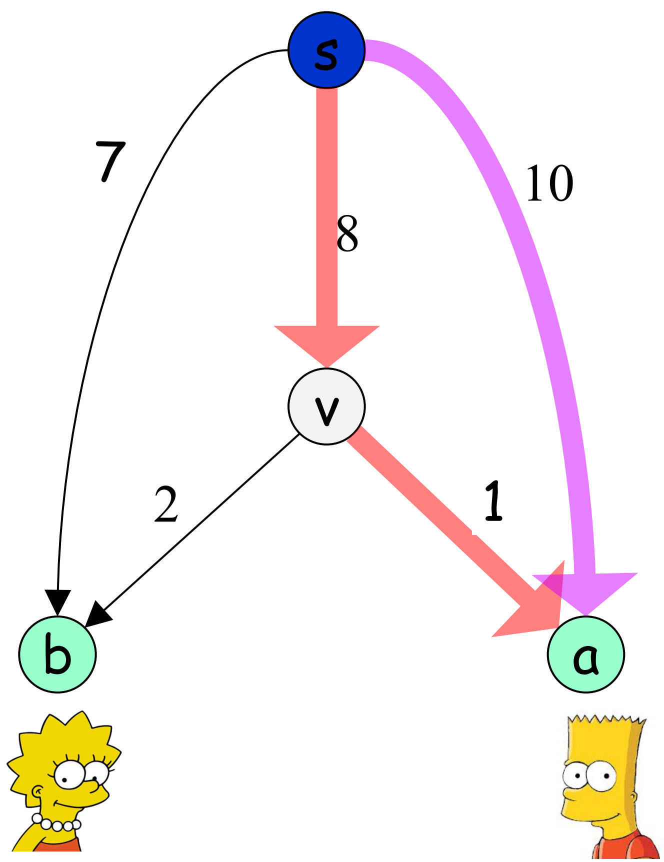
Two players need to transmit messages from s

Player 1 needs to reach a

Player 2 needs to reach b

The strategy space of Player 1:

$\{ \{\langle s,v\rangle,\langle v,a\rangle\} , \{\langle s,a\rangle\} \}$



A network formation game: example

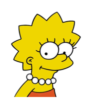
Two players need to transmit messages from s

Player 1  needs to reach a

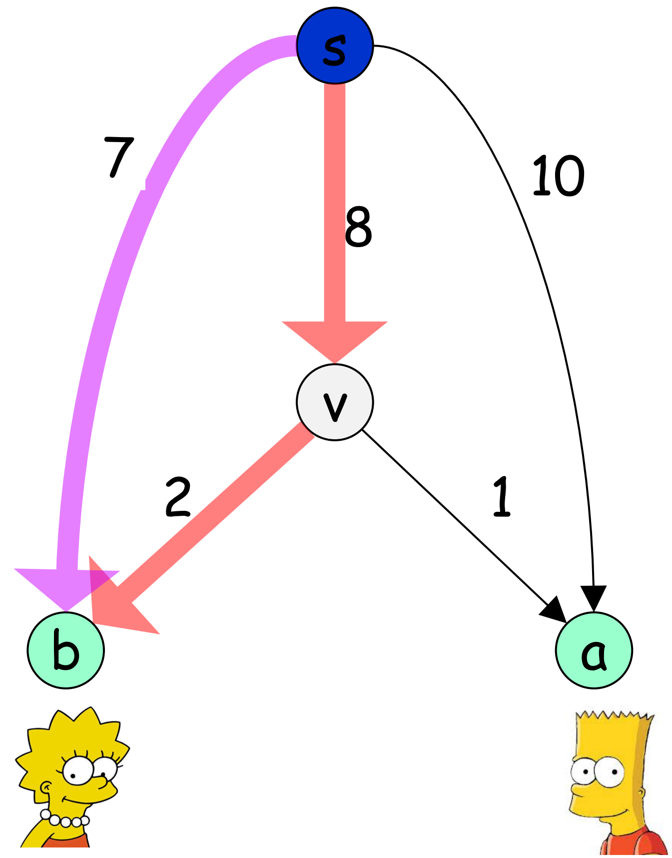
Player 2  needs to reach b

The strategy space of  :

$\{ \{\langle s,v\rangle,\langle v,a\rangle\} , \{\langle s,a\rangle\} \}$

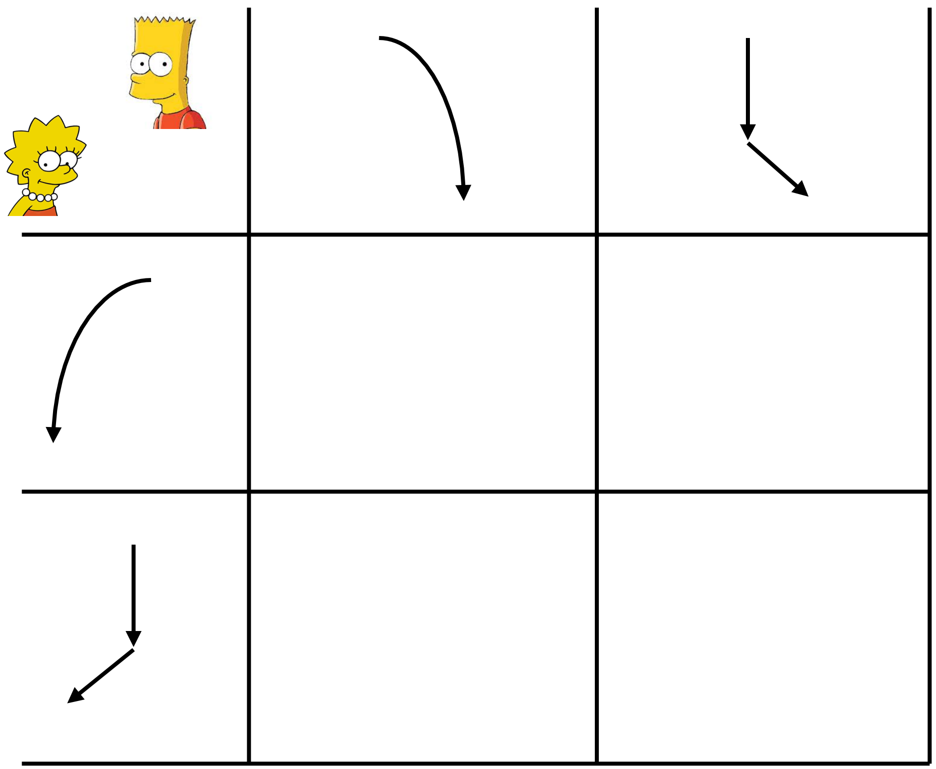
The strategy space of  :

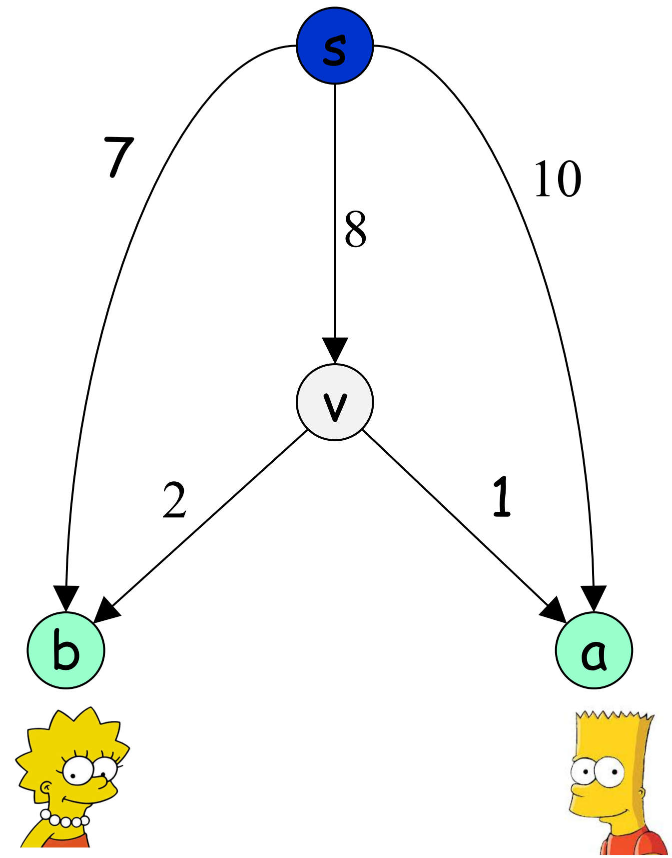
$\{ \{\langle s,b\rangle\} , \{\langle s,v\rangle, \langle v,b\rangle\} \}$



A profile is a choice of strategy for each player.

Four possible profiles in our example:





What are the payments?

A profile is a choice of strategy for each player.

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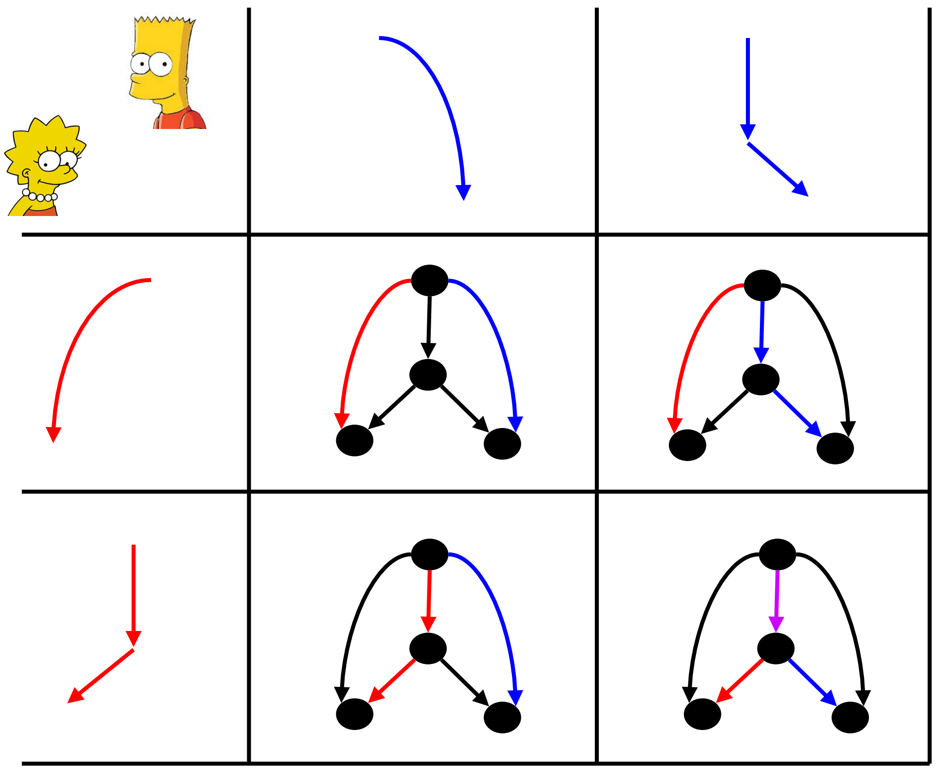

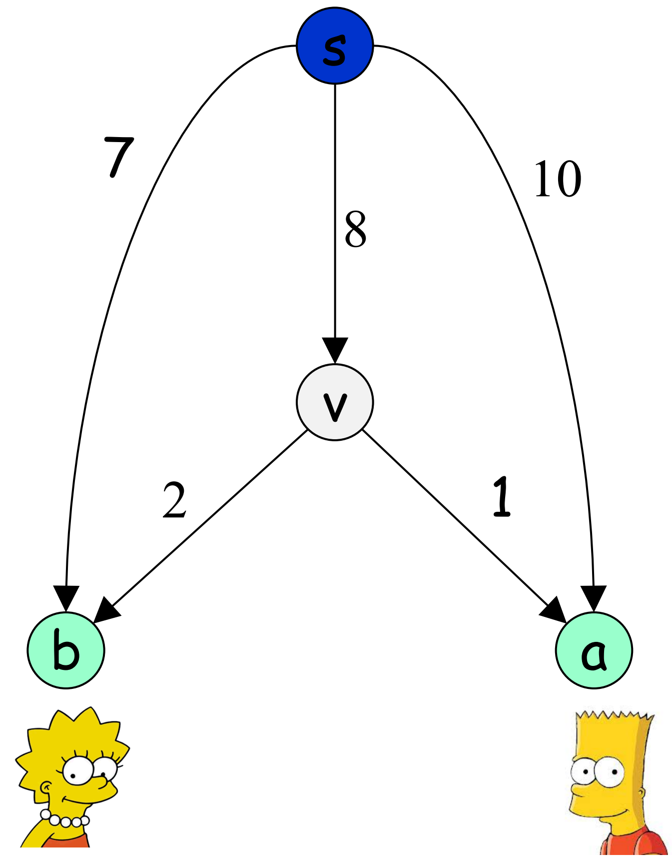

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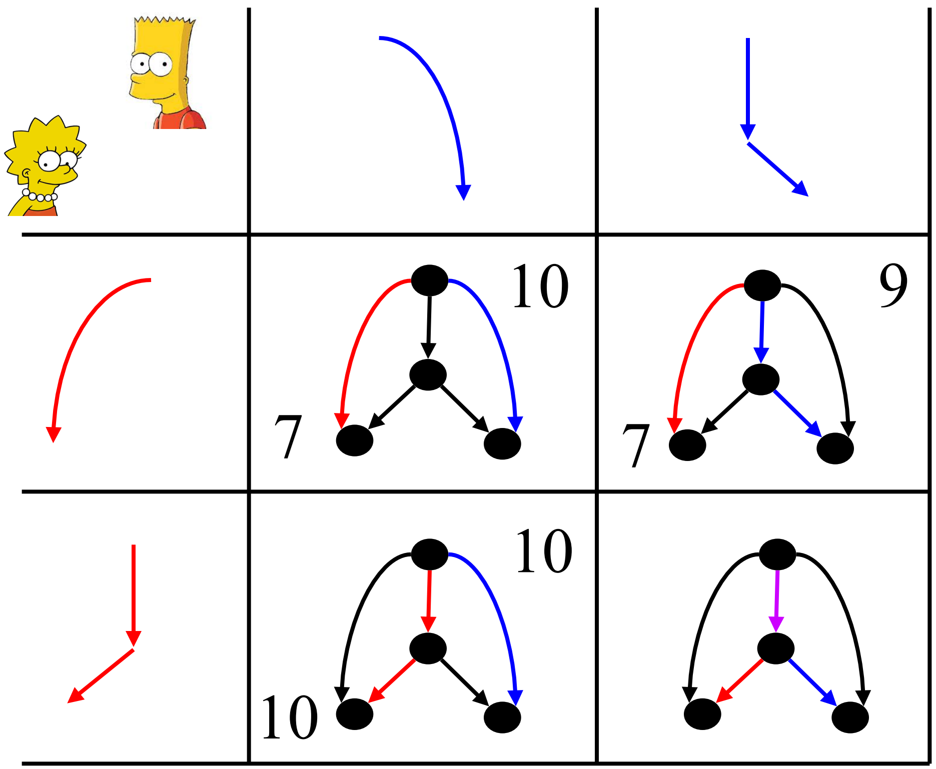
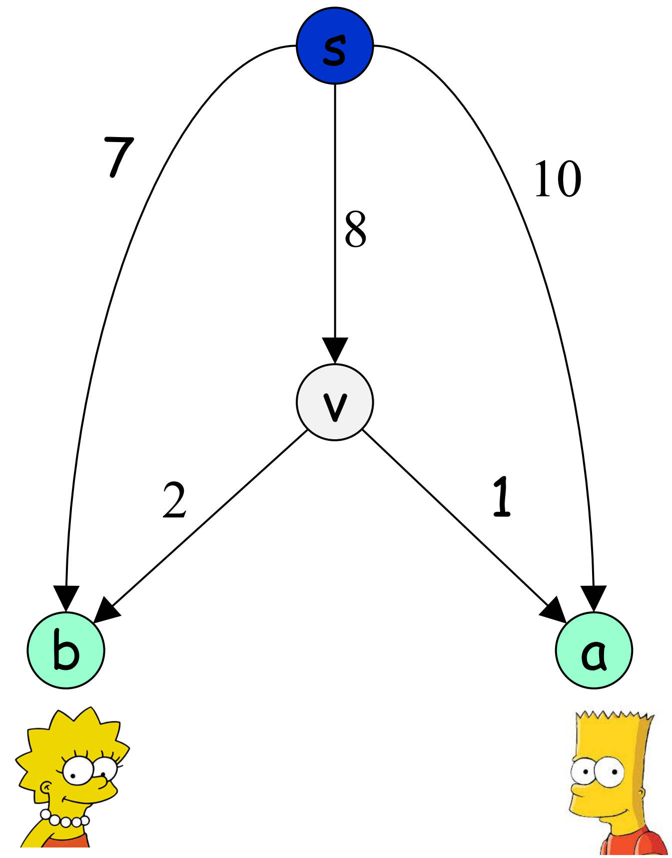
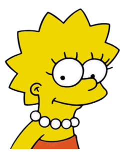




What are the payments?

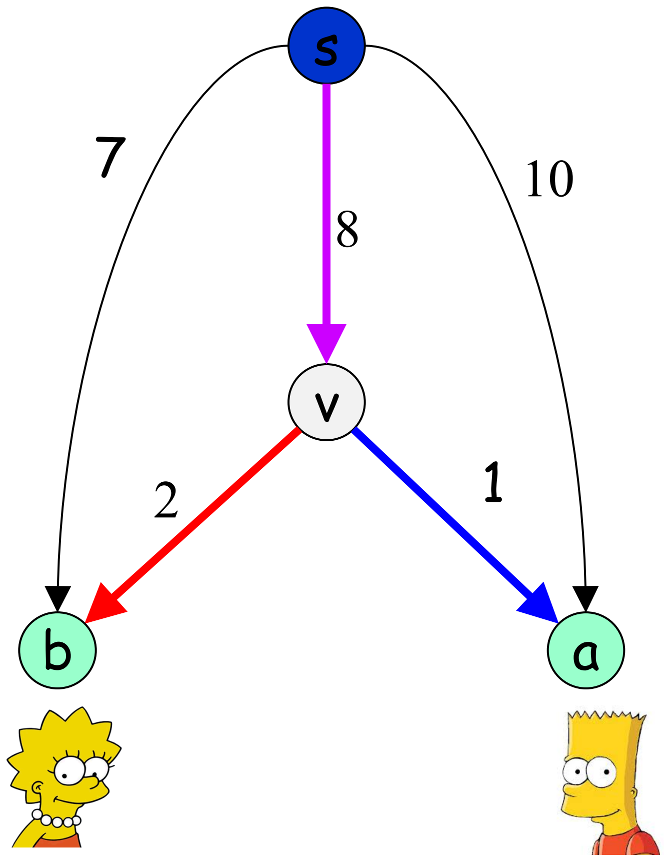
How is a cost shared?

Players that use the same channel share its cost:



$$\frac{8}{2}+2=6$$

$$\frac{8}{2}+1=5$$



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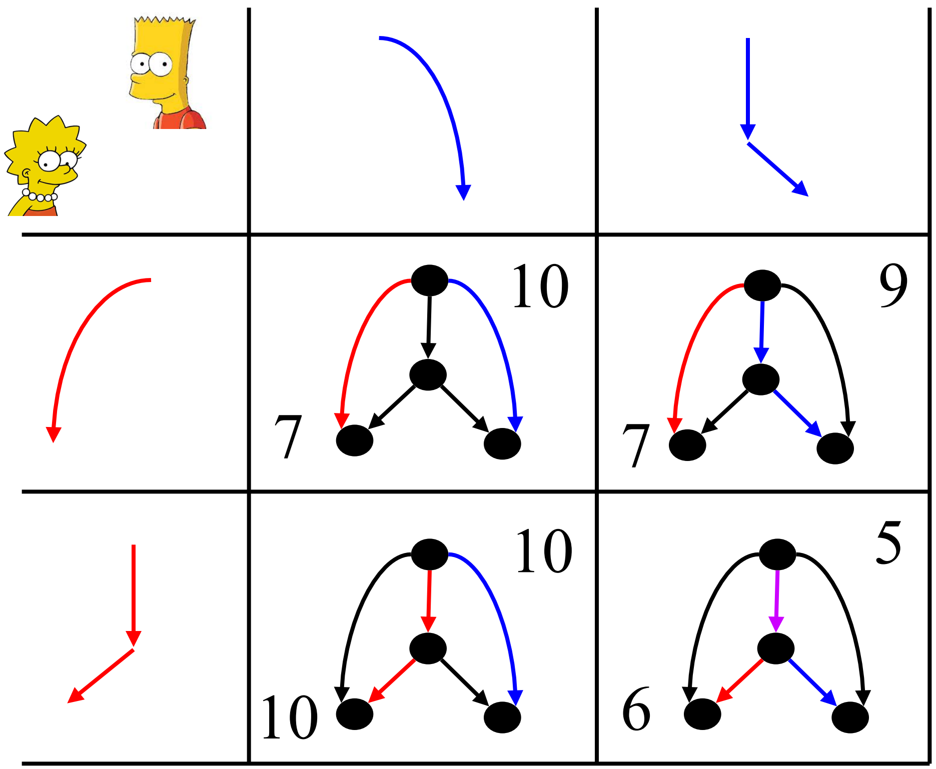
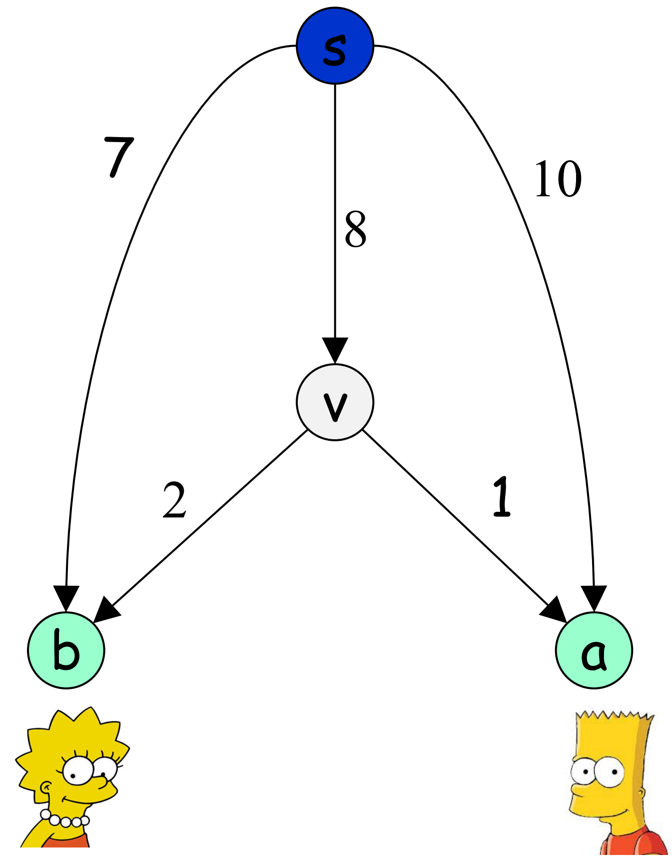


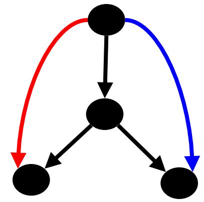


Best response dynamics (BRD):

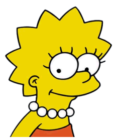
- A local search method: in each step some player is chosen and plays his best-response strategy, given the strategies of the others.

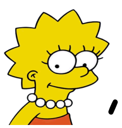
- BRD converges when no player wants to change his strategy.

Best response dynamics.

Example: starting from 

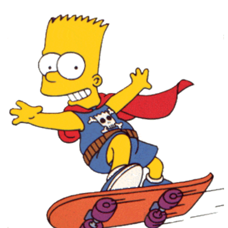
Cost for  :10

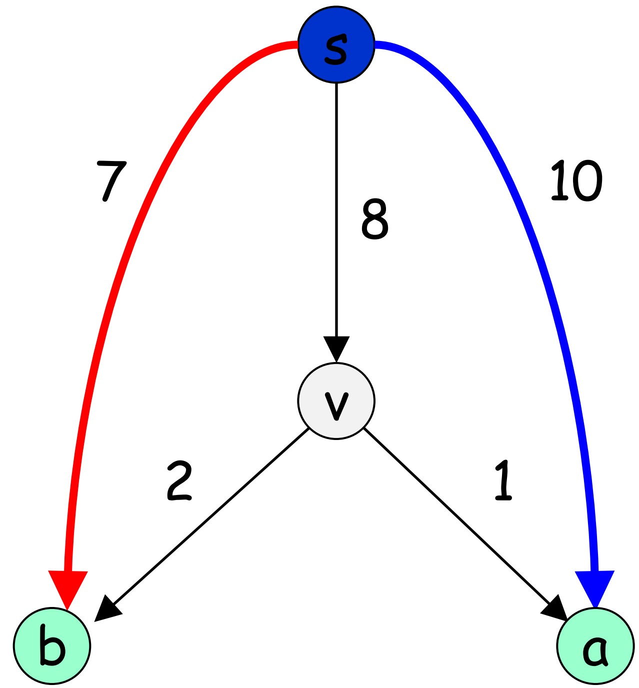
Cost for  :7

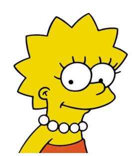
, want to change strategy?

No, 7 < 10

, want to change strategy?

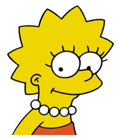
Sure ,9 < 10 

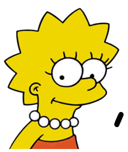




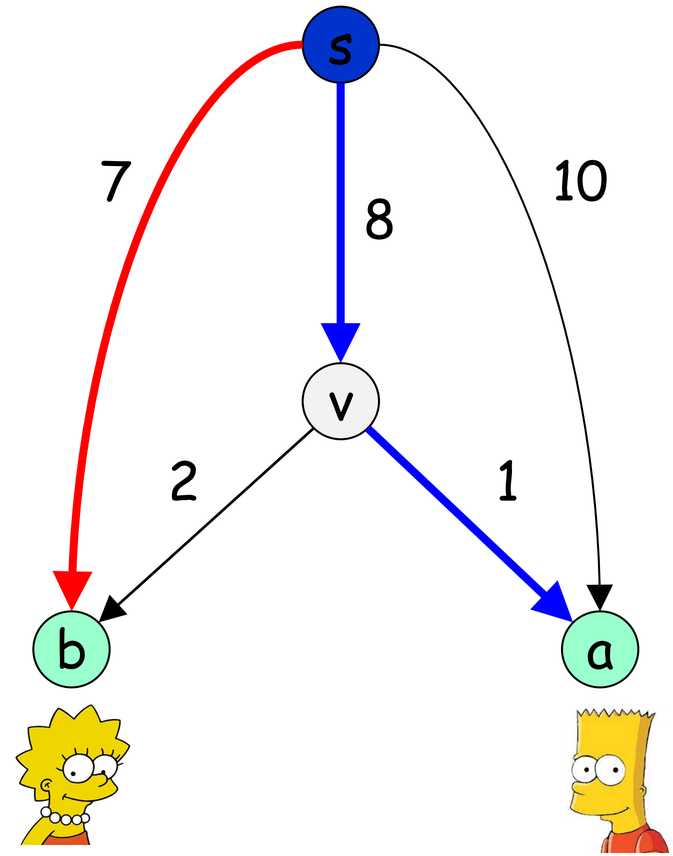
Best response dynamics.

Cost for  :9

Cost for  :7

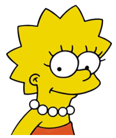
, want to change strategy?

Yes, 6 < 7



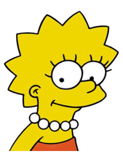
Best response dynamics.

Cost for  :5

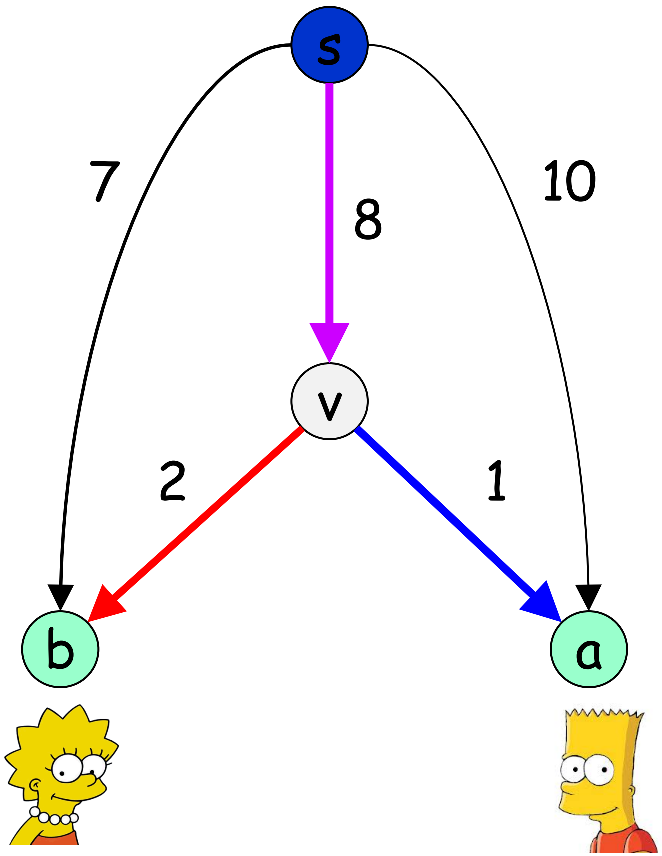
Cost for  :6

, want to change strategy?

No, 5 < 10

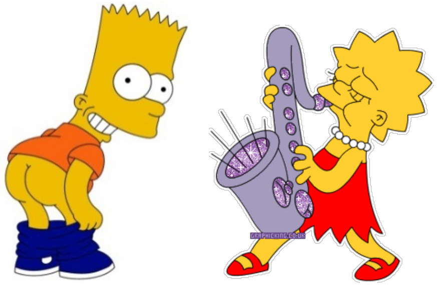
, want to change strategy?

No, 6 < 7



BRD halts, we've reached a stable profile.

Nash Equilibria (NE): a profile of strategies such that no player can benefit from changing to another strategy (assuming the other players stay with their strategies).



BRD halts, we've reached a stable profile.

Interesting questions:

- Does best response dynamics always converge?

Yes! In all network formation games.

Proof: potential functions.

If profile P' is obtained by applying a best-response in profile P, then $\Phi(P') < \Phi(P)$.

Interesting questions:

- Does best response dynamics always converge?

- Will we reach a good Nash equilibrium?

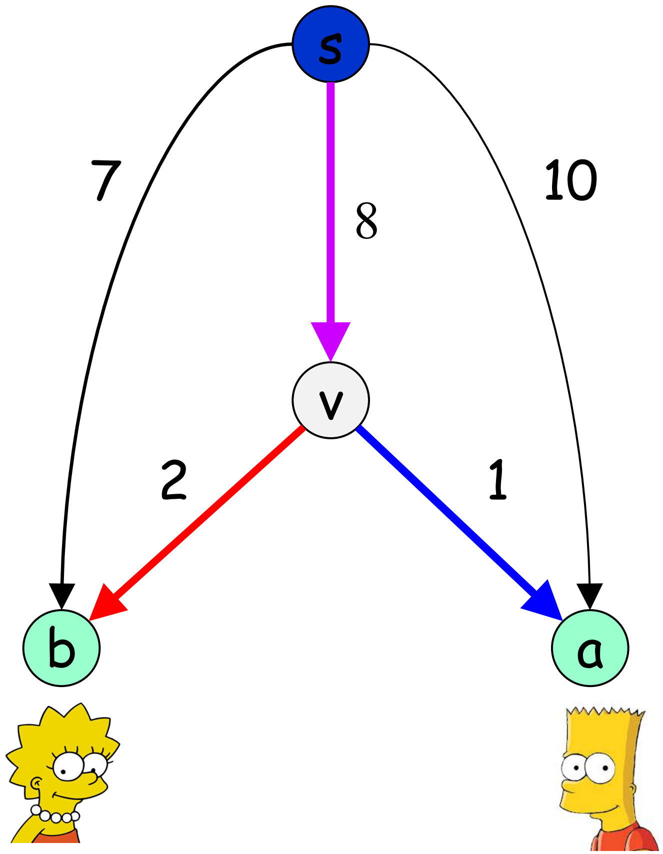
What is "good"?

Social optimum (SO): minimizes the sum of the payments of all players together.

Good: equal (or at least close) to the social optimum.

How much do we lose from the absence of a centralized authority?

In our example:



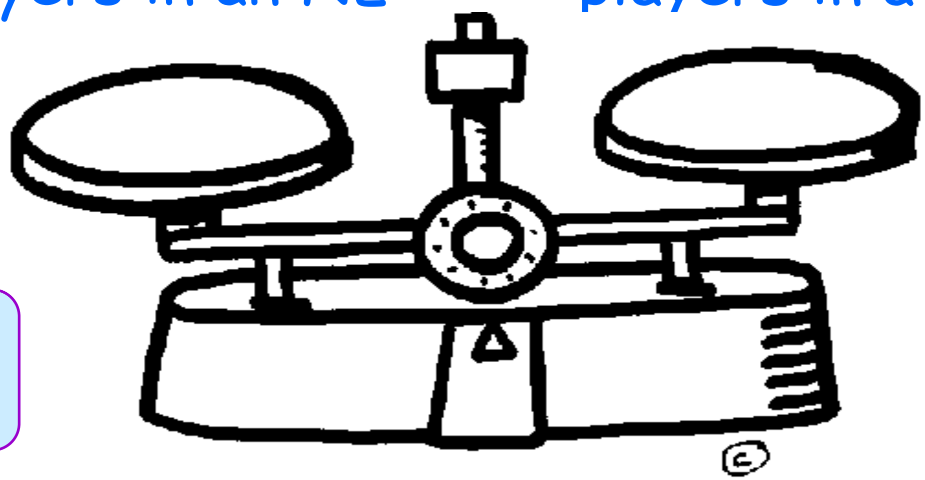
SO = NE = 11

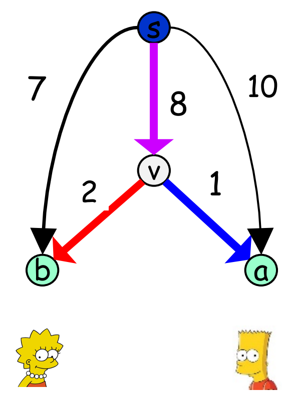
Interesting questions:

Will we reach a good Nash equilibrium?

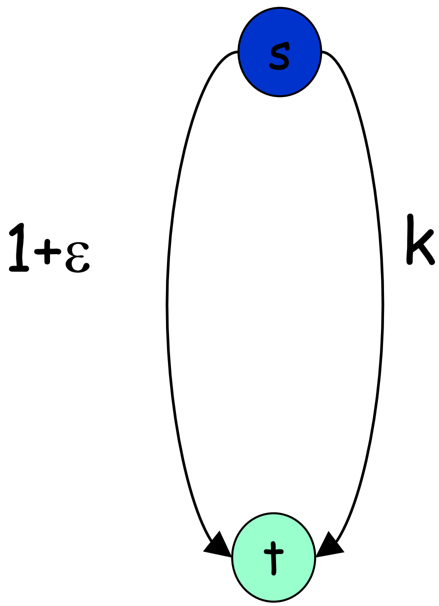
Payments of the players in an NE

Payments of the players in a SO



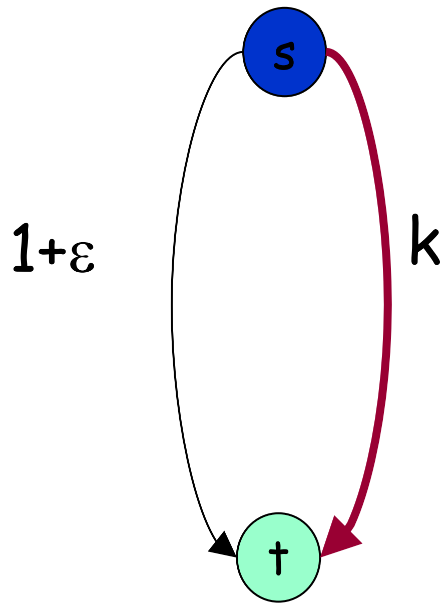


An NE may not be good!



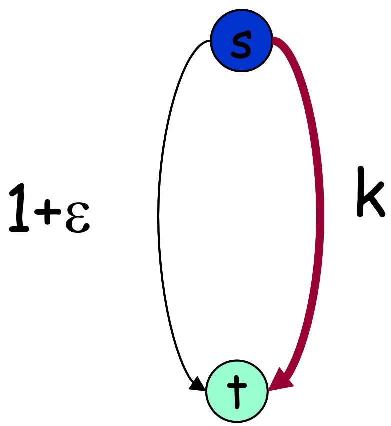
An NE may not be good!

- k players, all want to route from s to t
- All k players start in the channel that costs k.



Each player pays $\frac{k}{k}=1$

An NE may not be good!



Now I am paying 1.
If I switch I would
need to pay 1+ε

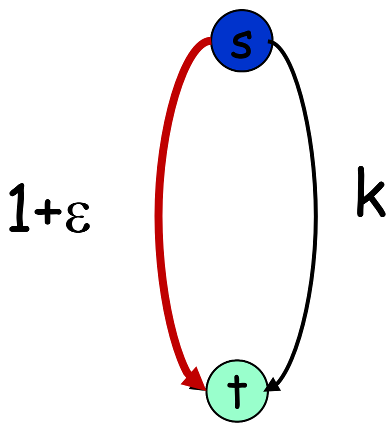
- No one wants to switch!

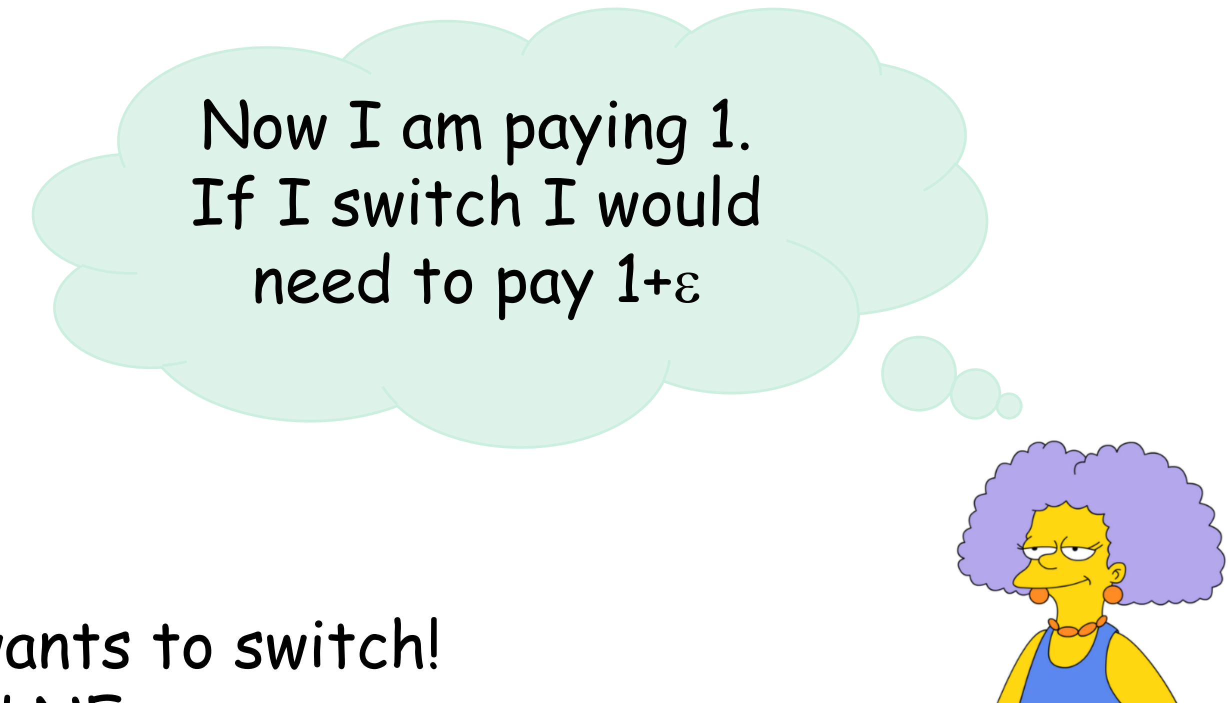
A very bad NE.

Price of Anarchy = k

PoA: worst NE / SO.

An NE may not be good!





- No one wants to switch!

A very bad NE.

Price of Anarchy = k

- But, a good NE does exist.

Does there always exist a good NE?

Does there always exist a good NE?

For every network formation game, there exists a good NE – one whose cost is at most $H_k \cdot SO$.

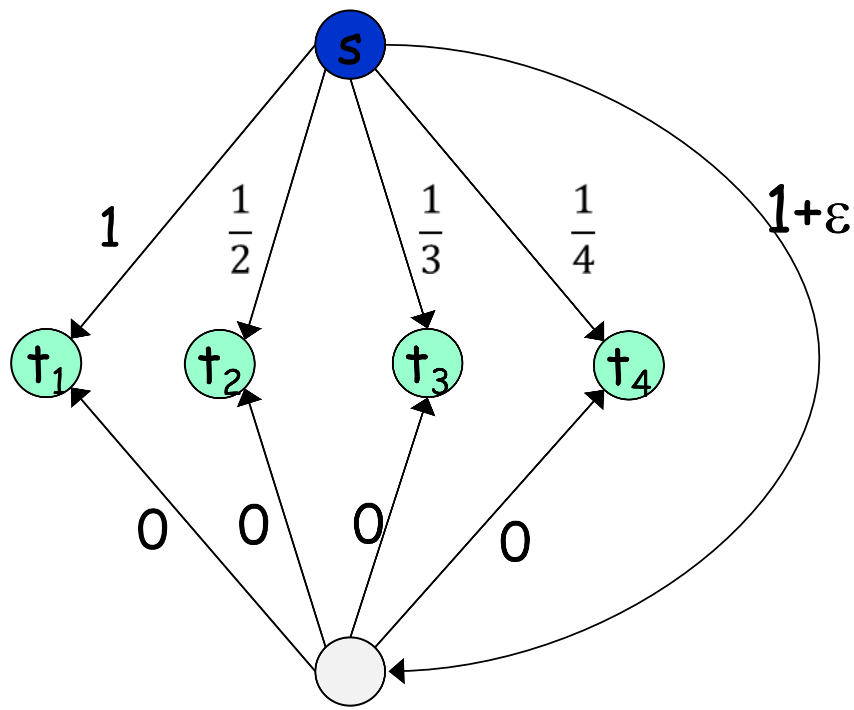
$$H_0 = 0,$$
$$H_k = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \approx \ln k$$

Price of stability: best NE / SO.

H_k is tight…

Does there always exist a good NE?

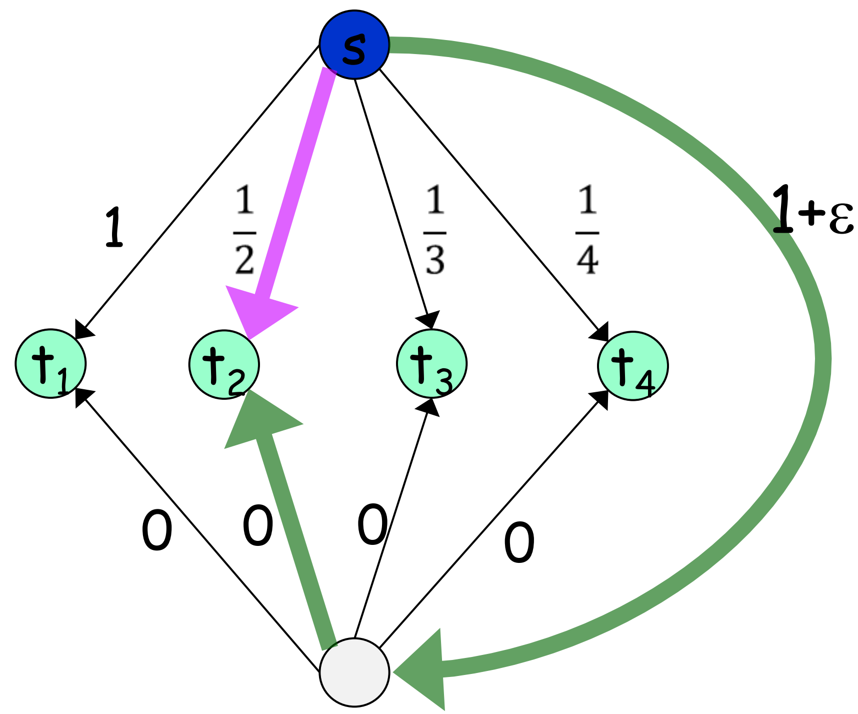
Four players want to route in the following network:



Does there always exist a good NE?

Four players want to route in the following network:

Each player has two possible strategies:
A direct edge or via the vertex at the bottom.

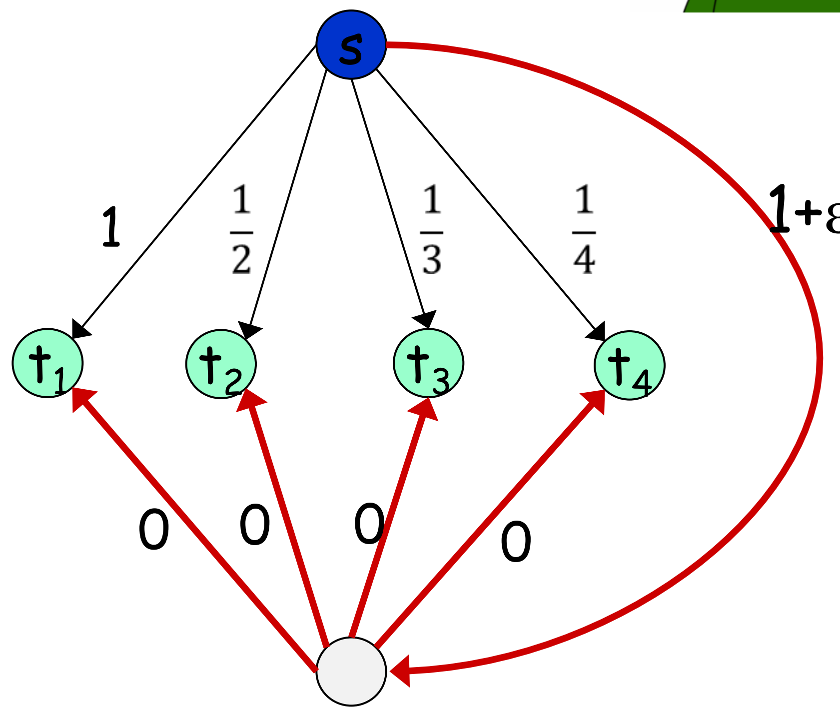


Does there always exist a good NE?

A profile that attains the social optimum:

Note: it costs 1+ε.

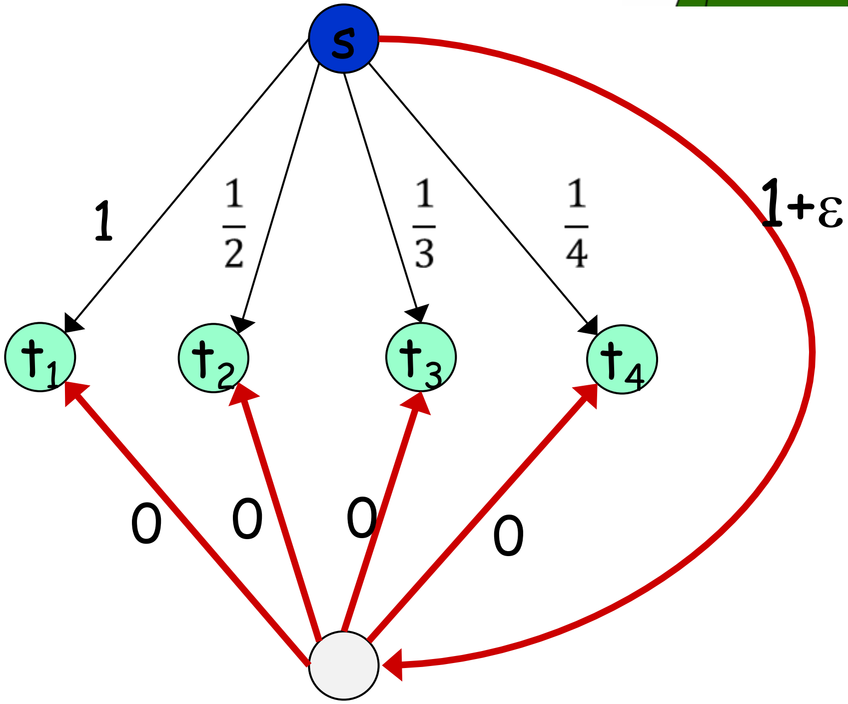
In this profile each player pays $\frac{1}{4}$+ε.



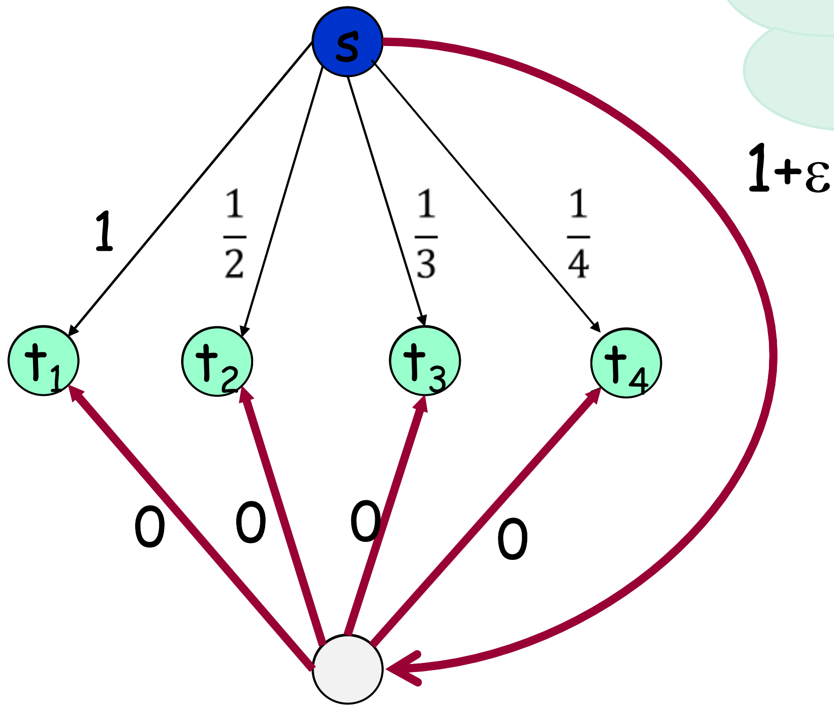
Does there always exist a good NE?

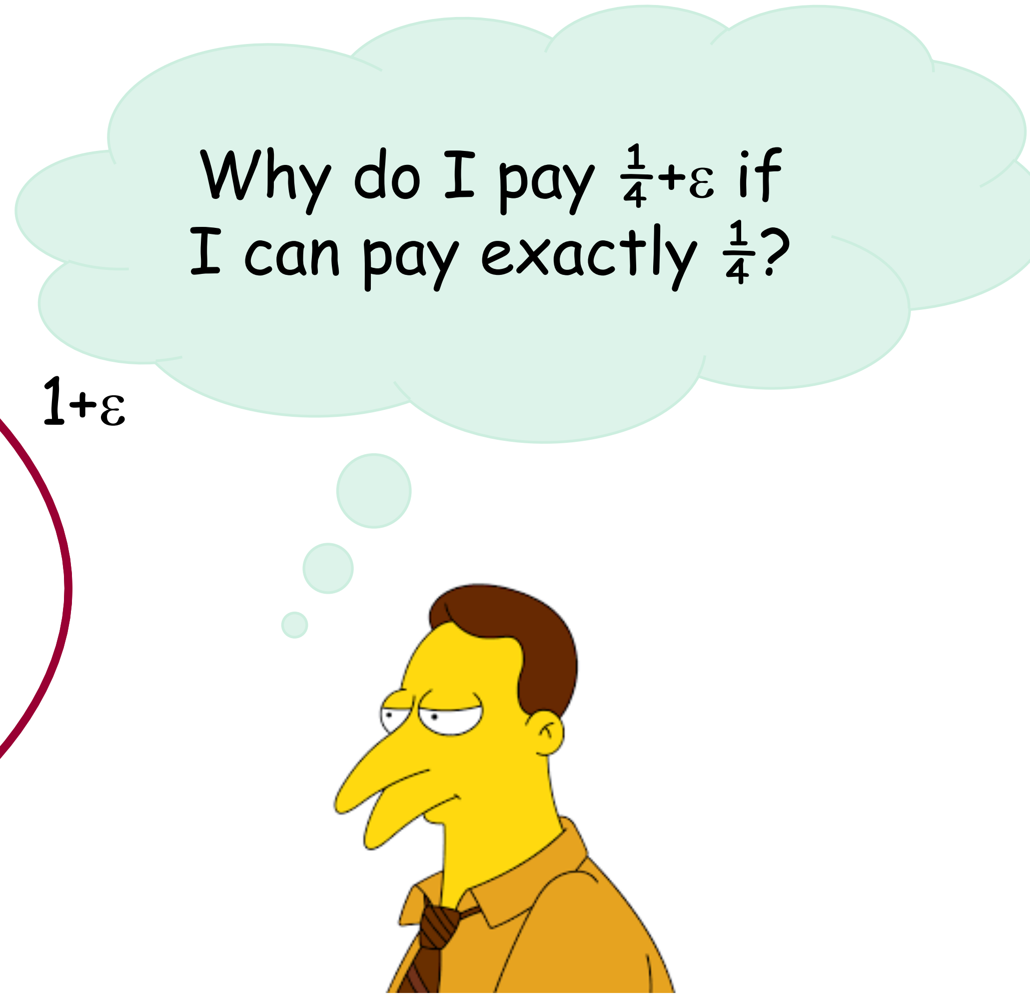
A profile that attains the social optimum:

But this is not an NE!



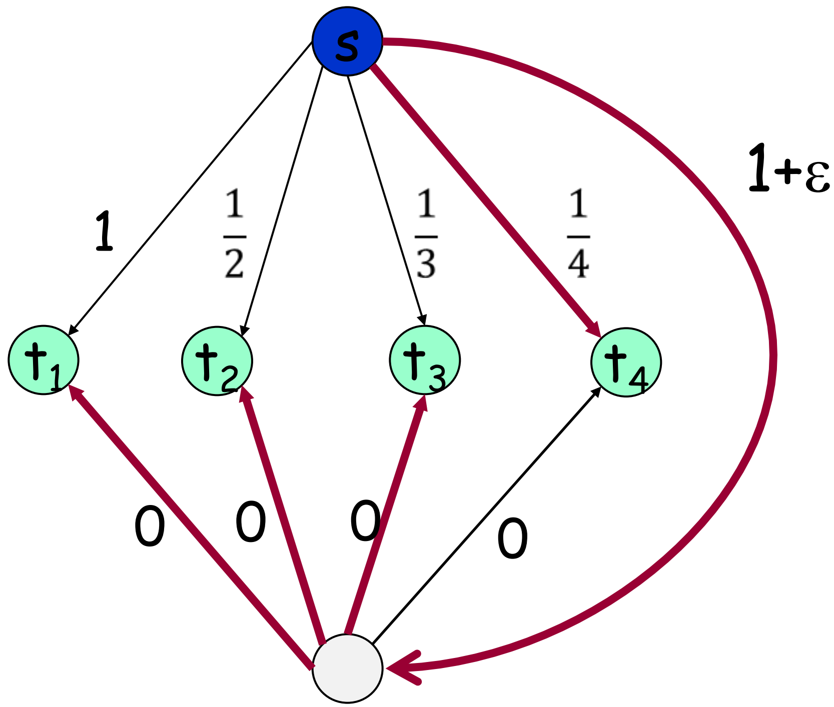
Does there always exist a good NE?





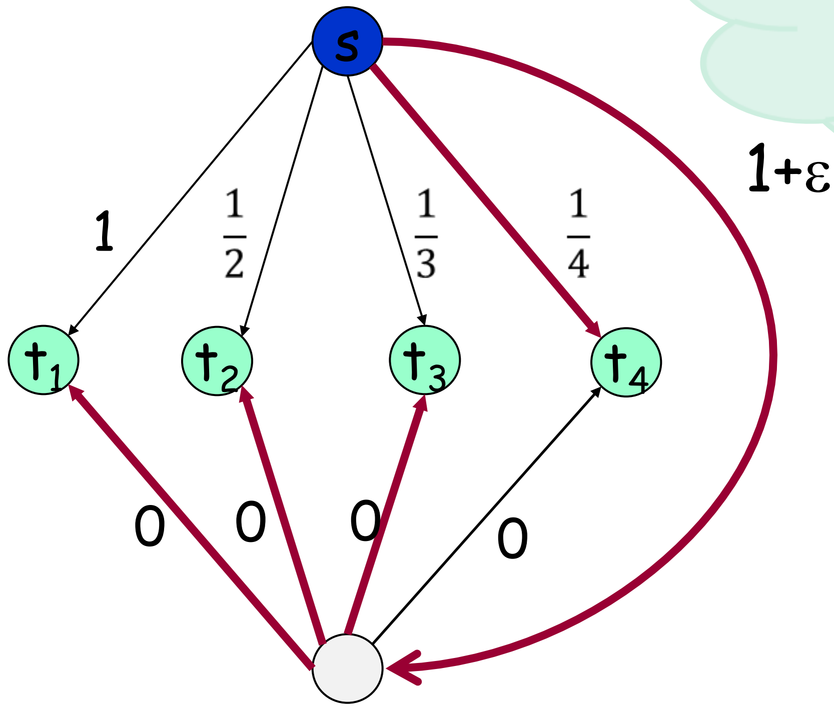
Player 4

Does there always exist a good NE?



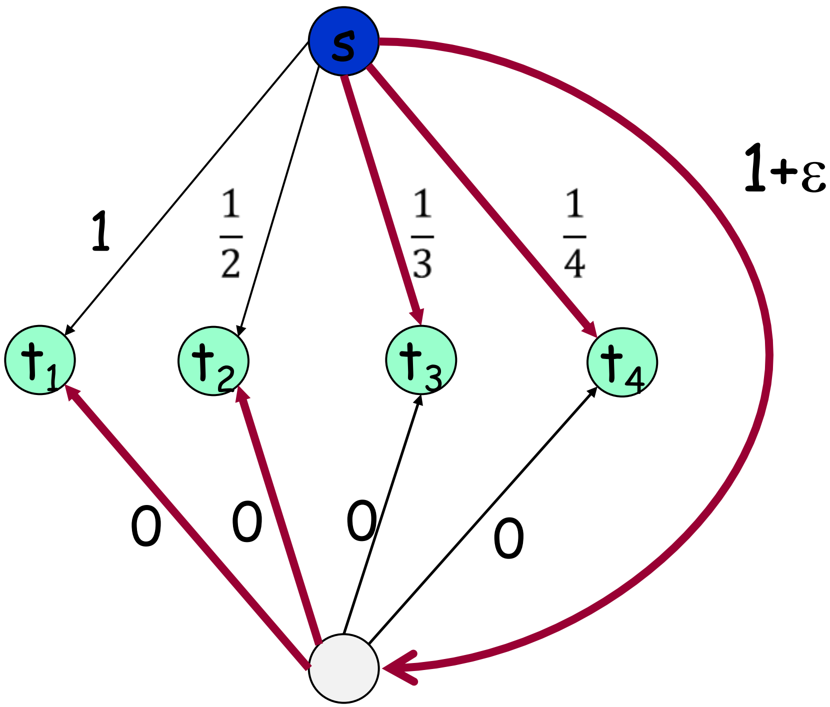
Does there always exist a good NE?

Why do I pay $\frac{1}{3}+\varepsilon$ if
I can pay exactly $\frac{1}{3}$?

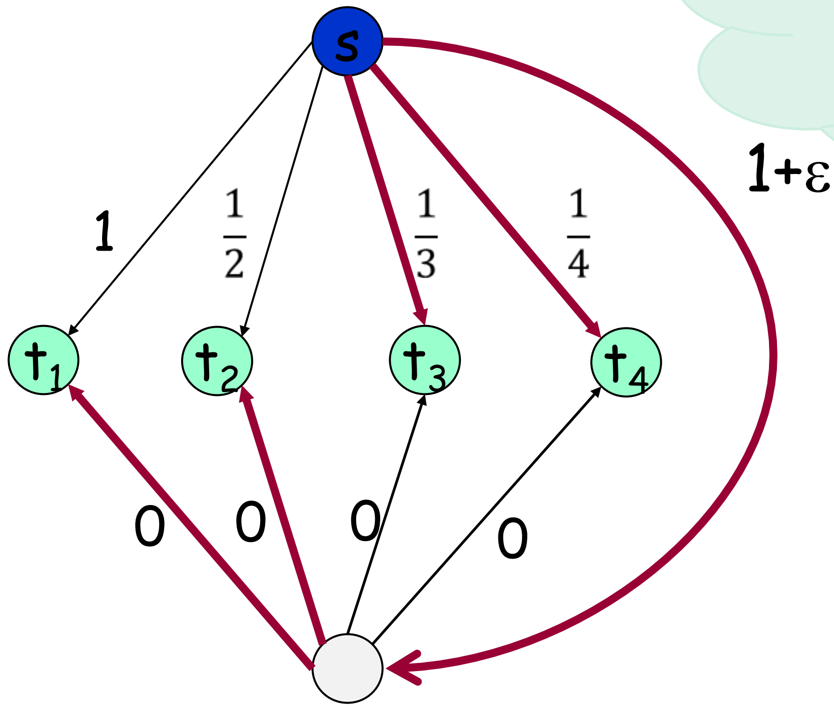


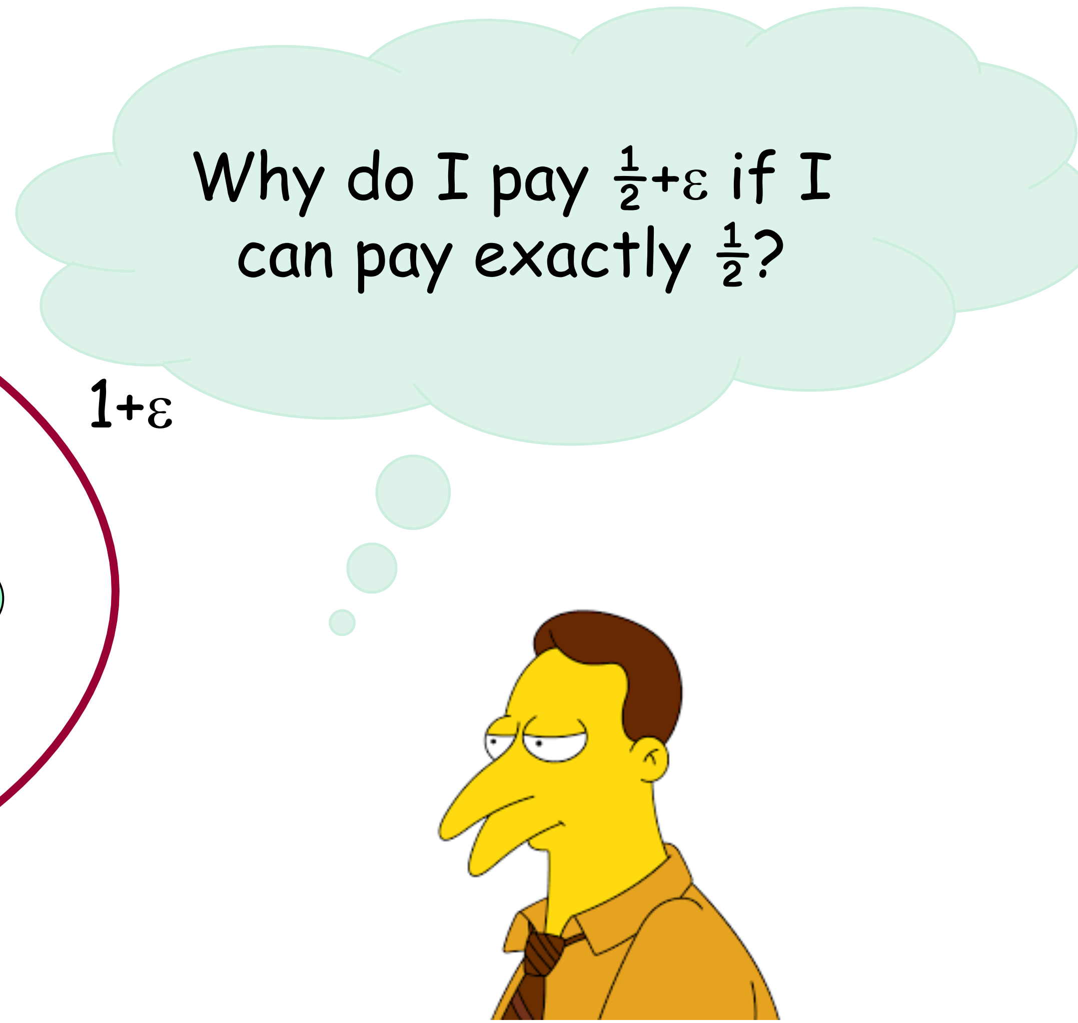
Player 3

Does there always exist a good NE?



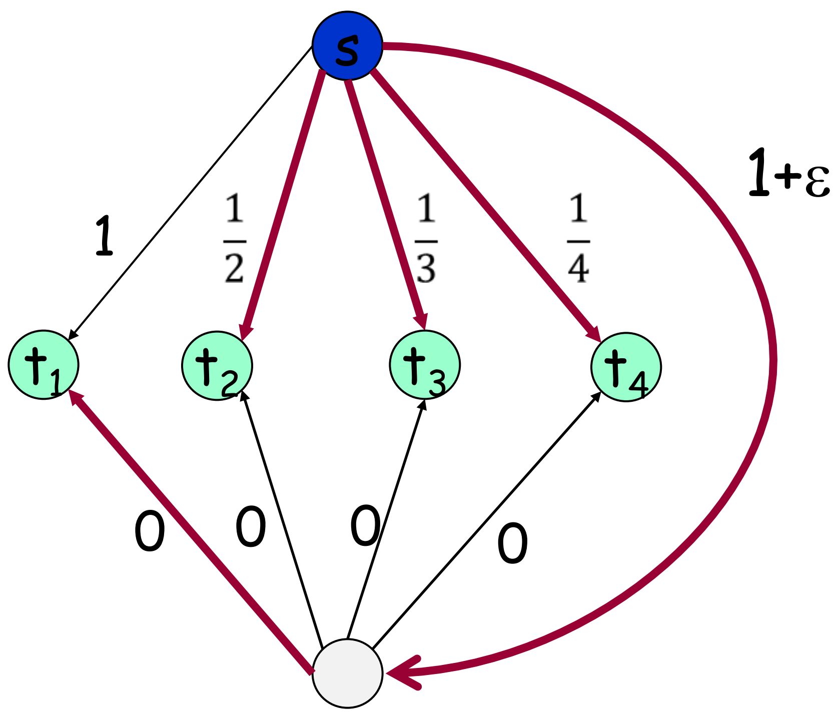
Does there always exist a good NE?



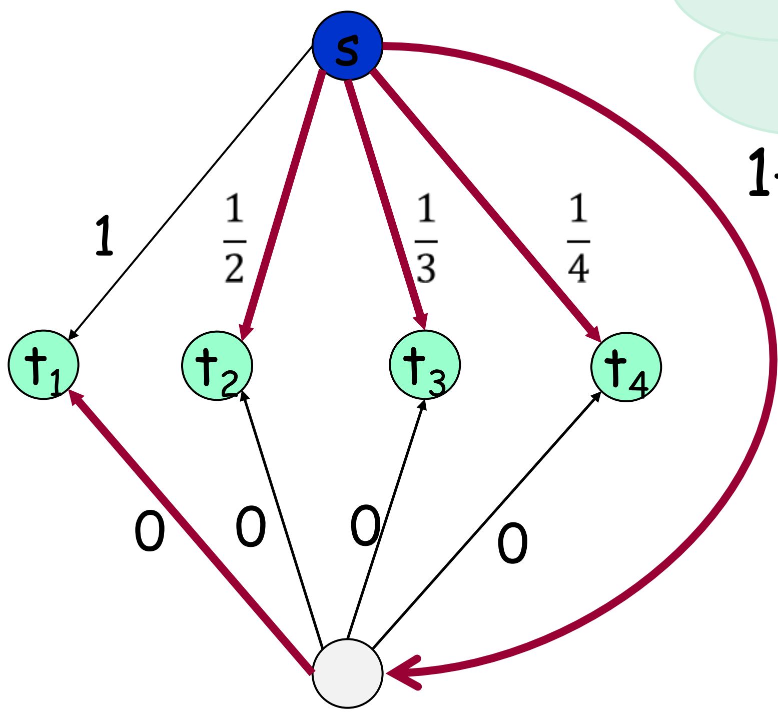


Player 2

Does there always exist a good NE?

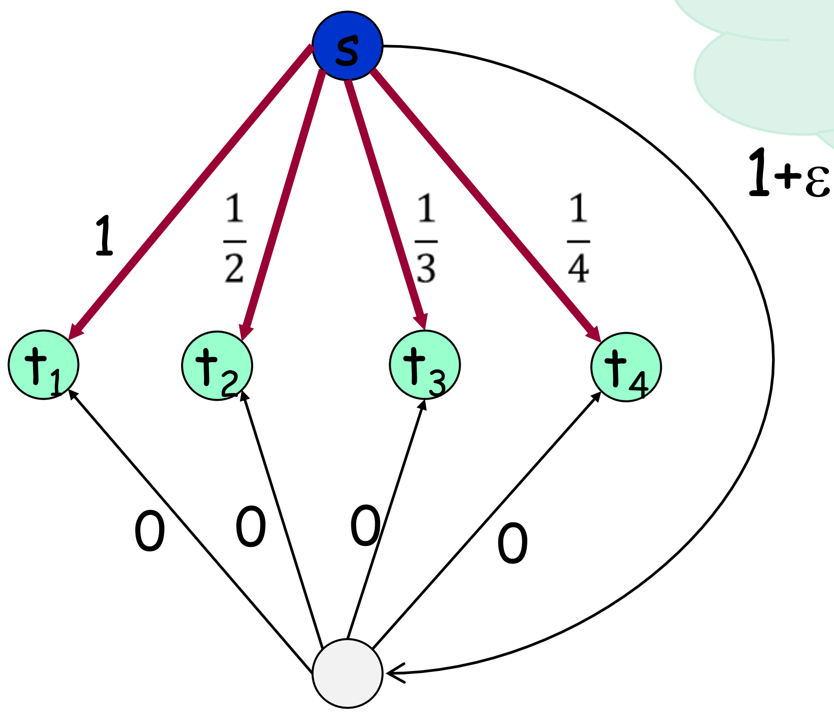


Does there always exist a good NE?



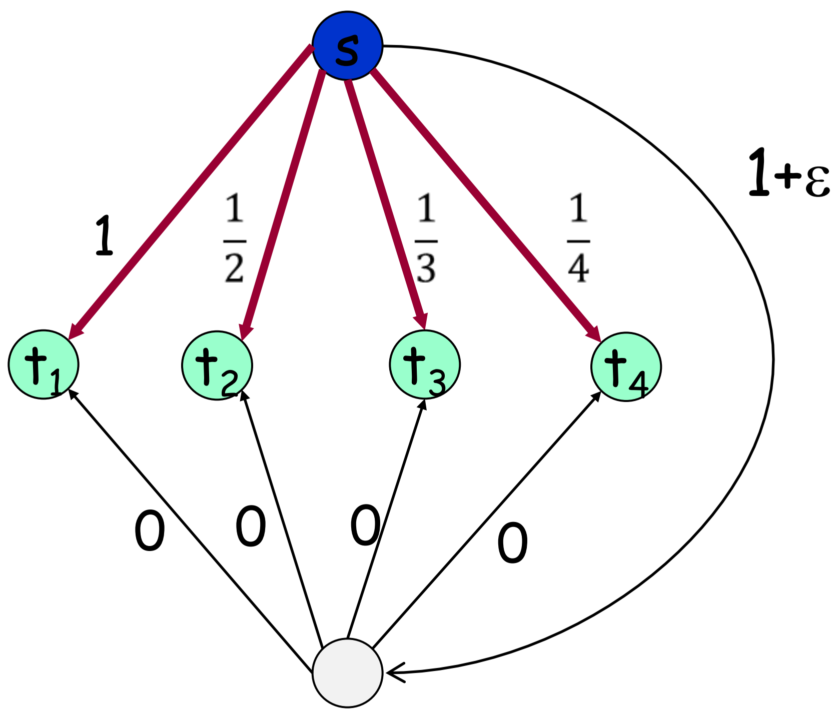
Player 1

Does there always exist a good NE?



Player 1

Does there always exist a good NE?

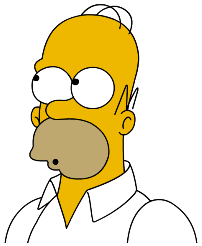


The price of the only stable (NE) profile:

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

There is no good NE!

So, network formation games:

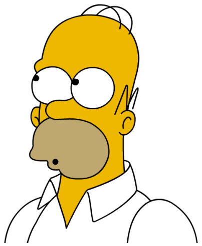


- Players have reachability objectives.
- Players that share a channel, share its cost.

- Nash Equilibrium (NE): a stable profile in which no player has an incentive to change his strategy – always exists in network formation games.
- Social Optimum (SO): a profile that minimizes the players' payments.

- Price of anarchy: worst NE/SO.
  PoA=k in network formation games.
- Price of stability: best NE /SO.
  PoS = $H_k \approx \log k$ in network formation games.

BTW: [Avni, Kupferman, Tamir, 2013]



- Players may have regular objectives (in a labeled network).
- Strategies: paths that need not be simple.
- Players that share a channel, share its cost proportionally.

- An NE need not exists
- PoS=PoA=k.
- ...

Back to Rational Synthesis

A stable (NE) profile $P=\langle f_0,\ldots,f_k\rangle$:

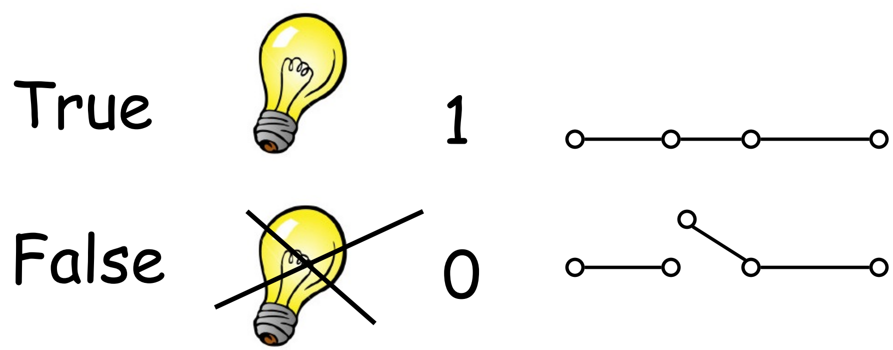
for every i, if φ_i is not satisfied in P, then φ_i is not satisfied also in $P[i \leftarrow f'_i]=\langle f_0,\ldots f'_i,\ldots,f_k\rangle$, for all alternative strategies f'_i for P_i.

The objectives are Boolean!

Network formation games: quantitative objectives!

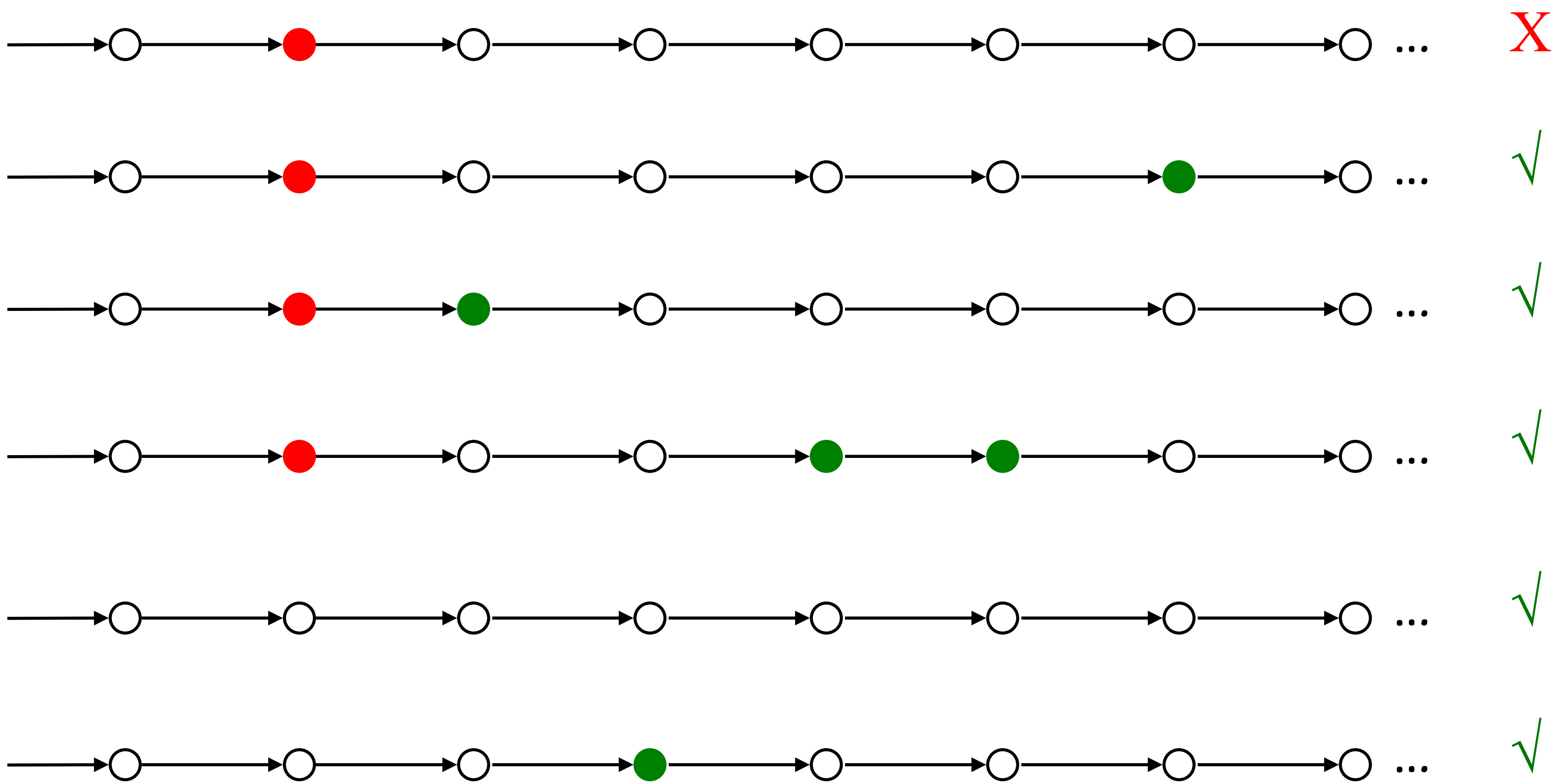
Logic is Boolean...

CS is Boolean...



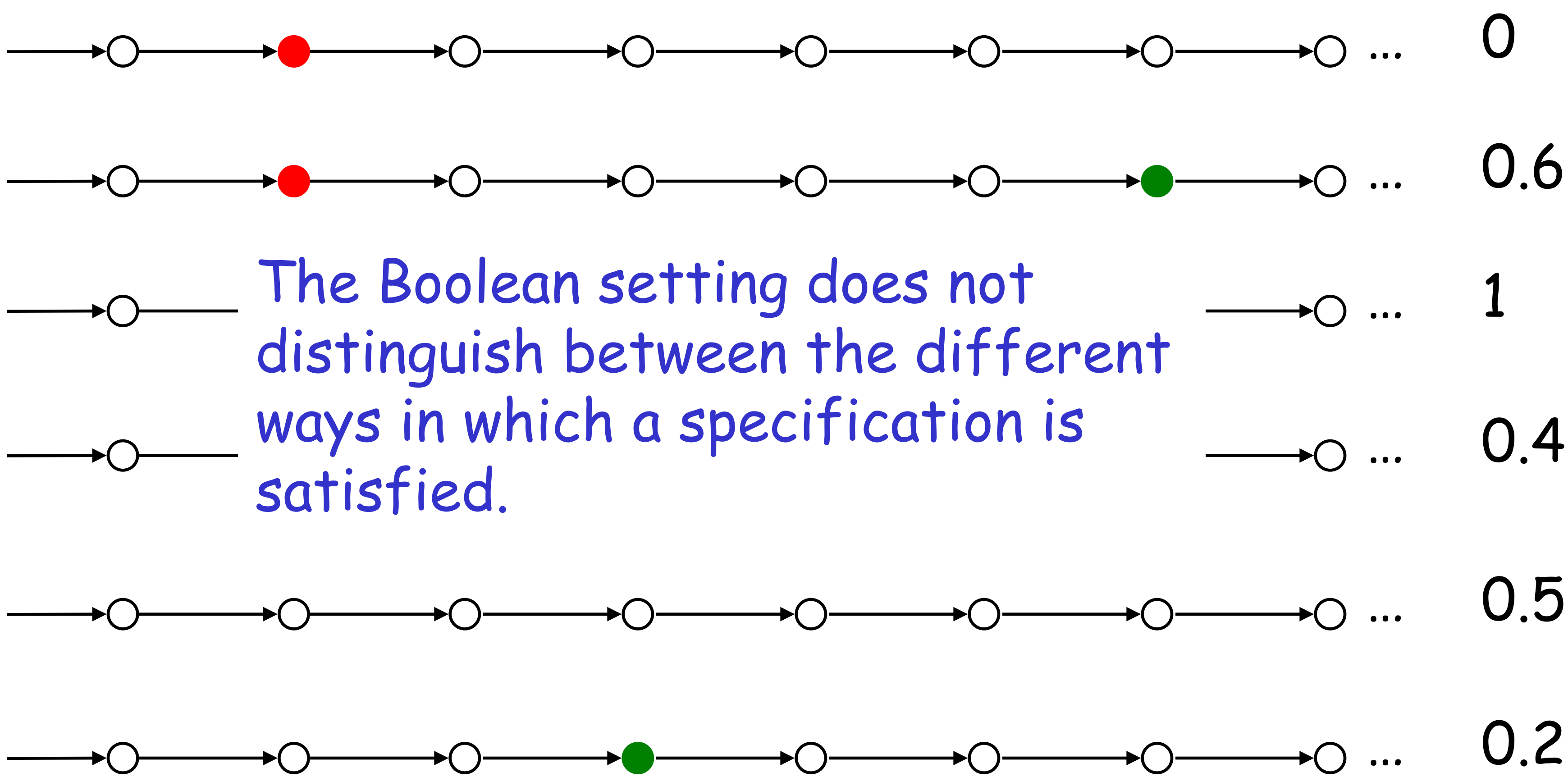
Is satisfaction really Boolean?

$\text{ALWAYS}(\textit{request} \rightarrow \text{EVENTUALLY } \textit{grant})$



Is satisfaction really Boolean?

$\text{ALWAYS}(\textit{request} \rightarrow \text{EVENTUALLY } \textit{grant})$



The Boolean setting does not distinguish between the different ways in which a specification is satisfied.

Behavioral quality: [Almagor,Boker,Kupferman 2014]

The logics LTL[F] and LTL[D]:
multi-valued extensions of LTL.

LTL[F]:

The satisfaction value of an LTL[F] formula is in [0,1].

0: "very bad". 1: very good.

F: a set of propositional-quality operators.

A k-ary operator $f:[0,1]^k \to [0,1]$

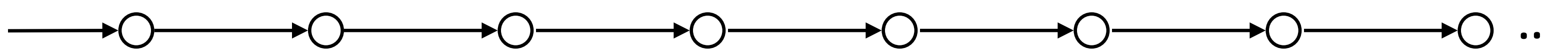
Examples: $x \wedge y$ min(x,y), $x \vee y$ max(x,y), $\neg x$ 1-x

Semantics of LTL[F]:

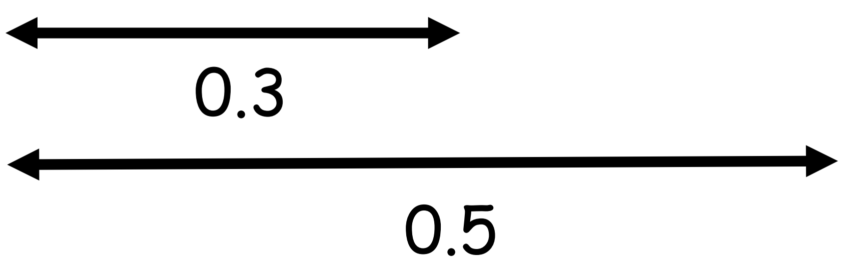
$[[\pi,\psi]]$: the satisfaction value of ψ in π.

Indeed only finitely many possible values

$$[[\pi, \varphi_1 U \varphi_2]] = \max_{i \geq 0} \{\min\{[[\pi^i, \varphi_2]], \min_{i > j \geq 0} \{[[\pi^j, \varphi_1]]\}\}\}$$

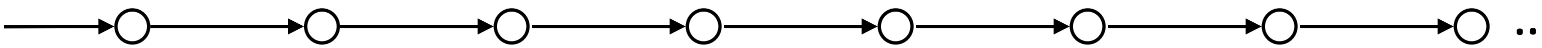


φ_2	0	0	0.3	0	0.6	0	0.8	0
φ_1	0.5	0.5	0.5	0.5	0.7	0.5	0.5	0.5



$$[[\pi, \varphi_1 U \varphi_2]] = \bigvee_{i \geq 0} \{ \bigwedge\{[[\pi^i, \varphi_2]], \bigwedge_{i > j \geq 0} [[\pi^j, \varphi_1]]\}\}$$

$$[[\pi,\varphi_1 U \varphi_2]] = \max_{i\geq 0} \{\min\{[[\pi^i, \varphi_2]], \min_{i>j\geq 0} \{[[\pi^j, \varphi_1]] \}\}\}$$



φ_2	0	0	0.3	0	0.6	0	0.8	0
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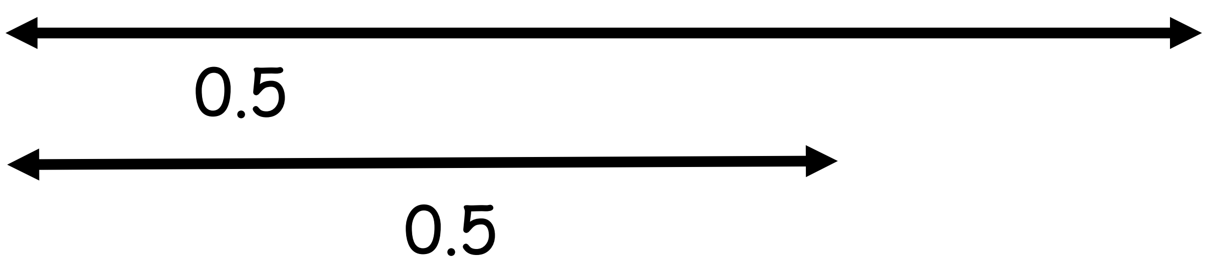


$$[[\pi,\varphi_1 U \varphi_2]] = \bigvee_{i\geq 0} \{ \bigwedge\{[[\pi^i, \varphi_2]], \bigwedge_{i>j\geq 0} [[\pi^j, \varphi_1]] \}\}$$

Two useful quality operators:

For a parameter λ in [0,1]:

$$[[\pi, \nabla_\lambda \varphi]] = \lambda \cdot [[\pi, \varphi]].$$

$$[[\pi, \varphi_1 \oplus_\lambda \varphi_2]] = \lambda \cdot [[\pi, \varphi_1]] + (1-\lambda) \cdot [[\pi, \varphi_2]].$$

Prioritize different behaviors

$\varphi_1 \vee \nabla_{3/4}\, \varphi_2$:

If φ_1 holds, the satisfaction value is 1.
If only φ_2 holds, the satisfaction value is ¾.
If none of them holds, the satisfaction value is 0.

Consider $G(p \rightarrow Xq \vee XXq)$.

LTL[F] variants:

$$G(p \rightarrow Xq \vee \nabla_{1/2} XXq)$$

Two q's: 1
Only the first: 1
Only the second: $\frac{1}{2}$

$$G(p \rightarrow Xq \oplus_{3/4} XXq)$$

Two q's: 1
Only the first: $\frac{3}{4}$
Only the second: $\frac{1}{4}$

Back to Rational Synthesis

A stable (NE) profile $P=\langle f_0,\ldots,f_k\rangle$:

for every i, if $[[P,\varphi_i]]=v$, then $[[P',\varphi_i]] \leq v$ for all profiles $P'=P[i \leftarrow f'_i]$.

Consider a profile $P=\langle f_0,\ldots,f_k\rangle$.

utility(P) = sum of satisfaction values =

$=[[P,\psi]]+[[P,\varphi_1]]+ \ldots + [[P,\varphi_k]]$.

- SO: max P {utility(P)}.
- PoS: SO/ utility of best NE.
- PoA: SO / utility of worst NE.

Note: in NFG it was dual

What are they in rational synthesis?

Cooperative vs. Non-cooperative RS

PoS vs. PoA

Input: objectives ψ and $\varphi_1,\ldots,\varphi_k$.

Cooperative rational synthesis:

Output: a stable profile $\langle f_0,\ldots,f_k \rangle$ that satisfies ψ.

best NE!

Non-cooperative rational synthesis:

Output: a strategy f_0 such that every stable profile $\langle f_0,\ldots,f_k \rangle$ satisfies ψ.

worst NE!

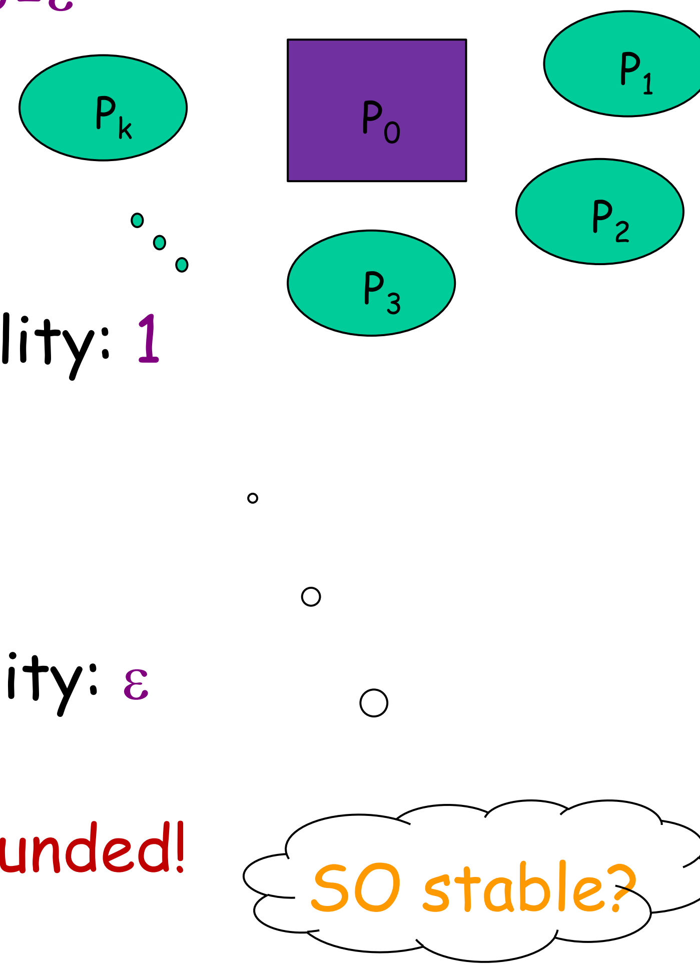
What are the prices of stability and anarchy in rational synthesis?

Price of Anarchy:

$P_1,\ldots,P_k$ assign values to $x_1,\ldots,x_k$

$\varphi_1,\ldots,\varphi_{k-1}$: $\varphi_i = \nabla_\alpha (x_i \wedge \neg x_k)$ $\alpha = (1-\varepsilon)/k-1$

$\varphi_k = \nabla_\beta(x_k \vee (x_1 \wedge x_2 \wedge \ldots \wedge x_{k-1}))$ $\beta = \varepsilon$



SO: T T T … T F

$\varphi_1,\ldots,\varphi_{k-1}$: $(1-\varepsilon)/k-1$ φ_k: ε utility: 1

Worst NE: F F F … F T

$\varphi_1,\ldots,\varphi_{k-1}$: 0 φ_k: ε utility: ε

PoA: SO/worst NE = $1/\varepsilon$ -- unbounded!

Price of Anarchy:

$P_1,\ldots,P_k$ assign values to $x_1,\ldots,x_k$

$\varphi_1,\ldots,\varphi_{k-1}$: $\varphi_i = \nabla_\alpha(x_i \wedge \neg x_k)$ $\alpha = (1-\varepsilon)/k-1$

$\varphi_k = \nabla_\beta(x_k \vee (x_1 \wedge x_2 \wedge \ldots \wedge x_{k-1}))$ $\beta = \varepsilon$

SO is stable --> SO is best NE.

SO: T T T ... T F

best/worst NE is unbounded.

$\varphi_1,\ldots,\varphi_{k-1}$: $(1-\varepsilon)/k-1$ φ_k: ε utility: 1

Cooperative RS may be unboundedly better than non-cooperative RS!

Worst NE: F F F ... F T

$\varphi_1,\ldots,\varphi_{k-1}$: 0 φ_k: ε utility: ε

PoA: SO/worst NE = $1/\varepsilon$ -- unbounded!

Price of Stability:

$P_1,\dots,P_k$ assign values to $x_1,\dots,x_k$

$\varphi_1,\dots,\varphi_{k-1}$: $\varphi_i = \nabla_\alpha (x_1 \wedge x_2 \wedge \dots \wedge x_{k-1} \wedge x_k)$ $\alpha = (1-\varepsilon)/k-1$

$\varphi_k = \nabla_\beta (x_1 \wedge x_2 \wedge \dots \wedge x_{k-1} \wedge \neg x_k)$ $\beta = \varepsilon$

SO: T T T … T stable?

no!

$\varphi_1,\dots,\varphi_{k-1}$: $(1-\varepsilon)/k-1$ φ_k: 0 utility: $1-\varepsilon$

Best NE: T T T … T F

$\varphi_1,\dots,\varphi_{k-1}$: 0 φ_k: ε utility: ε

PoS: SO/best NE = $(1-\varepsilon)/\varepsilon$ -- unbounded!

To Sum Up:

- Synthesis of open systems: winning strategy in a zero-sum game.

- Rationality assumption on the environment. Transition to non-zero-sum game.

- Classical game theory: quantitative utilities. Price of stability, price of anarchy.

- LTL[F]: quantitative specifications.

- Cooperative rational synthesis: PoS, unbounded.

- Non-cooperative rational synthesis: PoA, unbounded.

We did not see:

- Solving rational synthesis: connection with strategy logic.

- Rational verification: does S satisfy ψ in every rational? [Wooldridge, Gutierrez, Harrenstein, Marchioni, Perelli 2016]

- Fixing systems by making them stable.

- Richer settings: incomplete information, probability, other solution concepts.

Thank you