Homework CAO – Series C, November, 2010

Due: Turn in your solutions by 6 December, 11 p.m. Make precisely 4 out of 8 problems.

Problem 1. Let D be a symmetric and positive definite $n \times n$-matrix and let $d \in \mathbb{R}^n$. Consider the optimization problem (P): minimize $\frac{1}{2}x^t Dx + d^t x$ over all $x \in \mathbb{R}^n$ with $Ax \leq b$. Here A is an $m \times n$-matrix and $b \in \mathbb{R}^m$. Determine the Lagrangian dual of (P) and show that it can be expressed in the following form: maximize $\frac{1}{2}u^t Mu + u^t c - \frac{1}{2}d^t D^{-1}d$ for a suitable choice of the matrix M and the vector c.

Problem 2. Exercise 2.2 of "Perturbational duality (continued) ..."

Problem 3. Exercise 2.4 of "Perturbational duality (continued) ..."

Problem 4. Exercise 2.2 of "On generalized gradients and optimization"

Problem 5. Exercise 2.3 of "On generalized gradients and optimization"

Problem 6. Exercise 2.5 of "On generalized gradients and optimization"

Problem 7. Exercise 2.6 of "On generalized gradients and optimization"

Problem 8. Let $p : \mathbb{R}^n \to (-\infty, +\infty]$ be positively homogeneous and subadditive. Prove: there exists a closed convex set $C \subset \mathbb{R}^n$ such that $p(x) = \sup_{\xi \in C} \xi^t x$ for each $x \in \mathbb{R}^n$. *Hint:* It follows that the desired C must satisfy $p = \chi_C^*$.