

**Alternative take home exercise, to be handed in November 18**

**Exercise.**

We consider the differential operator  $P = -\Delta + e^{-\|x\|^2}$ , where

$$\Delta = \sum_{j=1}^n \frac{\partial^2}{\partial x_j^2}$$

denotes the Laplacian on  $\mathbb{R}^n$ .

- (a) Determine a symbol  $p \in S^2(\mathbb{R}^n)$  such that  $P = \Psi_p$ . Do not forget to show that  $p$  belongs to  $S^2(\mathbb{R}^n)$ .
- (b) Show that the operator  $P$  is properly supported.
- (c) Show that the function  $q(x, \xi) := (1 + \|\xi\|^2)^{-1}$  defines an element of  $S^{-2}(\mathbb{R}^n)$ .

Let  $\chi \in C_c^\infty(\mathbb{R}^n)$  and let  $\chi' \in C_c^\infty(\mathbb{R}^n)$  be such that  $\chi' = 1$  on an open neighborhood of  $\text{supp } \chi$ . Put  $Q = M_{\chi'} \circ \Psi_q \circ M_\chi$ .

- (d) Show that  $Q \in \Psi^{-2}$  and that  $Q$  is properly supported.
- (e) Show that

$$Q \circ P - M_\chi \in \Psi^{-1}(\mathbb{R}^n).$$

Hint: use the principal symbol.