"Many particle systems out of equilibrium" Problems, Series 3, 2006-07.

Problem 8. Time correlation functions for the random flight model

a) Find the velocity autocorrelation function for the random flight model of problem (6) and calculate the diffusion constant from this.

b) Calculate the generalized diffusion coefficient $U(\boldsymbol{k}, z)$ (the results from problem (6) may be used) and show that the particle density satisfies the equation

$$\frac{\partial n(x,t)}{\partial t} = \frac{\partial^2}{\partial x^2} \int_0^t d\tau \, e^{-\nu\tau} v^2 n(x, t-\tau). \tag{1}$$

What implicit assumption has been made about the initial velocity distribution?

c) What is the form of the function $C(\boldsymbol{k}, z)$? And what expression follows from this for the time correlation function $\langle (\hat{\boldsymbol{k}} \cdot \boldsymbol{v}_1(0))(\hat{\boldsymbol{k}} \cdot \boldsymbol{v}_1(t)) e^{-i\boldsymbol{k}\cdot(\boldsymbol{r}_1(t)-\boldsymbol{r}_1(0))} \rangle$?

Problem 9. Random flight with persistence

We consider again the random flight model of problem (6), but now assume that on scattering the probability for scattering in the opposite direction is p and that for scattering in the same direction $1-p$. Calculate the velocity autocorrelation function and the diffusion coefficient for this model.