"Many particle systems out of equilibrium" Problems, Series 6, 2006-07.

Problem 15. Shear viscosity of a dilute gas

In this problem the shear viscosity of a dilute gas should be evaluated approximately from the Green-Kubo expression for this transport coefficient.

a) Argue why for a dilute gas the shear current occurring in the Green-Kubo expression may be approximated as

$$J^{sh} = \mathsf{P}_{xy} = \sum_i m v_{ix} v_{iy}. \tag{1}$$

b) Show that the collision frequency of a dilute system of hard spheres/disks is given by

$$\nu^{3d} = 4na^2\sqrt{\frac{\pi k_B T}{m}}, \qquad \nu^{2d} = 2na\sqrt{\frac{\pi k_B T}{m}}. \tag{2}$$

c) Show that the contribution to the Green-Kubo integral from $t = 0$ up to the first collision is given approximately by nk_BT/ν.

d) make plausible that contributions from the time stretches between subsequent collisions sum up approximately to a correction factor $1/(1-p)$, with p given by the ratio

$$p = \frac{\langle m^2(v'_{1x}v'_{1y} + v'_{2x}v'_{2y})(v_{1x}v_{1y} + v_{2x}v_{2y})\rangle}{\langle m^2(v_{1x}v_{1y} + v_{2x}v_{2y})^2\rangle},$$

with $\langle\rangle$ an average over the precollisional equilibrium distribution plus over the collision parameters of a collision between particles 1 and 2 with precollisional velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ and postcollisional velocities $\boldsymbol{v}'_1$ and $\boldsymbol{v}'_2$. Calculate p for both disks and spheres. (Hint: It helps reexpressing the velocities of the colliding particles in terms of CM and relative velocities. And in 3 dimensions the post-collisional distribution of the relative velocity is isotropic (compare the 3d Lorentz gas).) What are the final expressions for the shear viscosity?

e) What are the sources of corrections to these expressions? Which of these become vanishing in the limit of vanishing density?

f) What does one find for the bulk viscosity in the low-density limit?