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Fluctuations in the Motions of Mass and of Patterns in One-Dimensional Driven Diffusive Systems

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The stochastic spreading of mass fluctuations in systems described by a fluctuating Burgers equation increases as $t^{2/3}$ with time. As a consequence the stochastic motion of a mass front, a point through which no excess mass current is flowing, is shown to increase as $t^{1/3}$. The same is true for the stochastic displacement of mass points and shock fronts with respect to their average drift, provided the initial configuration is fixed. An additional average over the stationary distribution of the initial configuration yields stochastic displacements, increasing with time as $t^{1/2}$.

KEY WORDS: Burgers equation (fluctuating); diffusion (anomalous); density fluctuations.

1. INTRODUCTION

A simple model for one-dimensional driven diffusion is the asymmetric simple exclusion model. In this model the sites of a one-dimensional lattice are either empty or occupied by a single particle and the particles may jump to empty neighboring sites with jump rates $p\Gamma$ and $(1-p)\Gamma$ for jumps to the right and to the left, respectively. To be specific I will assume $1/2 < p \leqslant 1$. The simplest stationary state for this system is given by the product measure, for which the probability of finding any site occupied is c, the average occupation number, independently of the occupation numbers (0 for an empty site, 1 for an occupied one) of all the other sites.

A number of results concerning the motions of particles as well as density patterns in this system have been obtained. The average motion of a tagged particle in the stationary state is a diffusive motion with

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diffusion constant $(p-\frac{1}{2})(1-c)\,\Gamma a^2$ around a drift with speed $v=(2p-1)(1-c)\,\Gamma a$.[1,2] The average is performed both over the random motion of the particles and over the random initial distribution of the untagged particles. On the other hand, if one fixes the initial configuration, the tagged particle, on a length scale that is scaled by $t^{1/2}$ with time, for $t\to\infty$ does not fluctuate around its average drift, as determined by the initial configuration.[3]

Similarly, the position of a shock wave, if averaged over both stochastic dynamics and initial distribution, performs a diffusive motion around an average drift, whereas for a fixed initial configuration its motion with respect to the average drift induced by the initial configuration becomes fixated for $t\to\infty$ on a length scale $\sim t^{1/2}$.[3]

Density fluctuations move at an average drift speed[2] $w=(\partial/\partial c)(cv)=(2p-1)(1-2c)\,\Gamma a$. The spreading of such a density fluctuation around its average drift is faster than diffusive; it increases as $t^{2/3}$, if one does a combined averaging over the stochastic dynamics and the initial distribution. This was shown nonrigorously by applying renormalization group methods[4] or a mode coupling expansion[5] to the fluctuating Burgers equation, which provides a macroscopic description of the asymmetric simple exclusion process. For fixed initial configurations the spreading of a density profile, to my knowledge, has not been studied before.

A microscopic way of describing these density fluctuations is the addition of so-called second class particles and holes (empty sites), which move and evolve in such a way that the motion of the remaining (first class) particles and holes remains completely unchanged, while the evolution of the system as a whole remains that of the simple exclusion model.[6] That is: second class particles may jump to neighboring empty sites at the same jump rates as the first class particles, a first class particle may change positions with a neighboring second class particle in the same way as though the latter were an empty site. If a second class particle and a second class hole exchange places, both are promoted to first class objects of the same kind as before. Finally, if a second class particle and a second class hole are on neighboring sites, the pair may be promoted to first class objects without exchanging positions at the rate $(1-p)\Gamma$, respectively $p\Gamma$, if the particle is to the left, respectively to the right, of the hole. The average motion of a single second class particle or hole is precisely that of a density fluctuation. However, if one has a finite density of them, the motion of a single second class object also depends on its interaction with the other ones. For a constant density of second class

[2] Compare to the case of ripples on a water surface, where the velocity of the ripples may be quite different from the velocity of the water.

particles, with no second class holes, the motion of a single second class particle is expected to have similar characteristics as the motion of a first class particle: diffusion around an average drift if one averages over dynamical fluctuations and initial conditions both, subdiffusive behavior if one fixes the initial configuration.[7]

In this paper I will use the methods and results of ref. 5 to investigate the properties summarized above. It will turn out that the diffusive behavior obtained for combined averages is entirely due to fluctuations in the mass train passing some given point, due to the fluctuations in the density pattern passing this point. If one fixes the initial configuration, or alternatively lets the point through which the mass train passes move with the average speed at which the density pattern moves, the fluctuations in the passing mass train are strongly reduced. Then one finds that the motion of a first class particle, or that of a second class particle constrained to move between second class neighbors at distances that on average remain constant in time, shows a spreading around the average drift, increasing with time as $t^{1/3}$. A simple physical argument allows one to relate this to the $t^{2/3}$ spreading of a density fluctuation.

2. FLUCTUATIONS IN THE MOTION OF MASS

Macroscopically the time evolution of the systems of interest to us can be described by the fluctuating Burgers equation

$$\frac{\partial \rho(x, t)}{\partial t} = \frac{-\partial \rho^2(x, t)}{\partial x} + D \frac{\partial^2 \rho(x, t)}{\partial x^2} - \frac{\partial \tilde{\jmath}(x, t)}{\partial x} \tag{1}$$

where $\rho(x, t)$ is the deviation at position x and time t from an average density. This equation describes the system in a coordinate system moving with the pattern velocity w and employs dimensionless variables. The random current $\tilde{\jmath}(x, t)$ is assumed to be Gaussian noise with variance satisfying

$$\langle \tilde{\jmath}(x, t)\, \tilde{\jmath}(x', t')\rangle = 2D\delta(t-t')\{\langle \rho(x, t)\, \rho(x', t)\rangle - \langle \rho(x, t)\rangle \langle \rho(x', t)\rangle\} \tag{2}$$

where the brackets denote an average over the stochastic dynamics. To apply this to the simple exclusion process, one has to choose for x and t the dimensionless variables x/a and $(2p-1)\, c\Gamma at$, respectively, in which case D in Eq. (1) takes the value $(1-c)/2c$.[5] In addition ρ is to be interpreted as the coarse-grained local deviation of the particle density from its average value c. In ref. 5, Eq. (1) was solved iteratively by applying a Fourier transform with respect to x, treating the nonlinear term $-\partial\rho^2/\partial x$ formally as a perturbation to the other two terms, and using (2). From the structure of the resulting equation it then followed that the average of the time correlation function of the Fourier transform of the density $\hat{\rho}(k, t)$ for long times and small k is of the form

$$\langle\langle \hat{\rho}(k, t)\, \hat{\rho}(k', 0)\rangle\rangle = 2\pi G(kt^{2/3})\, S\delta(k+k') \tag{3}$$

with $\lim_{x\to 0} G(x)=1$, $\lim_{x\to\infty} G(x)=0$. Now the inner brackets denote the average over the stochastic dynamics and the outer brackets that over the stationary Gaussian distribution for $\rho(x, 0)$. In order to obtain these results, one has to impose the condition that in the static correlation function $\langle\langle \hat{\rho}(k, 0)\, \hat{\rho}(k', 0)\rangle\rangle = S\delta(k+k')$ the factor S can be treated as a constant for small k [for the simple exclusion model one has in fact $S(k)= c(1-c)$ for all k].

Now consider the "mass" flowing through an interface moving at a constant speed s. Its current satisfies the continuity equation

$$\begin{aligned}\frac{\partial}{\partial t}\rho(x+st, t) &= -\frac{\partial}{\partial x} j_s(x, t)\\ &\equiv -\frac{\partial}{\partial x}\{j(x+st, t) - s\rho(x+st, t)\}\end{aligned} \tag{4}$$

where $j(x, t)$ and $j_s(x, t)$ are the current densities in a resting coordinate frame and in a coordinate frame moving with the speed of the interface, respectively. After subtracting from $j_s(x, t)$ its average value $\langle\langle j_s\rangle\rangle$ [for the simple exclusion model $\langle\langle j_s\rangle\rangle$ has the value $c(v-s)$, but for what follows this is not important] one finds its Fourier transform $\hat{j}_s(k, t)$ to be well-behaved at $k=0$. Equation (4) may then be represented as a Fourier integral, and an integration over time yields the result

$$M(x(t), t) \equiv \int_0^t d\tau\, \{j_s(0, \tau) - \langle\langle j_s\rangle\rangle\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} dk\, \frac{1}{ik}\{\hat{\rho}(k, 0) - e^{ikx(t)}\hat{\rho}(k, t)\} \tag{5}$$

where $x(t)=st$ is the position of the interface at time t. Obviously, $M(x(t), t)$ is the excess mass flowing through an interface moving from $x=0$ to $x=x(t)$ during the time period from 0 to t, and in fact the actual way in which this motion is performed is irrelevant. In the simple exclusion process this excess mass flow may be related to fluctuations in the positions of particles in the neighborhood of the interface. Let the particles be numbered sequentially $\cdots -1, 0, 1 \cdots$ from the left to the right and let $n_s(t)$ be the number of the rightmost particle to the left of the interface. Then $M_s(t) = -(n_s(t) - \langle\langle n_s(t) \rangle\rangle)$. However, since for all times the average spacing between neighboring particles is $1/c$, the position with respect to the interface of the particle labeled $ent(\langle\langle n_s(t) \rangle\rangle)$ equals $M_s(t)/c$, and if $|M_s(t)| \gg 1$, the fluctuations in this relative position are small, of $O(|M_s(t)|^{1/2})$. The same observation has been made before, in slightly different form, by Alexander and Pincus.[9] The right-hand side of (5) may be used to calculate the variance of M_s. If one averages both over the stochastic dynamics and the initial distribution, one obtains, with the aid of (3),

$$\begin{aligned}\langle\langle M_s^2(t) \rangle\rangle &= (S/\pi) \int_{-\pi}^{\pi} dk\, [1 - \cos(kst)\, G(kt^{2/3})]/k^2 \\ &= (Sst/\pi) \int_{-\pi st}^{\pi st} dx\, [1 - \cos xG(x/st^{1/3})]/x^2 \\ &= Sst[1 + O(s^{-2}t^{-2/3})] \end{aligned} \tag{6}$$

For $s \neq 0$ this implies that for long times (i.e., $t \gg s^{-3}$) the fluctuation in the excess mass passed through the interface increases as $t^{1/2}$. In particular, if in the simple exclusion process one considers the displacement $\Delta x(t)$ of a first class particle with respect to its average drift vt, one has to set $s = v - w = c$ and $S = c(1 - c)$. This yields, in unscaled units,

$$\langle\langle (\Delta x(t))^2 \rangle\rangle = (1 - c)(2p - 1)\, \Gamma a^2 t \tag{7}$$

in agreement with the results of refs. 1 and 2. For $s = 0$, substitution of $x = kt^{2/3}$ in (6) yields

$$\langle\langle M_s^2(t) \rangle\rangle = \frac{St^{2/3}}{\pi} \int_{-\pi t^{2/3}}^{\pi t^{2/3}} dx\, [1 - G(x)]\, x^2 \tag{8}$$

Hence the excess mass flow through an interface moving at speed w [corresponding to zero speed in the coordinate frame of (1)] increases with time as $t^{1/3}$. To obtain the average variance of $M_s(t)$ for fixed initial configurations one may consider two different realizations $\rho_1(x, t)$ and $\rho_2(x, t)$ that start out from the same initial configuration, but have independent stochastic currents $\tilde{j}_1(x, t)$ and $\tilde{j}_2(x, t)$. Then the difference between the mass flows in 1 and 2 satisfies the identity

$$\begin{aligned}&\langle\langle [M_1(x, t) - M_2(x, t)]^2 \rangle\rangle \\ &\quad = \langle\langle \{[M_1(xt) - \langle M_1(xt) \rangle] - [M_2(x, t) - \langle M_2(x, t) \rangle]\}^2 \rangle\rangle \\ &\quad = 2\langle\langle [M(x, t) - \langle M(x, t) \rangle]^2 \rangle\rangle \end{aligned} \tag{9}$$

Inserting (5), one may derive

$$
\begin{aligned}
&\langle\!\langle [M_1(x,t)-M_2(x,t)]^2 \rangle\!\rangle \\
&= \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi} dk \int_{-\pi}^{\pi} dk' \frac{-1}{kk'} \\
&\quad \times \langle\!\langle \{\hat\rho_1(k,0)-e^{ikx}\hat\rho_1(k,t)-\hat\rho_2(k,0)+e^{ikx}\hat\rho_2(k,t)\} \\
&\quad \times \{\hat\rho_1(k',0)-e^{ik'x}\hat\rho_1(k',t)-\hat\rho_2(k',0)+e^{ik'x}\hat\rho_2(k',t)\}\rangle\!\rangle \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi} dk \frac{1}{k^2} \langle\!\langle \hat\rho_1(k,t)\,\hat\rho_1(-k,t)-\hat\rho_1(k,t)\,\hat\rho_2(-k,t)\rangle\!\rangle \\
&= \frac{S}{\pi}\int_{-\pi}^{\pi} dk \frac{1}{k^2}\{1-H(kt^{2/3})\} \\
&= \frac{St^{2/3}}{\pi}\int_{-\pi t^{2/3}}^{\pi t^{2/3}} d\kappa \frac{1}{\kappa^2}\{1-H(\kappa)\} \qquad (10)
\end{aligned}
$$

Here I used the identity $\hat\rho_1(k,0)=\hat\rho_2(k,0)$ and introduced $H(k,t)=\langle|\langle\hat\rho_1(k,t)\rangle|^2\rangle/S$. Further I used that for large t, $H(k,t)$ satisfies the same scaling behavior as the function G introduced in (3), as follows from a scaling analysis similar to the one performed in ref. 5.

The final result in (10) is independent of x! It confirms the finding of ref. 3 that for fixed initial configuration the fluctuation in the displacement of a tagged particle scaled by $t^{1/2}$ goes to zero for $t\to\infty$. Comparing (10) and (6), one sees that both the fluctuations in the initial distribution and those caused by the dynamics give rise to fluctuations $\sim t^{1/3}$ in the mass flow through an interface at rest with respect to the pattern velocity. The calculation of the magnitude of these fluctuations is not simple and seems to require the summation of infinite graph expansions.

3. MOTION OF SHOCK FRONTS

A shock front is a sharply defined area between two homogeneous phases of different densities ρ_1 and ρ_2. The relative location of these two phases must be such that the mass flow toward the shock is larger than the mass flow away from it. For example, for the simple exclusion process with $p>1/2$ this means that the less dense phase has to be to the left of the denser phase. As a result of mass conservation the shock moves at an average speed

$$v_{sh}=\frac{\rho_2 v_2-\rho_1 v_1}{\rho_2-\rho_1}=\left(\frac{d(\rho v)}{d\rho}\right)_{\rho=\rho_1+\xi(\rho_2-\rho_1)}$$

with $0 < \xi < 1$. Hence v_{sh} is in between the pattern velocities of ρ_1 and ρ_2; seen from the density pattern in 1 it moves to the left at an average speed $v_{sh} - w_1$ and seen from the density pattern in 2 it moves to the right at average speed $v_{sh} - w_2$. Seen from the shock itself the front eats up both the density patterns coming toward it from the left and the right, and it will do it in such a way that the mass flows through interfaces somewhat to the left and to the right of it will be equal. So the following conclusions can be drawn: if one averages both stochastic dynamics and initial configurations, one finds excess mass currents fluctuating $\sim t^{1/2}$, hence the position of the shock front will also fluctuate as $t^{1/2}$ around its average value; if one fixes the initial configuration, the fluctuations on the mass flows, and hence in the position of the shock, will be $\sim t^{1/3}$. These predictions are in full agreement with the results of Gärtner and Presutti.[3]

4. FLUCTUATIONS IN THE EVOLUTION OF DENSITY PATTERNS

From (3) and the property

$$G(x) = 1 - \frac{\alpha}{2} x^2 + \mathcal{O}(x^4)$$

for x tending to zero, we may derive that the mean square displacement of the center of mass with respect to its average drift is given by[5]

$$\langle\!\langle [X(t) - X(0)]^2 \rangle\!\rangle = -\left\{ \frac{\partial^2}{\partial k^2} \log G(kt^{2/3}) \right\}_{k=0} = \alpha t^{4/3} \tag{11}$$

Similarly, one may obtain the average mean square displacement of the center of mass with respect to its average drift for fixed initial configuration by considering the difference in displacement between two independent stochastic evolutions starting from the same initial configuration. One finds

$$\begin{aligned} \langle\!\langle [X(t) - \langle X(t) \rangle]^2 \rangle\!\rangle &= \frac{1}{2} \langle\!\langle [X_1(t) - X_2(t)]^2 \rangle\!\rangle \\ &= -\frac{1}{2} \left\{ \frac{D^2}{Dk^2} \log \langle\!\langle n_1(k, t)\, n_2(-k, t) \rangle\!\rangle \right\}_{k=0} \\ &= t^{4/3} \lim_{x \to 0} \{1 - H(x)\}/x^2 \end{aligned} \tag{12}$$

So again the effects of initial fluctuations and those of dynamical fluctuations are of the same order in t.

Next I want to address the question of how a density change imposed at the initial time will evolve in time. In the simple exclusion process this will correspond to the time behavior of second class particles.

Let the total density be $\rho_1(x,t)+\rho_2(x,t)=\rho(x,t)$, where ρ_1 is the unperturbed density and ρ_2 the perturbation. Both ρ and ρ_1 satisfy (1). Hence by subtraction one obtains for ρ_2 the equation

$$\frac{\partial\rho_2}{\partial t}=-\frac{\partial}{\partial x}\{\rho_2(2\rho-\rho_2)\}+D\frac{\partial^2\rho_2}{\partial x^2}-\frac{\partial\tilde{j}_2}{\partial x} \tag{13}$$

with $\tilde{j}_2$ again Gaussian noise, satisfying

$$\langle\tilde{j}_2(x,t)\,\tilde{j}(x',t')\rangle$$
$$=2D\delta(t-t')\{\langle\rho_2(x,t)\,\rho(x',t)\rangle-\langle\rho_2(x,t)\rangle\langle\rho(x',t)\rangle\} \tag{14a}$$

$$\langle\tilde{j}_2(x,t)\,\tilde{j}_2(x',t')\rangle$$
$$=2D\delta(t-t')\{\langle\rho_2(x,t)\,\rho_2(x',t)\rangle-\langle\rho_2(x,t)\rangle\langle\rho_2(x',t)\rangle\} \tag{15b}$$

These equations are to be combined with Eqs. (1) and (2) for ρ. If one averages over stochastic dynamics and initial distribution $\rho(x,0)$, keeping $\rho_2(x,0)$ fixed, one readily finds that $\langle\!\langle\hat{\rho}_2(k,t)\rangle\!\rangle$ satisfies the relation

$$\langle\!\langle\rho_2(\hat{k},t)\rangle\!\rangle=G(kt^{2/3})\,\hat{\rho}_2(k,0) \tag{15}$$

which of source is no surprising result. As a consequence, the center of mass of an initially localized density disturbance with nonvanishing total mass will spread again $\sim t^{2/3}$. In particular, this means that the spatial distribution of a single second class particle inserted in a simple exclusion chain will spread according to the same power law. It is interesting to consider what happens to a localized initial disturbance of positive mass, corresponding to the insertion of a finite number of second class particles in a restricted region in the simple exclusion process. As an example, consider the case that N particles are distributed uniformly in the interval $[-l,l]$. According to (5) and (15), the average excess current through an interface at position x is given as

$$\langle\!\langle M(x,t)\rangle\!\rangle=\frac{1}{2\pi}\int_{-\pi}^{\pi}dk\,\frac{e^{ikx}}{ik}\,[1-G(kt^{2/3})]\,\hat{\rho}_2(k,0) \tag{16}$$

For the variance of this quantity one obtains

$$\langle\!\langle M^2(x,t)\rangle\!\rangle-\langle\!\langle M(x,t)\rangle\!\rangle^2$$
$$=\frac{-1}{(2\pi)^2}\int_{-\pi}^{\pi}dk\int_{-\pi}^{\pi}dk'\,\frac{e^{i(k+k')x}}{kk'}\,\{G^{(2)}(kt^{2/3},k't^{2/3})$$
$$-G(kt^{2/3})\,G(k't^{2/3})\}\,\hat{\rho}_2(k,0)\,\hat{\rho}_2(k',0) \tag{17}$$

Here I introduced the pair Green function $G^{(2)}$ through

$$\langle\langle \rho_2(k,t)\,\rho_2(k',t)\rangle\rangle = G^{(2)}(kt^{2/3}, k't^{2/3})\,\rho_2(k,0)\,\rho_2(k',0) \tag{18}$$

For large l one may distinguish two typical time regimes. For $t \ll l^{3/2}$ the relevant values of k and k' are $\gg l^{-1}$ and one approximately has $\hat{\rho}_2(k,0)\,\hat{\rho}_2(k',0) = 2\pi\delta(k+k')\,2lS_2$, where, for the simple exclusion process, $S_2 = c_2(1-c_2)$, with c_2 the average density of second class particles between $-l$ and $+l$ at $t=0$. Here the results of Section 2 may be applied and one finds an excess mass flow growing as $t^{2/3}$ through an interface at rest and, averaging over the initial distribution, one that increases as $t^{1/2}$ for an interface moving at a constant speed, as long as this interface remains within the interval $[-l, l]$. On the other hand, for $t \gg l^{3/2}$ the relevant k values are $\ll l^{-1}$. Then $\hat{\rho}_2(k,0)$ and $\hat{\rho}_2(k',0)$ may be replaced by N and the variance of $M(x,t)$ reduces to N times a constant function of $(x/t^{2/3})$. This means that on this time scale the particles move like independent particles.

If one inserts second class particles at a homogeneous density c_2 their behavior is similar to that of first class particles. The average speed of density patterns in ρ_1 is $w(c)$, that of patterns in the total density is $w(c+c_2)$, and, by conservation of mass, the average drift velocity of second class particles equals $\{(c+c_2)\,v(c+c_2) - cv(c)\}/c_2$. In general all three will be different, so again a second class particle moves diffusively with respect to its average drift if one averages over both initial and dynamical fluctuations, whereas its displacement with respect to the average drift increases as $t^{1/3}$ if one fixes the initial configuration.

If one adds positive and negative second class mass with an average density zero, still the excess flow of second class mass through an interface will behave in the same ways as found for the first class mass flow in Section 2. However, if one now realizes this by inserting second class particles and holes in the simple exclusion process, it becomes impossible to follow the motion of a tagged second class particle or hole, because of the possibility that these objects will annihilate each other.

5. INTUITIVE ARGUMENTS

At first sight it may look rather surprising that at the same time the position of the center of mass of a density disturbance spreads faster than diffusively and the position of a specific mass point (or second class particle) in this disturbance spreads slower than diffusively. However, the following argument may shed light on this: Consider a closed chain of length N on which initially mass is distributed according to the stationary distribution. After a time t, according to the scaling properties of the stochastic Burgers equation, disturbances in the density distribution of coherence length $\sim t^{2/3}$ will have formed, so the total number of independent disturbances will be $\sim t^{-2/3}N$. The displacements of these disturbances with respect to their average drift will be randomly directed toward the left or the right, so, according to the law of large numbers, the excess number of disturbance regions displaced in either direction wil be $\sim t^{-1/3}N^{1/2}$. Suppose the typical displacement of such a disturbance is of magnitude $a(t)$; then the displacement of the center of mass will be proportional to $t^{-1/3}N^{1/2}t^{2/3}a(t)/N$, the factor $t^{2/3}$ coming from the total mass in a typical disturbance. Hence in order that this displacement is $\sim t^{2/3}$, $a(t)$ has to be $\sim t^{1/3}$, implying that also the mass points within the disturbance are displaced in this way. The same type of argument, applied to simple exclusion systems satisfying an ordinary one-dimensional fluctuating diffusion equation, predicts a random displacement of mass fronts (which may be identified in this case with the positions of tagged particles) proportional to $t^{1/4}$, in agreement with the known behavior for such systems.[8,9]

The notion of a correlation length growing as $t^{2/3}$ may be strengthened by looking at the difference in excess mass flows through different points x and y. From (15) one obtains for this

$$\langle\!\langle [M(x, t) - M(y, t)]^2 \rangle\!\rangle = \frac{1}{2\pi^2} \int dk \, \frac{1 - \cos k(x-y)}{k^2} \langle\!\langle |\hat{\rho}(k, t) - \hat{\rho}(k, 0)|^2 \rangle\!\rangle \tag{19}$$

which, by the usual scaling arguments, becomes a function of $(x-y)/t^{2/3}$ increasing from 0 for $x-y=0$ to an asymptotic value for $(x-y) \to \infty$.

The diffusive behavior of a first class particle on averaging over initial configurations is also easy to understand. During a time t such a particle samples a stretch of length $(v-w)t$ of the moving density pattern in the system. Dividing this stretch into segments of length λ larger than the microscopic correlation length, one has that each segment adds a stochastic amount to the excess drift of the particle. Since the particle travels at a faster speed than the density disturbances in the system, these stochastic additions will be uncorrelated and, again according to the law of large numbers, the excess drift after a time t will increase as $t^{1/2}$.

On the other hand, if one fixes the mass distribution for all times, the excess drift of the particle is completely determined and does not fluctuate. In reality, however, the mass distribution does fluctuate in time. For the simple exclusion process the excess drift of a first class particle is proportional to the excess mass it passes during its motion. We saw before that for fixed initial condition this excess mass contains a stochastic contribution that grows as $t^{1/3}$. Additional contributions from diffusion around the average drift remain bounded in time, as can be seen readily from the arguments put forward in refs. 1 and 2.

6. CONCLUSION

The spreading of mass patterns in systems described by a one-dimensional fluctuating Burgers equation inherently goes $\sim t^{2/3}$. As a consequence the random displacement of a mass front, which may be defined as an interface moving in such a way that at no time is there an excess mass current running through it, is proportional to $t^{1/3}$. Due to fluctuations in the initial configuration the random displacement from its average position of an actual mass point, which moves at a different average speed than the mass front defined above, increases as $t^{1/2}$. However, if one fixes the initial configuration, that increase is reduced to one $\sim t^{1/3}$. Similar conclusions were drawn for shock fronts as well as for second class particles in the simple exclusion model at a uniform nonzero average density. Disturbances consisting of a finite number of second class particles after some time show quasi-independent motion for all of these particles.

Quantitative results, such as the amplitudes of variances and the form of Green functions, are not very simple to obtain. Using graph techniques for solving the fluctuating Burgers equation, one may obtain increasingly better successive approximations that could be solved numerically. Rigorous proofs for the various scaling relations derived in this paper might be even harder to obtain.

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NOTE ADDED IN PROOF

After completing this paper I received a preprint by S. N. Majumdar and M. Barma on Tag Diffusion in Driven Systems, Growing Interfaces and Anomalous Fluctuations, that contains related and, partly, equivalent results.

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