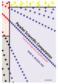
Experiments with `bsplu`
(PSC §2.5–2.6)

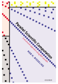


Broadcast function

```
void bsp_broadcast(double *x, int n, int src,
                   int s0, int stride, int p0,
                   int s, int phase){
/* Broadcast the vector x of length n
from processor src to processors s0+t*stride,
0 <= t < p0. x has already been registered.

s = local processor identity.
phase= phase of two-phase broadcast (0 or 1)
Only one phase is performed, without sync. */
```

- Standard 1D–2D identification $P(s,t) \equiv P(s+tM)$.
- $\texttt{stride} = 1$, $\texttt{p0} = M$: broadcast within processor column.
  $\texttt{stride} = M$, $\texttt{p0} = N$: broadcast within processor row.
- No sync to allow combining supersteps.

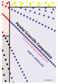


Phase 0: source processor spreads the data

```
b= (n%p0==0 ?  n/p0 : n/p0+1); /* block size */

if (phase==0 && s==src){
    for (t=0; t<p0; t++){
        dest= s0+t*stride;
        nbytes= MIN(b,n-t*b)*SZDBL;
        if (nbytes>0)
            bsp_put(dest,&x[t*b],x,t*b*SZDBL,nbytes);
}}
```

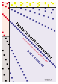
Data is put in the same location $t \cdot b$ of array `x` in the destination processor as in the source processor.



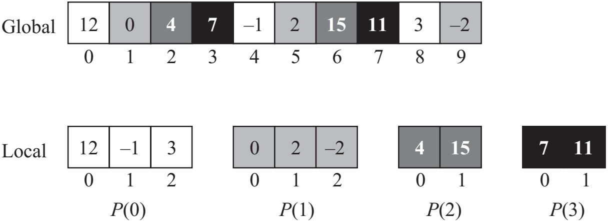
Phase 1: participating processors perform broadcast

```
if (phase==1 && s%stride==s0%stride){
    t=(s-s0)/stride; /* s = s0+t*stride */
    if (0<=t && t<p0){
        nbytes= MIN(b,n-t*b)*SZDBL;
        if (nbytes>0){
            for (t1=0; t1<p0; t1++){
                dest= s0+t1*stride;
                if (dest!=src)
                    bsp_put(dest,&x[t*b],x,
                            t*b*SZDBL,nbytes);
}}}}
```

Data is not sent back to the source. No influence on BSP cost, but it reduces the communication volume. This cannot be bad.



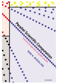
Local and global indices for cyclic distribution



Global index: i Local index on $P(s)$: $\mathtt{i}$

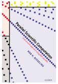
Relation: $i = \mathtt{i} \cdot p + s$

```
/* Initialise permutation vector pi */
nlr=  nloc(M,s,n); /* number of local rows */
if (t==0)
    for(i=0; i<nlr; i++)
        pi[i]= i*M+s; /* global row index */
```



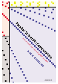
Putting data directly into a 2D array

```
a = matallocd(nlr, nlc); /* in bsplu_test.c */
void bsplu( ..., int *pi, double **a){
   double *pa= NULL;
   if (nlr>0)
      pa= a[0];
   bsp_push_reg(pa,nlr*nlc*SZDBL);
   bsp_push_reg(pi,nlr*SZINT);
   ...
   if (k%M==s){
      /* Store pi(k) in pi(r) on P(r%M,0) */
      if (t==0)
         bsp_put(r%M,&pi[k/M],pi,(r/M)*SZINT,SZINT);
      /* Store row k of A in row r on P(r%M,t) */
      bsp_put(r%M+t*M,a[k/M],pa,
                      (r/M)*nlc*SZDBL,nlc*SZDBL);
   } ...
```



Two-phase broadcast of column k

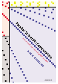
```
double *lk;
nlr= nloc(M,s,n); /* number of local rows */
kr=  nloc(M,s,k); /* first local row
                     with global index >= k */
kc=  nloc(N,t,k);
kr1= nloc(M,s,k+1);
lk= vecallocd(nlr); bsp_push_reg(lk,nlr*SZDBL);
...
if (k%N==t) /* Store new column k in lk */
     for(i=kr1; i<nlr; i++)
          lk[i-kr1]= a[i][kc];
bsp_broadcast(lk,nlr-kr1,s+(k%N)*M,s,M,N,s+t*M,0);
bsp_sync();
bsp_broadcast(lk,nlr-kr1,s+(k%N)*M,s,M,N,s+t*M,1);
bsp_sync();
```



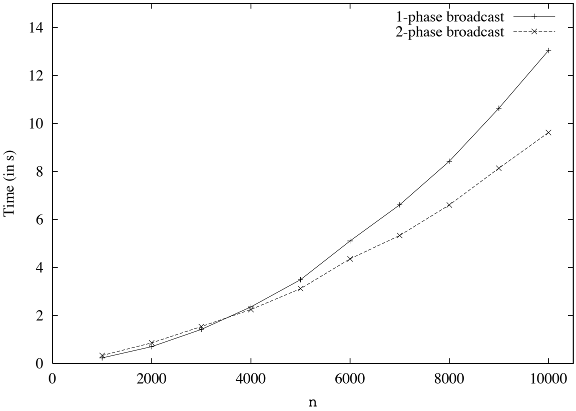
Time (in s) of LU decomposition

n	one-phase	two-phase
1 000	1.21	1.33
2 000	7.04	7.25
3 000	21.18	21.46
4 000	47.49	47.51
5 000	89.90	89.71
6 000	153.23	152.79
7 000	239.21	238.25
8 000	355.84	354.29
9 000	501.92	499.74
10 000	689.91	689.56

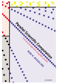
Cray T3E with $p = 64$, $r = 38.0$ Mflop/s, $g = 87$, $l = 2718$ (measured by `bspbench`). 8×8 cyclic distribution.



Total broadcast time of LU decomposition

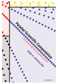


Cray T3E with $p = 64$, $r = 38.0$ Mflop/s, $g = 87$, $l = 2718$.

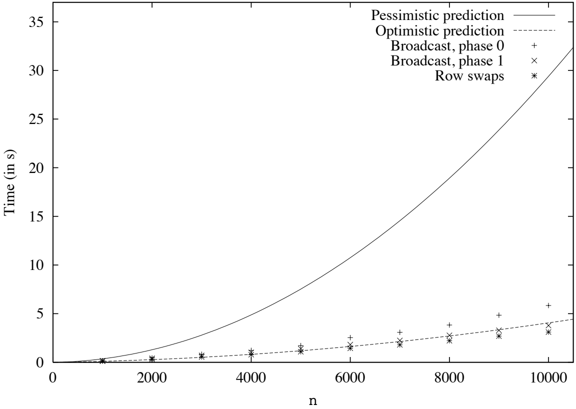


Any actual savings by two-phase broadcast?

- Not much difference in total time between one-phase and two-phase approach.
- For $n < 4000$, with local broadcast length < 500, one-phase is better.
- For $n > 4000$, two-phase is better. But savings are insignificant compared to computation time. Total broadcast time is $< 5\%$ of overall time.
- BSP analysis gives insight and explains results, even if they are surprising/disappointing/...
- On a different machine with slower communication, such as a PC cluster, the savings will be significant. Try it!

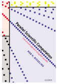


Total measured and predicted time

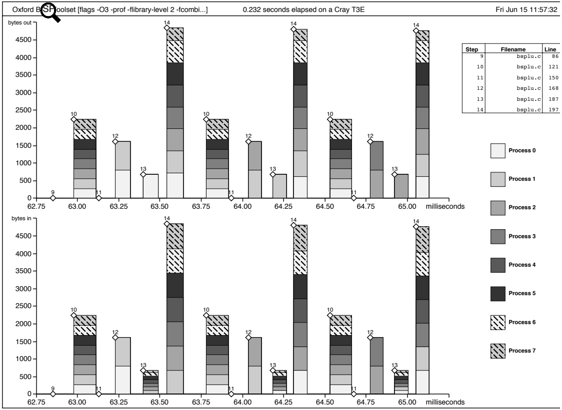


Optimistic prediction is right

- BSP model predicts: row swaps, phase 0 of the broadcast, and phase 1 all take the same time. Measurements validate this.
- Very different communication patterns: row swaps and phase 0 are very unbalanced, phase 1 is well-balanced.
- Pessimists are usually wrong. The pessimistic g-value (for puts of single data words) is far off.
- You need to plug the right g-value into the BSP cost formula to obtain meaningful predictions. `bsplu` puts elements from row and column k as large data packets. Therefore, we should use the optimistic g-value.

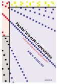


Profile of stages $k = 0, 1, 2$ of an LU decomposition



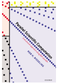
Step	Filename	Line
9	bsplu.c	86
10	bsplu.c	121
11	bsplu.c	150
12	bsplu.c	168
13	bsplu.c	187
14	bsplu.c	197

Cray T3E: $n = 100$, $M = 8$, $N = 1$. Obtained by `bspprof`.



Game: recognise the supersteps

- $M = 8$, $N = 1$: row distribution of the matrix.
- Column broadcast is for free.
- Row swap involves two processors; each time a different pair. This must be superstep 12.
- Phase 0 of row broadcast has 1 sender, 7 receivers. This must be superstep 13.
- Phase 1 has 8 senders, 7 receivers, and takes about the same time (bar width) as superstep 13. This must be superstep 14.
- The wide gap between supersteps 14 and 10 is a big computation superstep. This must be the matrix update.
- Superstep 10 must be the exchange of local winners in the pivot search with 8 senders and 8 receivers. Relatively costly, because the problem size is only $n = 100$.



Summary

- We use global indices in the description of an algorithm, but local indices in an actual program.
- We understand the behaviour of our program, though we may not always like it.
- Very different communication patterns with the same BSP cost take about the same time on an actual parallel computer, the Cray T3E.
- Profiling is a way of getting intimate knowledge of your program. The superstep concept makes this very easy.