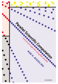
Experimental results for the FFT
(PSC §3.7)

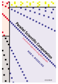


Test computer: SGI Origin 3800

Photo: Walter de Jong

`http://www.xs4all.nl/~walterj/sara`

- Teras, the national supercomputer in the Netherlands, located in Amsterdam. Installed in 2000; overtaken by an SGI Altix 3700 (Aster) in 2003. Machines come and go.
- Named after Teraflop/s computing rate (10^{12} flop/s) and after the Greek word for 'monster', $\tau\epsilon\rho\alpha\varsigma$.
- 1024 processors, split into 6 partitions with 512, 256, 128, 64, 32, 32 processors.



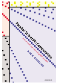
SGI Origin 3800 is a CC-NUMA machine

- Each processor has:
  - MIPS RS14000 CPU with a clock rate of 500 MHz and a theoretical peak performance of 1 Gflop/s
  - primary data cache of 32 Kbyte
  - secondary cache of 8 Mbyte
  - memory of 1 Gbyte.
- Cache Coherent Non-Uniform Memory Access:
  - cache is kept coherent, so user views a shared memory
  - physically, the memory is distributed; hence, access time to local and remote memory differs

Benchmarked BSP parameters of SGI Origin 3800

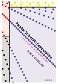
p	g	l	$T_{\mathrm{comm}}(0)$
1	99	55	378
2	75	5118	1414
4	99	12743	2098
8	126	32742	4947
16	122	93488	15766

$r = 285$ Mflop/s.
$T_{\mathrm{comm}}(0)$ is the time of a 0-relation.

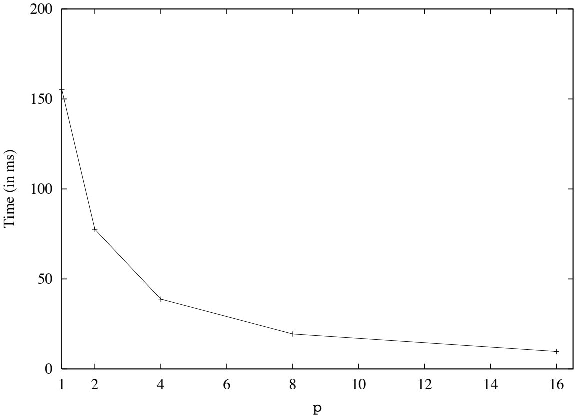


Aggressive optimisation

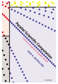
- Initial tests: maximum optimisation level -O3 for newly installed C compiler gave benchmark rate 981 Mflop/s.
- This is almost the theoretical peak rate. For a DAXPY, such a speed is impossible.
- The new compiler discovered our true intention of just measuring the computing rate, and cleverly removed some unnecessary statements.
- We reduced the optimisation level for benchmarking to -O2.
- We may have been fooled before (predecessor Origin 2000, Chapter 1), with a measured rate of 326 Mflop/s. This high rate is partly due to having the machine to ourselves, but perhaps also to overly aggressive optimisation.
- Always be cautious about benchmark results!



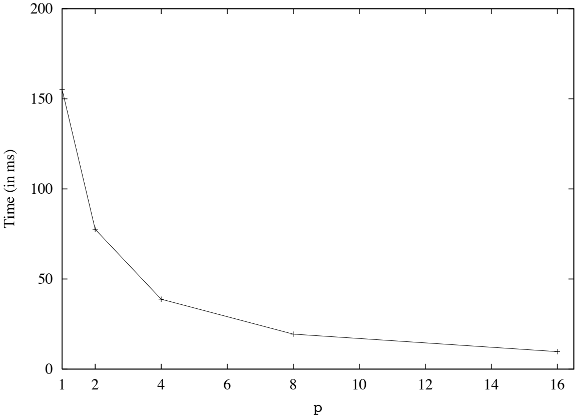
Time of a parallel FFT of length 262144



What do you think? Good or bad?



Time of a parallel FFT of length 262144



What do you think? Good or bad?

Surprise! This is the time of a theoretical, perfectly parallelised FFT, based on a time of 155.2 ms for $p = 1$.

Measured time $T_p(n)$ of sequential and parallel FFT

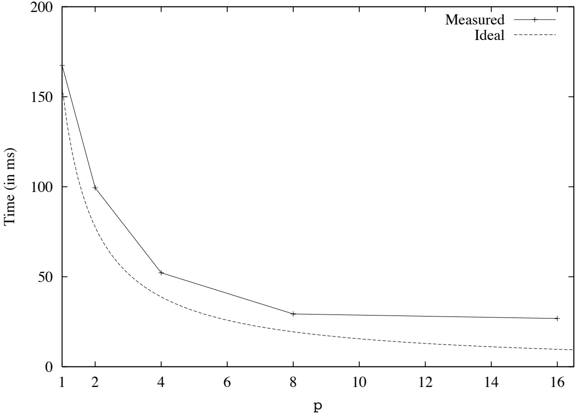
p		Length n 4096	16384	65536	262144
1	(seq)	1.16	5.99	26.6	155.2
1	(par)	1.32	6.58	29.8	167.4
2		1.06	4.92	22.9	99.4
4		0.64	3.15	13.6	52.2
8		1.18	2.00	8.9	29.3
16		8.44	11.07	9.9	26.8

Time in ms.

Time measurements are difficult on the Origin

- Timings may suffer from interference by other programs (caused e.g. by sharing of communication links).
- Best of three: we run each experiment 3 times, and take the best result.
- Often, the best two timings are within 5% of each other, and the third result is worse.

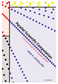
Time T_p of actual parallel FFT of length 262144



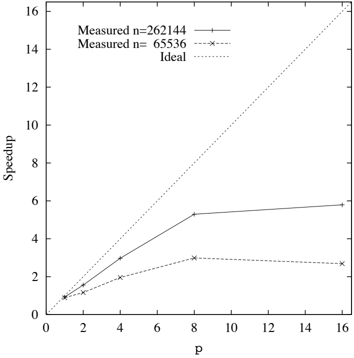
Warning: this kind of picture gives some insight, but it is not the best representation of the results.

Speedup

- The speedup $S_p(n)$ of a parallel program is the increase in speed of the program running on p processors compared to the speed of a sequential program with the same level of optimisation,
$$S_p(n) = \frac{T_{\mathrm{seq}}(n)}{T_p(n)}.$$
- Do not compare with T_1 instead of T_{seq}, since this may be too flattering. The parallel program run with $p = 1$ may have much overhead. Here: 8%.
- Often, it is easy to simplify a parallel program into a sequential one by removing overhead.
- If this is too much work, then be at least clear about the reference 'sequential' program.



Speedup $S_p(n)$ of parallel FFT



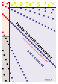
This kind of picture gives much more insight. It allows comparison for different problem sizes.

Superlinear speedup

- Bound on speedup:

$$0 \leq S_p(n) \leq p.$$

- $S_p(n) < 1$ is called a slowdown. It usually happens for $p = 1$, and sometimes for $p = 2$.
- $S_p(n) > p$ is called superlinear speedup. In theory, this cannot happen, but in practice it does. Possible causes:
  - Cache effects: in the parallel case, each processor has less data to handle than in the sequential case, so that the local data may fit in the cache.
  - Different order of the computations: less work in the parallel case. Example: search algorithms, where the search stops when one processor finds a solution. (Trick often used in demos by parallel computer vendors.)

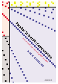


Superlinear speedup: blessing or curse?

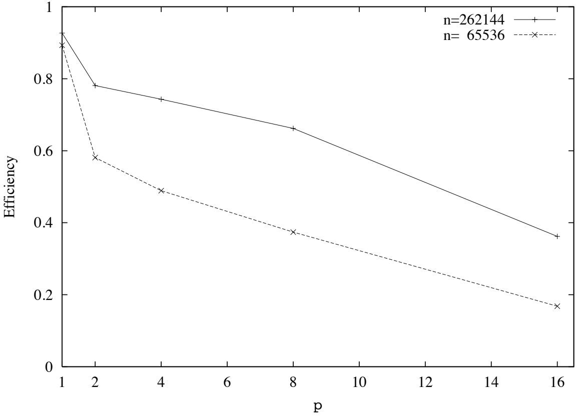
- Effects that cause superlinear speedups make it difficult to judge the quality of the parallelisation. Even if no actual superlinear speedups are observed ...
- Still, a faster computation is always welcome. Besides, you paid for the multiple caches of a parallel computer.

Efficiency

- The efficiency $E_p(n)$ of a parallel program is the fraction of the total computing power that is usefully employed. It is defined by
$$E_p(n) = \frac{S_p(n)}{p} = \frac{T_{\mathrm{seq}}(n)}{pT_p(n)}.$$
- Bound on efficiency:
$$0 \leq E_p(n) \leq 1.$$



Measured efficiency $E_p(n)$ of parallel FFT



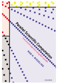
The ideal value is 1.

Inefficiency

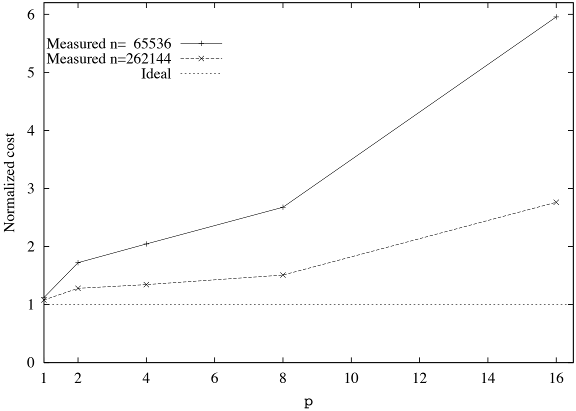
- The normalised cost (or inefficiency) $C_p(n)$ is the ratio between the time of the parallel program and the time of a perfectly parallelised version of the sequential program. It is defined by

$$C_p(n) = \frac{T_p(n)}{T_{\mathrm{seq}}(n)/p} = \frac{pT_p(n)}{T_{\mathrm{seq}}(n)} = \frac{1}{E_p(n)}.$$

- Bound on the inefficiency: $C_p(n) \geq 1$.
- The parallel overhead equals $C_p(n) - 1$. It usually consists of:
  - load imbalance
  - communication time
  - synchronisation time



Normalised cost $C_p(n)$ of parallel FFT

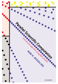


Breakdown of predicted execution time

p	T_{Comp}	T_{Comm}	T_{Sync}	T_{FFT} (pred.)	T_p (meas.)
1	82.78	0.00	0.00	82.78	167.4
2	41.39	68.99	0.05	110.43	99.4
4	20.70	45.53	0.13	66.36	52.2
8	10.35	28.97	0.35	39.67	29.3
16	5.17	14.03	0.98	20.18	26.8

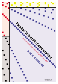
Time in ms. $n = 262144$.
Prediction is based on time

$$T_p(n) = 5\frac{n}{p}\log_2 n + 2\frac{n}{p}g + 3l.$$



Insights gained from breakdown

- It is difficult to predict the total time correctly, mainly due to misprediction of the sequential computation time.
- $n = 1024$ DAXPY benchmark fits in cache, but $n = 262144$ FFT does not. This reduces the rate from 285 Mflop/s to 144 Mflop/s.
- Benchmark of computing rate r can be adapted to application, if desired.
- Communication is the bottleneck, even though we perform only one data permutation.
- Prediction overestimates the communication time, being based on a pessimistic g-value, but the actual parallel FFT was optimised to send data in packets.
- Synchronisation is unimportant for this problem size.



Total computing rate $R_p(n)$

- The total computing rate of the FFT is defined by

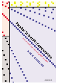
$$R_p(n) = \frac{5n \log_2 n}{T_p(n)}.$$

- The rate is based on the sequential flop count $5n \log_2 n$. This count is commonly used to measure FFT rates, even for FFT variants with fewer actual flops.
- Radix-4 FFTs have $4.25n \log_2 n$ flops.

Computing rate $R_p(n)$ of sequential and parallel FFT

p		Length n 4096	16384	65536	262144
1	(seq)	220	197	202	155
1	(par)	193	179	180	144
2		239	240	234	243
4		397	375	395	462
8		216	591	607	824
16		30	107	545	900

Rate in Mflop/s. Measured on SGI Origin 3800.
Note: we need at least 4 processors to exceed sequential benchmark speed of 285 Mflop/s.



Summary

- We have introduced several metrics to express the performance of a parallel program:
  - $T_p(n)$, the time (in s)
  - $S_p(n) = T_{\mathrm{seq}}(n)/T_p(n)$, the speedup
  - $E_p(n) = S_p(n)/p$, the efficiency
  - $C_p(n) = 1/E_p(n)$, the normalised cost or inefficiency
  - $C_p(n) - 1$, the overhead
  - $R_p(n) = (5n \log_2 n)/T_p(n)$, the total computing rate (in flop/s).
- Speedup plots give much insight.
- Always take a critical look at benchmark results obtained on a parallel computer.