Covid-19 Spreading on Networks

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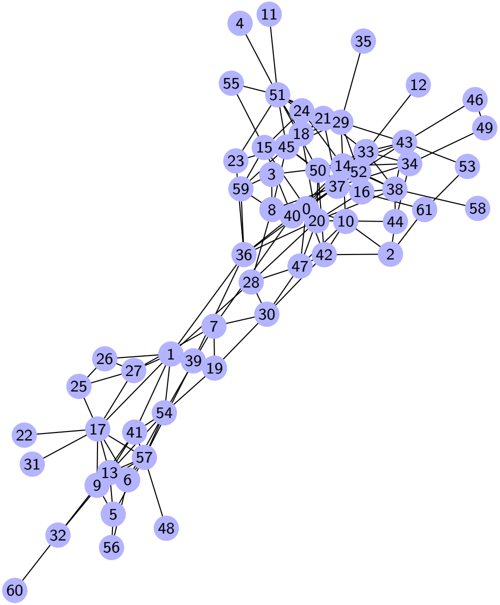
SIR model

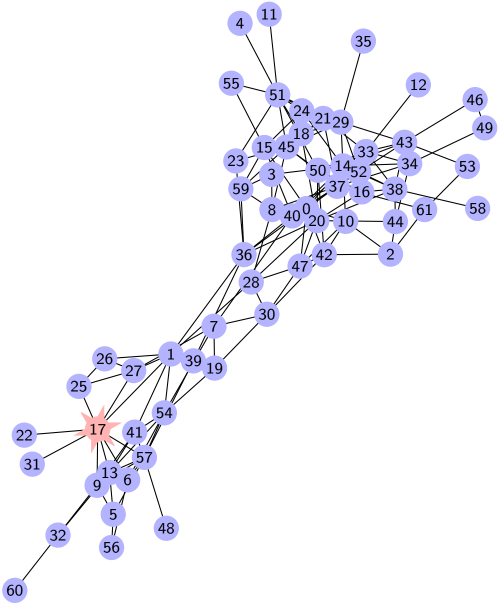
Vaccination

Mixed vaccination strategy

Network of 62 persons susceptible (S) to a disease



One person becomes infected (I)



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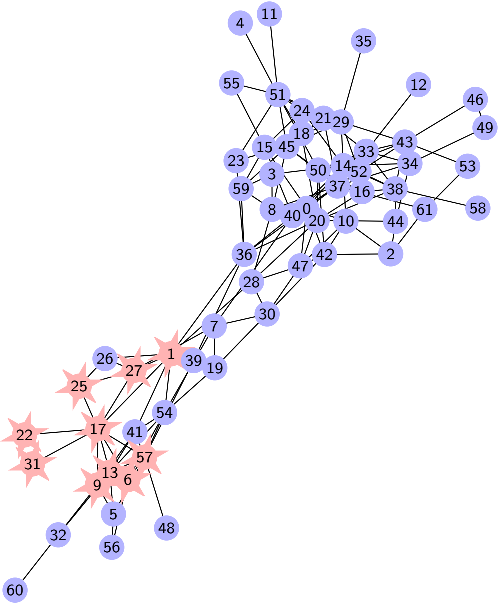
infects 9 others



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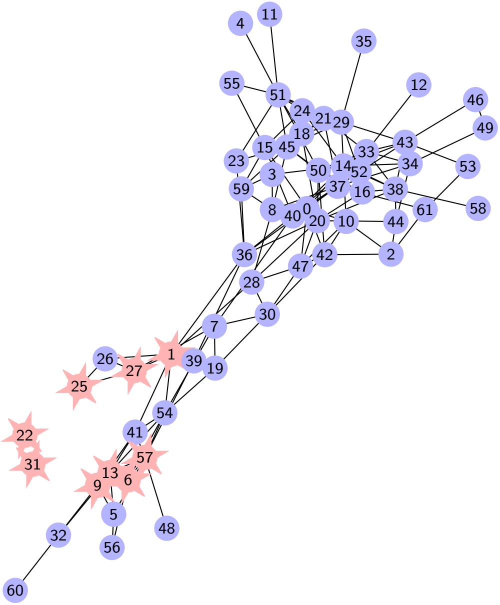
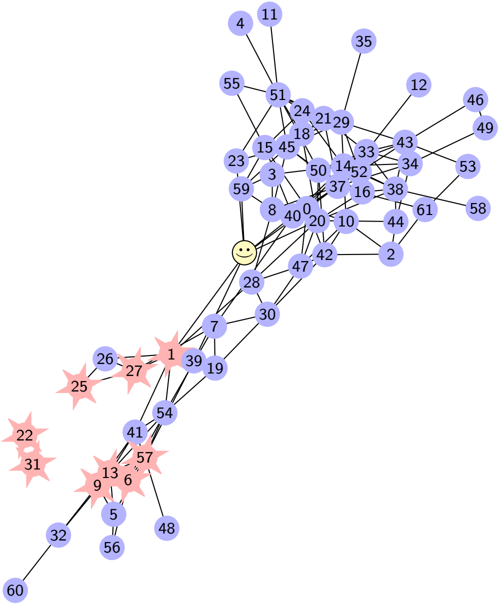
and recovers (R)



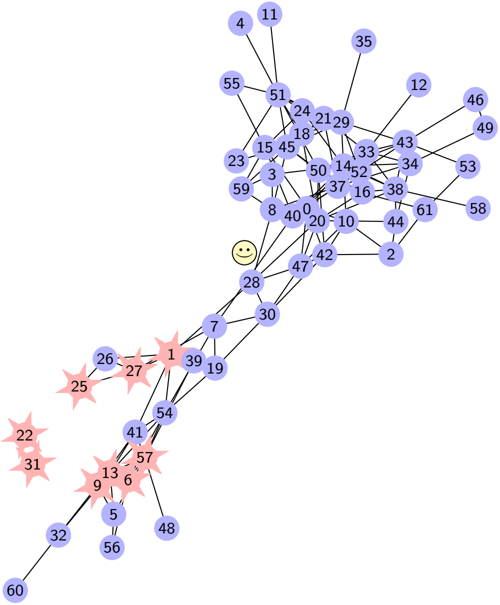
The SIR model

- A vertex can be either: Susceptible, Infectious, Recovered.
- The total number of vertices is $S + I + R = n$.
- This is the simplest possible model of disease spreading.
- The R can also stand for Removed, because people can either recover and become immune (for a certain long period of time), or die.
- In both cases, the vertex and its edges can be removed from the graph.

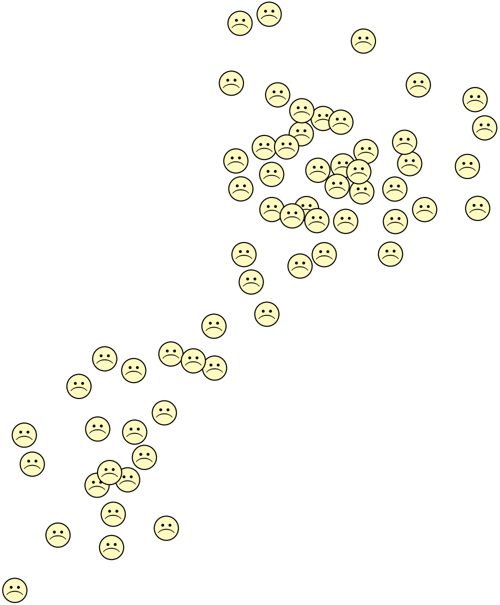
One person gets vaccinated



and cannot infect others any more



Lockdown can temporarily achieve the same



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Storing vertex data

- For vertex i of the graph $(0 \leq i < n)$, we can store:
  - x_i infected (1 if infected, 0 otherwise);
  - r_i recovered and alive (1 or 0);
  - z_i dead (1 or 0);
  - d_i degree of i (number of contacts);
  - n_i number of days infected.
- The vertices may be renumbered, e.g. by decreasing degree, putting high-degree vertices (hubs) first.
- Other data may also be useful.

Storing edge data

- For edge (i, j) of the graph $(0 \leq i, j < n)$, we can store:
  - $a_{ij} = 1$ if i and j are connected by frequent contact, 0 otherwise.
- The graph is sparse, since people have only a limited number of contacts.
- The new value of $\mathbf{x}$ after one time step (day) is computed by the sparse matrix–vector multiplication

$$\mathbf{y} = A\mathbf{x},$$

  followed by the operation

$$x_i^{\text{new}} = \max(y_i, 1), \quad \text{for } 0 \leq i < n.$$

- The maximum is taken to handle the case of multiple infections of the same person on the same day.

Probability parameters

- We can modify the model to introduce probabilities:
  - p with $0 \leq p \leq 1$ is the probability that a contact along an edge of the graph leads to an infection.
  - t with $0 \leq t \leq 1$ is the probability that a vaccinated person still transmits the disease.
- In the model shown we assumed $p = 1$ and $t = 0$, but:
  - people will not have contact every day, so $p < 1$;
  - vaccines will not always protect against transmission of the disease, so $t > 0$.
- Both can be incorporated by drawing random numbers and, based on this, deciding whether to create new infections.

Creating A as a model of the population

- Some people are hubs and have many contacts, but many people have fewer contacts.
- This can be modeled by a power law, where the probability of having a degree d equals

$$\mathbf{P}[d_i = d] = c d^{-\alpha}.$$

The exponent often has a value in the range $2 \leq \alpha \leq 3$ and the constant c is such that the total probability is 1.

- A random graph (and its adjacency matrix A) can be created that takes this degree distribution into account.
- An alternative is to use real-world data for the degree distribution: how many contacts do people have, perhaps as a function of age? What is the age distribution?

Goals: prevent spreading and becoming seriously ill

- Assume that $t > 0$, so that vaccination helps in preventing spreading, and not only prevents people from getting ill.
- The exact value of t for the various approved vaccines is still unknown.
- The chance of becoming seriously ill with Covid-19 increases with age, and rises steeply after age 60.
- The chance of spreading the disease is related to the number of contacts, which in first approximation decreases with age.
- A mixed strategy, starting at both ends of the age range, could attempt to achieve both goals.

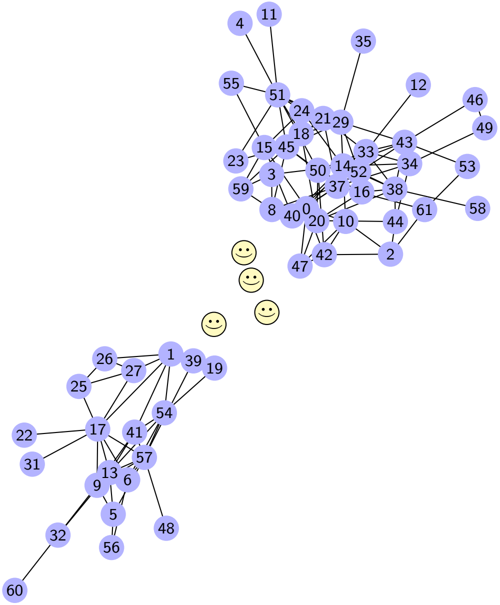
Which mix of ages is optimal?

- Assume that the number of vaccines available is much less than the total number $2n$ needed, based on 2 doses per person and n adults (18+).
- The main question for a good vaccination strategy is: what is the optimal fraction $\beta > 0$ of the vaccines that should be given to the young, starting from the 18-year olds?
- Note that reducing the spreading will help protect the whole population that has not yet been vaccinated, including older nonvaccinated people.
- To evaluate success, various metrics can be used:
  - number of deaths (based on an age-related probability of dying after infection);
  - number of serious illness cases;
  - the general state of happiness of the population (related to the need for lockdown, mental health).

Alternative strategies for preventing spreading

- Vaccinate specific groups of people, identified by:
  - highest degree;
  - highest estimated number of acquaintances: choose a random neighbour of randomly chosen vertices, see Havlin 2003
  - PageRank: the symmetric matrix A can be viewed as a weblink matrix where all links are mutual;
  - betweenness centrality: being situated on many shortest paths;
  - nested dissection: people separating regions

Nested dissection vaccination



Summary

- We can design vaccination strategies against the Covid-19 pandemic by network simulations.
- The simulations require strong simplifying assumptions to enable the computations, but they can still give useful insights.
- The simulations can be made more realistic by using real-world data, provided by RIVM, WHO, corona dashboard.
- Starting by vaccinating both the young and the old in the population might be a superior strategy, provided the vaccination has the right target mix of young and old.
- The final vaccination percentage achieved may become higher by starting also with the young, as they may benefit from the incentive of an earlier exit from restrictions.