11g. Eigenfunction equations, non-abelian eigenfunction module

In[ ]:=
```
Clear[f, tht, ell, eps, dtt, m, j, nu]
F = tht[m[r]] × f[r, t] × Phi[h, p, r, p]
```

Out[ ]= f[r, t] × Phi[h, p, r, p] × tht[m[r]]

The sign off **ell** is denoted by **eps**. We use **ddt** like in 10a.

Computation of eigenfunction equations. This takes very long; that is why we put the result in a routine, in 11h.

In[ ]:=
```
efeq = {eR[CasZ, F, subnab] - ld2[j, nu] F, eR[Dt3Z, F, subnab] - ld3[j, nu] F} // Simplify
```

Out[ ]=

$$\Big\{-\Big(\Big(-4+\frac{j^2}{3}+nu^2\Big) f[r,t]\times Phi[h,p,r,p]\times tht[m[r]]\Big)+$$
$$4\,\pi\,t^2\,Abs[ell]\,f[r,t]\,(-1+eps-2\,m[r])\,Phi[h,p,r,p]\times tht[m[r]]-i\,\sqrt{2\,\pi}\;t\,\sqrt{Abs[ell]}\;f[r,t]$$
$$\Big(\sqrt{m[r]}\;((1+eps)\times(2+p-r)\,Phi[h,p,-2+r,p]-(-1+eps)\times(2+p+r)\,Phi[h,p,2+r,p])$$
$$tht[-1+m[r]]+\sqrt{1+m[r]}\;((-1+eps)\times(2+p-r)\,Phi[h,p,-2+r,p]-$$
$$(1+eps)\times(2+p+r)\,Phi[h,p,2+r,p])\,tht[1+m[r]]\Big)+\frac{1}{12}\,Phi[h,p,r,p]\times tht[m[r]]$$
$$((h^2+9\,r^2-24\,ell\,\pi\,r\,t^2-48\,ell\,\pi\,t^2\,(1+ell\,\pi\,t^2)-6\,h\,(r+4\,ell\,\pi\,t^2))\,f[r,t]+$$
$$12\,t\,(-3\,f^{(0,1)}[r,t]+t\,f^{(0,2)}[r,t])),$$
$$-\Big((3+j)\times\Big(-\frac{1}{9}\,(-6+j)^2+nu^2\Big)\,f[r,t]\times Phi[h,p,r,p]\times tht[m[r]]\Big)+$$
$$2\,\pi\,(6+h+3\,r)\,t^2\,Abs[ell]\,f[r,t]\,(-1+eps-2\,m[r])\,Phi[h,p,r,p]\times tht[m[r]]+$$
$$\frac{1}{2}\,i\,\sqrt{\frac{\pi}{2}}\;t\,\sqrt{Abs[ell]}$$
$$\Big(f[r,t]\Big(\sqrt{m[r]}\;((1+eps)\times(2+p-r)\,(h-3\,r+12\,ell\,\pi\,t^2)\,Phi[h,p,-2+r,p]+$$
$$(-1+eps)\times(2+p+r)\,(-h+3\times(8+r-4\,ell\,\pi\,t^2))\,Phi[h,p,2+r,p])\,tht[-1+m[r]]+$$
$$\sqrt{1+m[r]}\;((-1+eps)\times(2+p-r)\,(h-3\,r+12\,ell\,\pi\,t^2)\,Phi[h,p,-2+r,p]+$$
$$(1+eps)\times(2+p+r)\,(-h+3\times(8+r-4\,ell\,\pi\,t^2))\,Phi[h,p,2+r,p])\,tht[1+m[r]]\Big)-$$
$$6\,t\Big(\sqrt{m[r]}\;((1+eps)\times(2+p-r)\,Phi[h,p,-2+r,p]+(-1+eps)\times(2+p+r)\,Phi[h,p,2+r,p])$$
$$tht[-1+m[r]]+\sqrt{1+m[r]}\;((-1+eps)\times(2+p-r)\,Phi[h,p,-2+r,p]+$$
$$(1+eps)\times(2+p+r)\,Phi[h,p,2+r,p])\,tht[1+m[r]]\Big)\,f^{(0,1)}[r,t]\Big)-$$
$$\frac{1}{72}\,Phi[h,p,r,p]\times tht[m[r]]\,((h^3-9\,h^2\,(2+r-8\,ell\,\pi\,t^2)-27\,(r^3+r^2\,(6-8\,ell\,\pi\,t^2)+$$
$$16\,ell\,\pi\,t^2\,(-2+2\,p+p^2-2\,ell\,\pi\,t^2)+16\,r\,(-1-2\,ell\,\pi\,t^2+ell^2\,\pi^2\,t^4))+$$
$$9\,h\,(3\,r^2-4\,r\,(-3+4\,ell\,\pi\,t^2)+16\times(-1+4\,ell\,\pi\,t^2+ell^2\,\pi^2\,t^4)))\,f[r,t]-$$
$$36\times(6+h-3\,r)\,t\,(-3\,f^{(0,1)}[r,t]+t\,f^{(0,2)}[r,t]))\Big\}$$

Separate treatment for different values of Sign[ell]

```
In[ ]:= efeqp = efeq /. eps → 1 // Simplify;
        efeqm = efeq /. eps → -1 // Simplify;
```

Bringing all terms with the same value of r together

```
In[ ]:= efqp = ({efeqp[[1]] // compr, efeqp[[2]] // compr}) / (tht[m[r]] × Phi[h, p, r, p]) /.
           {m[r + 2] → m[r] + 1, m[r - 2] → m[r] - 1} // Simplify
        efqm = ({efeqm[[1]] // compr, efeqm[[2]] // compr}) / (tht[m[r]] × Phi[h, p, r, p]) /.
           {m[r + 2] → m[r] - 1, m[r - 2] → m[r] + 1} // Simplify
```

Out[ ]=

$$\Big\{\frac{1}{12}\, f[r,t]\big(48+h^2-4\,j^2-12\,\mathrm{nu}^2+9\,r^2-48\,\mathrm{ell}\,\pi\,t^2-24\,\mathrm{ell}\,\pi\,r\,t^2-48\,\mathrm{ell}^2\,\pi^2\,t^4-6\,h\,(r+4\,\mathrm{ell}\,\pi\,t^2)-96\,\pi\,t^2\,\mathrm{Abs}[\mathrm{ell}]\,m[r]\big)+t\Big(2\,i\,\sqrt{2\,\pi}\,\sqrt{\mathrm{Abs}[\mathrm{ell}]}\,\Big((p+r)\,f[-2+r,t]\,\sqrt{m[r]}+(-p+r)\,f[2+r,t]\,\sqrt{1+m[r]}\Big)-3\,f^{(0,1)}[r,t]+t\,f^{(0,2)}[r,t]\Big),$$

$$-\frac{1}{72}\,\big(h^3+72\,j^2-8\,j^3+72\,j\,\mathrm{nu}^2-9\,h^2\,(2+r-8\,\mathrm{ell}\,\pi\,t^2)-27\times(-8\,\mathrm{nu}^2+r^3+r^2\,(6-8\,\mathrm{ell}\,\pi\,t^2)+16\times(2+\mathrm{ell}\,(-2+2\,p+p^2)\,\pi\,t^2-2\,\mathrm{ell}^2\,\pi^2\,t^4)+16\,r\,(-1-2\,\mathrm{ell}\,\pi\,t^2+\mathrm{ell}^2\,\pi^2\,t^4))+9\,h\,(3\,r^2-4\,r\,(-3+4\,\mathrm{ell}\,\pi\,t^2)+16\times(-1+4\,\mathrm{ell}\,\pi\,t^2+\mathrm{ell}^2\,\pi^2\,t^4))\big)\,f[r,t]-4\,\pi\,(6+h+3\,r)\,t^2\,\mathrm{Abs}[\mathrm{ell}]\,f[r,t]\times m[r]-i\,\sqrt{\frac{\pi}{2}}\,t\,\sqrt{\mathrm{Abs}[\mathrm{ell}]}\,\Big((p+r)\,(h-3\times(6+r-4\,\mathrm{ell}\,\pi\,t^2))\,f[-2+r,t]\,\sqrt{m[r]}+(p-r)\,(-h+3\times(2+r-4\,\mathrm{ell}\,\pi\,t^2))\,f[2+r,t]\,\sqrt{1+m[r]}+6\,t\,\Big((p+r)\,\sqrt{m[r]}\,f^{(0,1)}[-2+r,t]+(p-r)\,\sqrt{1+m[r]}\,f^{(0,1)}[2+r,t]\Big)\Big)+\frac{1}{2}\times(6+h-3\,r)\,t\,(-3\,f^{(0,1)}[r,t]+t\,f^{(0,2)}[r,t])\Big\}$$

Out[ ]=

$$\Big\{\frac{1}{12}\, f[r,t]\big(48+h^2-4\,j^2-12\,\mathrm{nu}^2+9\,r^2-48\,\mathrm{ell}\,\pi\,t^2-24\,\mathrm{ell}\,\pi\,r\,t^2-48\,\mathrm{ell}^2\,\pi^2\,t^4-6\,h\,(r+4\,\mathrm{ell}\,\pi\,t^2)-96\,\pi\,t^2\,\mathrm{Abs}[\mathrm{ell}]\,(1+m[r])\big)+t\Big(2\,i\,\sqrt{2\,\pi}\,\sqrt{\mathrm{Abs}[\mathrm{ell}]}\,\Big((p-r)\,f[2+r,t]\,\sqrt{m[r]}-(p+r)\,f[-2+r,t]\,\sqrt{1+m[r]}\Big)-3\,f^{(0,1)}[r,t]+t\,f^{(0,2)}[r,t]\Big),$$

$$-\frac{1}{72}\,\big(h^3+72\,j^2-8\,j^3+72\,j\,\mathrm{nu}^2-9\,h^2\,(2+r-8\,\mathrm{ell}\,\pi\,t^2)-27\times(-8\,\mathrm{nu}^2+r^3+r^2\,(6-8\,\mathrm{ell}\,\pi\,t^2)+16\times(2+\mathrm{ell}\,(-2+2\,p+p^2)\,\pi\,t^2-2\,\mathrm{ell}^2\,\pi^2\,t^4)+16\,r\,(-1-2\,\mathrm{ell}\,\pi\,t^2+\mathrm{ell}^2\,\pi^2\,t^4))+9\,h\,(3\,r^2-4\,r\,(-3+4\,\mathrm{ell}\,\pi\,t^2)+16\times(-1+4\,\mathrm{ell}\,\pi\,t^2+\mathrm{ell}^2\,\pi^2\,t^4))\big)\,f[r,t]-4\,\pi\,(6+h+3\,r)\,t^2\,\mathrm{Abs}[\mathrm{ell}]\,f[r,t]\,(1+m[r])+i\,\sqrt{\frac{\pi}{2}}\,t\,\sqrt{\mathrm{Abs}[\mathrm{ell}]}\,\Big((p-r)\,(-h+3\times(2+r-4\,\mathrm{ell}\,\pi\,t^2))\,f[2+r,t]\,\sqrt{m[r]}+(p+r)\,(h-3\times(6+r-4\,\mathrm{ell}\,\pi\,t^2))\,f[-2+r,t]\,\sqrt{1+m[r]}+6\,t\,\Big((p+r)\,\sqrt{1+m[r]}\,f^{(0,1)}[-2+r,t]+(p-r)\,\sqrt{m[r]}\,f^{(0,1)}[2+r,t]\Big)\Big)+\frac{1}{2}\times(6+h-3\,r)\,t\,(-3\,f^{(0,1)}[r,t]+t\,f^{(0,2)}[r,t])\Big\}$$

Simplification of the second component.

In[ ]:=
```
Coefficient[efqp, f^(0,2)[r, t]] // Simplify
Coefficient[efqm, f^(0,2)[r, t]] // Simplify
```

Out[ ]= $\left\{t^2,\ \frac{1}{2} \times (6 + h - 3\ r)\ t^2\right\}$

Out[ ]= $\left\{t^2,\ \frac{1}{2} \times (6 + h - 3\ r)\ t^2\right\}$

In[ ]:=
```
eiap = 2 efqp[[2]] - (6 + h - 3 r) efqp[[1]] // Simplify;
eiam = 2 efqm[[2]] - (6 + h - 3 r) efqm[[1]] // Simplify;
```

In[ ]:=
```
eqp = {efqp[[1]], eiap}
eqm = {efqm[[1]], eiam}
```

Out[ ]=
```
{1/12 f[r, t] (48 + h^2 - 4 j^2 - 12 nu^2 + 9 r^2 - 48 ell π t^2 -
      24 ell π r t^2 - 48 ell^2 π^2 t^4 - 6 h (r + 4 ell π t^2) - 96 π t^2 Abs[ell] m[r]) +
   t (2 i Sqrt[2 π] Sqrt[Abs[ell]] ((p + r) f[-2 + r, t] Sqrt[m[r]] + (-p + r) f[2 + r, t] Sqrt[1 + m[r]]) -
      3 f^(0,1)[r, t] + t f^(0,2)[r, t]),
  f[r, t] (-h^3/9 + (2 j^3)/9 - 2 j nu^2 + h^2 r - j^2 r - 3 nu^2 r + 3 r^3 + h (j^2/3 + nu^2 - 3 r^2) + 24 ell p π t^2 +
      12 ell p^2 π t^2 - 24 ell π r t^2 - 12 ell π r^2 t^2 - 48 π r t^2 Abs[ell] m[r]) -
   3 i Sqrt[2 π] t Sqrt[Abs[ell]] ((p + r) (-2 + h - 3 r + 4 ell π t^2) f[-2 + r, t] Sqrt[m[r]] -
      (p - r) (2 + h - 3 r + 4 ell π t^2) f[2 + r, t] Sqrt[1 + m[r]] +
      2 t ((p + r) Sqrt[m[r]] f^(0,1)[-2 + r, t] + (p - r) Sqrt[1 + m[r]] f^(0,1)[2 + r, t]))}
```

Out[ ]=
```
{1/12 f[r, t] (48 + h^2 - 4 j^2 - 12 nu^2 + 9 r^2 - 48 ell π t^2 -
      24 ell π r t^2 - 48 ell^2 π^2 t^4 - 6 h (r + 4 ell π t^2) - 96 π t^2 Abs[ell] (1 + m[r])) +
   t (2 i Sqrt[2 π] Sqrt[Abs[ell]] ((p - r) f[2 + r, t] Sqrt[m[r]] - (p + r) f[-2 + r, t] Sqrt[1 + m[r]]) -
      3 f^(0,1)[r, t] + t f^(0,2)[r, t]), -1/9 f[r, t]
   (h^3 - 2 j^3 + 18 j nu^2 - 9 h^2 r + 9 j^2 r + 27 nu^2 r - 27 r^3 - 3 h (j^2 + 3 nu^2 - 9 r^2) - 216 ell p π t^2 -
      108 ell p^2 π t^2 + 216 ell π r t^2 + 108 ell π r^2 t^2 + 432 π r t^2 Abs[ell] (1 + m[r])) -
   3 i Sqrt[2 π] t Sqrt[Abs[ell]] ((p - r) (2 + h - 3 r + 4 ell π t^2) f[2 + r, t] Sqrt[m[r]] +
      (p + r) (2 - h + 3 r - 4 ell π t^2) f[-2 + r, t] Sqrt[1 + m[r]] -
      2 t ((p + r) Sqrt[1 + m[r]] f^(0,1)[-2 + r, t] + (p - r) Sqrt[m[r]] f^(0,1)[2 + r, t]))}
```

Comparison with routine **efeqn** in 12f. (That routine was taken from a previous version; so this is a check of the result.)

In[ ]:=
```
Clear[h, p, r, f, ell]
efeqn[h, p, r, f, ell, m[r], 1] == eqp // Simplify
efeqn[h, p, r, f, ell, m[r], -1] == eqm // Simplify
```

Out[ ]= True

Out[ ]= True