13a. N-trivial Fourier term modules

In[ ]:=
```
Clear[f, r, p]
F = f[r, t] × Phi[h, p, r, p]
```

Out[ ]= f[r, t] × Phi[h, p, r, p]

Eigenfunction equation

In[ ]:=
```
efeq = efeqt[h, p, r, f] // Simplify
```

Out[ ]= $\left\{\frac{1}{12} \times \left(-4\left(j^2+3\times\left(-4+nu^2\right)\right)+(h-3r)^2\right) f[r,t]+t\left(-3 f^{(0,1)}[r,t]+t f^{(0,2)}[r,t]\right),\ (h-2j-3r)(h+j+3nu-3r)(h+j-3(nu+r))\, f[r,t]\right\}$

First downward shift operator

We solve the relation for a non-trivial kernel

In[ ]:=
```
sh[3, -1, F, subtriv] // Simplify
fra = f^(0,1)[r, t] /. Solve[% == 0, f^(0,1)[r, t]][[1]] // Simplify
```

Out[ ]= $\frac{1}{4\times(1+p)} p\, Phi[3+h, -1+p, 1+r, -1+p]\left((-4+h-2p-r) f[r,t]+2t f^{(0,1)}[r,t]\right)$

Out[ ]= $-\frac{(-4+h-2p-r) f[r,t]}{2t}$

Substitute in eigenfunction equation

In[ ]:=
```
efeq /. f^(0,2)[r, t] → D[fra, t] /. f^(0,1)[r, t] → fra // Simplify
% /. r → (h - 2 j) / 3 // Simplify
% // Factor
```

Out[ ]= $\left\{\frac{1}{3}\left(h^2-j^2-3h(p+r)+3\left(-nu^2+p^2+pr+r^2\right)\right) f[r,t],\ (h-2j-3r)(h+j+3nu-3r)(h+j-3(nu+r))\, f[r,t]\right\}$

Out[ ]= $\left\{\frac{1}{9}\left(h^2+j^2-9nu^2+2h(j-3p)-6jp+9p^2\right) f\left[\frac{1}{3}(h-2j), t\right], 0\right\}$

Out[ ]= $\left\{\frac{1}{9}(h+j-3nu-3p)(h+j+3nu-3p)\, f\left[\frac{1}{3}(h-2j), t\right], 0\right\}$

Second downward shift operator

```
In[ ]:= sh[-3, -1, F, subtriv] // Simplify
        fra = f^(0,1)[r, t] /. Solve[% == 0, f^(0,1)[r, t]][[1]] // Simplify
```

Out[ ]= $\frac{1}{4\times(1+p)}\,p\,\mathrm{Phi}[-3+h,\,-1+p,\,-1+r,\,-1+p]\left(-((4+h+2p-r)\,f[r,t])+2\,t\,f^{(0,1)}[r,t]\right)$

Out[ ]= $\frac{(4+h+2p-r)\,f[r,t]}{2t}$

```
In[ ]:= efeq /. f^(0,2)[r, t] → D[fra, t] /. f^(0,1)[r, t] → fra // Simplify
        % /. r → (h - 2 j) / 3 // Simplify // Factor
```

Out[ ]= $\left\{\frac{1}{3}\left(h^2-j^2+3h(p-r)+3\left(-nu^2+p^2-p\,r+r^2\right)\right)f[r,t],\;(h-2j-3r)(h+j+3nu-3r)(h+j-3(nu+r))\,f[r,t]\right\}$

Out[ ]= $\left\{\frac{1}{9}(h+j-3nu+3p)(h+j+3nu+3p)\,f\left[\frac{1}{3}(h-2j),\,t\right],\,0\right\}$