13c. Non-abelian case

First downward shift operator

Get the kernel relations

In[ ]:=
```
Clear[sum, f, r, m, p, h]
F = tht[m[h, r]] × f[r, t] × Phi[h, p, r, p]
s3m1 = sh[3, -1, F, subnab]
```

Out[ ]= f[r, t] × Phi[h, p, r, p] × tht[m[h, r]]

Out[ ]= $-\frac{1}{4\times(1+p)}\, p\,\Big(2\,i\,\sqrt{2\,\pi}\;t\,\sqrt{\text{Abs[ell]}}\;\text{f[r, t]}\times\text{Phi[3 + h, -1 + p, -1 + r, -1 + p]}\Big((1+\text{eps})\sqrt{\text{m[h, r]}}\;\text{tht[-1 + m[h, r]]}+(-1+\text{eps})\sqrt{1+\text{m[h, r]}}\;\text{tht[1 + m[h, r]]}\Big)+\text{Phi[3 + h, -1 + p, 1 + r, -1 + p]}\times\text{tht[m[h, r]]}\,\big((4-h+2\,p+r-4\,\text{ell}\,\pi\,t^2)\,\text{f[r, t]}-2\,t\,f^{(0,1)}\text{[r, t]}\big)\Big)$

Separate values 1 and -1 of **eps**=Sign[**ell**]

Shift the summation variable r such that Phi[hh,pp,r,pp] occurs

In[ ]:=
```
Clear[relp]
s3m1 /. eps → 1 // compr
relp[r_] = Solve[% == 0, f[r + 1, t]][[1]] /. r → r - 1 /. m[h, r - 2] → m[h, r] - 1 // Simplify
```

Out[ ]= $-\frac{1}{4\times(1+p)}\,p\,\text{Phi[3 + h, -1 + p, r, -1 + p]}\Big((3-h+2\,p+r-4\,\text{ell}\,\pi\,t^2)\,\text{f[-1 + r, t]}\times\text{tht[m[h, -1 + r]]}+4\,i\,\sqrt{2\,\pi}\;t\,\sqrt{\text{Abs[ell]}}\;\text{f[1 + r, t]}\,\sqrt{\text{m[h, 1 + r]}}\;\text{tht[-1 + m[h, 1 + r]]}-2\,t\,\text{tht[m[h, -1 + r]]}\,f^{(0,1)}\text{[-1 + r, t]}\Big)$

Out[ ]= $\Big\{\text{f[r, t]}\to-\Big(\big(i\,((-2+h-2\,p-r+4\,\text{ell}\,\pi\,t^2)\,\text{f[-2 + r, t]}+2\,t\,f^{(0,1)}\text{[-2 + r, t]})\big)\Big/\Big(4\,\sqrt{2\,\pi}\;t\,\sqrt{\text{Abs[ell]}}\;\sqrt{\text{m[h, r]}}\Big)\Big)\Big\}$

In[ ]:=
```
Clear[relm]
s3m1 /. eps → -1 // compr
relm[r_] = Solve[% == 0, f[r + 1, t]][[1]] /. r → r - 1 /. m[r - 2] → m[r] + 1 // Simplify
```

Out[ ]= $\frac{1}{4\times(1+p)}\,p\,\text{Phi[3 + h, -1 + p, r, -1 + p]}\Big((-3+h-2\,p-r+4\,\text{ell}\,\pi\,t^2)\,\text{f[-1 + r, t]}\times\text{tht[m[h, -1 + r]]}+2\,t\Big(2\,i\,\sqrt{2\,\pi}\;\sqrt{\text{Abs[ell]}}\;\text{f[1 + r, t]}\,\sqrt{1+\text{m[h, 1 + r]}}\;\text{tht[1 + m[h, 1 + r]]}+\text{tht[m[h, -1 + r]]}\,f^{(0,1)}\text{[-1 + r, t]}\Big)\Big)$

Out[ ]= $\Big\{\text{f[r, t]}\to\big(i\,\text{tht[m[h, -2 + r]]}\,((-2+h-2\,p-r+4\,\text{ell}\,\pi\,t^2)\,\text{f[-2 + r, t]}+2\,t\,f^{(0,1)}\text{[-2 + r, t]})\big)\Big/\Big(4\,\sqrt{2\,\pi}\;t\,\sqrt{\text{Abs[ell]}}\;\sqrt{1+\text{m[h, r]}}\;\text{tht[1 + m[h, r]]}\Big)\Big\}$

Insert in eigenfunction equations for lowest value of r.

First for eps=1, r0<-p, and r=-p.

```
In[ ]:= f2p = f[2 - p, t] /. relp[2 - p]
eip1 = efeqn[h, p, -p, f, ell, m[h, -p], 1] /. f^(0,1)[2 - p, t] → D[f2p, t] /. f[2 - p, t] → f2p /.
    m[h, 2 - p] → m[h, -p] + 1 // Simplify
Coefficient[%, f^(0,2)[-p, t]] // Simplify
```

Out[ ]= $$-\frac{i\left(\left(-4+h-p+4\,\text{ell}\,\pi\,t^2\right)f[-p,t]+2\,t\,f^{(0,1)}[-p,t]\right)}{4\sqrt{2\pi}\;t\sqrt{\text{Abs}[\text{ell}]}\;\sqrt{m[h,2-p]}}$$

Out[ ]= $$\Big\{f[-p,t]\left(4+\frac{h^2}{12}-\frac{j^2}{3}-\text{nu}^2+4\,p+\frac{7\,p^2}{4}-4\,\text{ell}\,\pi\,t^2-2\,\text{ell}\,p\,\pi\,t^2-4\,\text{ell}^2\,\pi^2\,t^4-\frac{1}{2}\,h\left(p+4\,\text{ell}\,\pi\,t^2\right)-8\,\pi\,t^2\,\text{Abs}[\text{ell}]\,m[h,-p]\right)+t\left(-\left((3+2\,p)\,f^{(0,1)}[-p,t]\right)+t\,f^{(0,2)}[-p,t]\right),$$
$$f[-p,t]\left(-\frac{h^3}{9}+\frac{2\,j^3}{9}-2\,j\,\text{nu}^2-24\,p+\frac{h^2\,p}{2}+j^2\,p+3\,\text{nu}^2\,p-24\,p^2-\frac{15\,p^3}{2}+24\,\text{ell}\,p\,\pi\,t^2+12\,\text{ell}\,p^2\,\pi\,t^2+24\,\text{ell}^2\,p\,\pi^2\,t^4+h\left(\frac{j^2}{3}+\text{nu}^2+12\,\text{ell}\,p\,\pi\,t^2\right)+48\,p\,\pi\,t^2\,\text{Abs}[\text{ell}]\,m[h,-p]\right)+6\,p\,t\left((3+2\,p)\,f^{(0,1)}[-p,t]-t\,f^{(0,2)}[-p,t]\right)\Big\}$$

Out[ ]= $\{t^2, -6\,p\,t^2\}$

```
In[ ]:= 6 p eip1[[1]] + eip1[[2]] // Factor
```

Out[ ]= $$-\frac{1}{9}\,(h-2\,j-3\,p)\,(h+j-3\,\text{nu}-3\,p)\,(h+j+3\,\text{nu}-3\,p)\,f[-p,t]$$

Next for eps = 1, p≤r0<p, and r = r0.

In[ ]:=
```
Clear[r0]
fr0p = f[2 + r0, t] /. relp[2 + r0]
eip2 = efeqn[h, p, r0, f, ell, 0, 1] /. f^(0,1)[2 + r0, t] → D[fr0p, t] /. f[2 + r0, t] → fr0p /.
    m[h, r0 + 2] → 1 // Simplify
Coefficient[%, f^(0,2)[r0, t]] // Simplify
```

Out[ ]= $-\left(\left(i\left((-4+h-2p-r0+4\,ell\,\pi\,t^2)\,f[r0,t]+2\,t\,f^{(0,1)}[r0,t]\right)\right)\Big/\left(4\sqrt{2\pi}\,t\sqrt{\mathrm{Abs}[ell]}\sqrt{m[h,2+r0]}\right)\right)$

Out[ ]= $\Big\{\frac{1}{12}\,(h^2-4j^2-6h\,(p+4\,ell\,\pi\,t^2)-$
$3\times(-16+4\,nu^2-4p^2+8\,r0-r0^2+16\,ell\,\pi\,t^2+16\,ell^2\,\pi^2\,t^4+2p\,(-4+r0+4\,ell\,\pi\,t^2)))$
$f[r0,t]+t\,((-3-p+r0)\,f^{(0,1)}[r0,t]+t\,f^{(0,2)}[r0,t]),$
$-\frac{1}{9}\,(h^3-2j^3+18\,j\,nu^2-9h^2\,r0+9j^2\,r0-3h\,(j^2+3\,nu^2-9\,r0^2)-$
$27\,(-nu^2\,r0+r0^3+4\,ell\,p\,(2+p)\,\pi\,t^2-8\,ell\,\pi\,r0\,t^2-4\,ell\,\pi\,r0^2\,t^2))\,f[r0,t]+\frac{3}{4}\,(p-r0)$
$((-16+h^2+8\,r0+3\,r0^2-16\,ell\,\pi\,t^2-16\,ell\,\pi\,r0\,t^2+16\,ell^2\,\pi^2\,t^4+p\,(-8+6\,r0-8\,ell\,\pi\,t^2)-$
$2h\,(p+2\,r0-4\,ell\,\pi\,t^2))\,f[r0,t]-4t\,((-3-p+r0)\,f^{(0,1)}[r0,t]+t\,f^{(0,2)}[r0,t]))\Big\}$

Out[ ]= $\{t^2,\ 3\,(-p+r0)\,t^2\}$

In[ ]:=
```
3 (r0 - p) eip2[[1]] - eip2[[2]] // Factor
```

Out[ ]= $\frac{1}{9}\,(h-2j-3p)\,(h+j-3\,nu-3p)\,(h+j+3\,nu-3p)\,f[r0,t]$

For eps=1, r0 = p, no substitution is necessary.

In[ ]:=
```
efeqn[h, p, p, f, ell, 0, 1] // Simplify
%[[2]] // Factor
```

Out[ ]= $\Big\{\frac{1}{12}\,(h^2-4j^2-6h\,(p+4\,ell\,\pi\,t^2)-3\times(4\,nu^2-3p^2+8\,ell\,p\,\pi\,t^2+16\times(-1+ell\,\pi\,t^2+ell^2\,\pi^2\,t^4)))$
$f[p,t]+t\,(-3\,f^{(0,1)}[p,t]+t\,f^{(0,2)}[p,t]),$
$-\frac{1}{9}\,(h-2j-3p)\,(h^2+j^2-9\,nu^2+2h\,(j-3p)-6\,j\,p+9p^2)\,f[p,t]\Big\}$

Out[ ]= $-\frac{1}{9}\,(h-2j-3p)\,(h+j-3\,nu-3p)\,(h+j+3\,nu-3p)\,f[p,t]$

For eps = -1, r=-p, r0>-p

In[ ]:=
```
f2p = f[2 - p, t] /. relm[2 - p]
eip3 = efeqn[h, p, -p, f, ell, m[h, -p], -1] /. f^(0,1)[2 - p, t] → D[f2p, t] /. f[2 - p, t] → f2p /.
    m[h, 2 - p] → m[h, -p] - 1 // Simplify
Coefficient[%, f^(0,2)[-p, t]] // Simplify
```

Out[ ]=
$$\left(i\, \mathrm{tht}[m[h,-p]]\left(\left(-4+h-p+4\,\mathrm{ell}\,\pi t^2\right) f[-p,t]+2\,t\, f^{(0,1)}[-p,t]\right)\right) \Big/ \left(4\sqrt{2\pi}\, t\sqrt{\mathrm{Abs}[\mathrm{ell}]}\sqrt{1+m[h,2-p]}\;\mathrm{tht}[1+m[h,2-p]]\right)$$

Out[ ]=
$$\Big\{\frac{1}{12} f[-p,t]\left(48+h^2-4j^2-12\,nu^2+48p+21p^2-48\,\mathrm{ell}\,\pi t^2-24\,\mathrm{ell}\,p\,\pi t^2-48\,\mathrm{ell}^2\pi^2 t^4-6h\left(p+4\,\mathrm{ell}\,\pi t^2\right)-96\pi t^2\,\mathrm{Abs}[\mathrm{ell}]\left(1+m[h,-p]\right)\right)+t\left(-\left((3+2p)\, f^{(0,1)}[-p,t]\right)+t\, f^{(0,2)}[-p,t]\right),$$
$$f[-p,t]\left(-\frac{h^3}{9}+\frac{2j^3}{9}-2j\,nu^2-24p+\frac{h^2 p}{2}+j^2 p+3\,nu^2 p-24p^2-\frac{15p^3}{2}+24\,\mathrm{ell}\,p\,\pi t^2+12\,\mathrm{ell}\,p^2\pi t^2+24\,\mathrm{ell}^2 p\,\pi^2 t^4+h\left(\frac{j^2}{3}+nu^2+12\,\mathrm{ell}\,p\,\pi t^2\right)+48p\,\pi t^2\,\mathrm{Abs}[\mathrm{ell}]\left(1+m[h,-p]\right)\right)+6pt\left((3+2p)\, f^{(0,1)}[-p,t]-t\, f^{(0,2)}[-p,t]\right)\Big\}$$

Out[ ]=
$$\left\{t^2,\,-6\,p\,t^2\right\}$$

In[ ]:=
```
6 p eip3[[1]] + eip3[[2]] // Factor
```

Out[ ]=
$$-\frac{1}{9}(h-2j-3p)(h+j-3\,nu-3p)(h+j+3\,nu-3p)\, f[-p,t]$$

Last case eps = -1, r0=-p

In[ ]:=
```
efeqn[h, p, -p, f, ell, 0, -1][[2]] /. Abs[ell] → -ell // Factor
```

Out[ ]=
$$-\frac{1}{9}(h-2j+3p)(h+j-3\,nu+3p)(h+j+3\,nu+3p)\, f[-p,t]$$

Second downward shift operator

Get the kernel relations

In[ ]:=
```
F = tht[m[h, r]] × f[r, t] × Phi[h, p, r, p]
sm3m1 = sh[-3, -1, F, subnab]
```

Out[ ]=
$$f[r,t]\times \mathrm{Phi}[h,p,r,p]\times \mathrm{tht}[m[h,r]]$$

Out[ ]=
$$\frac{1}{4\times(1+p)}\,p\Big(2i\sqrt{2\pi}\,t\sqrt{\mathrm{Abs}[\mathrm{ell}]}\; f[r,t]\times \mathrm{Phi}[-3+h,-1+p,1+r,-1+p]\left((-1+eps)\sqrt{m[h,r]}\;\mathrm{tht}[-1+m[h,r]]+(1+eps)\sqrt{1+m[h,r]}\;\mathrm{tht}[1+m[h,r]]\right)-\mathrm{Phi}[-3+h,-1+p,-1+r,-1+p]\times \mathrm{tht}[m[h,r]]\left(\left(4+h+2p-r+4\,\mathrm{ell}\,\pi t^2\right) f[r,t]-2t\, f^{(0,1)}[r,t]\right)\Big)$$

Separate values 1 and -1 of **eps**=Sign[**ell**]

Shift the summation variable r such that Phi[hh,pp,r,pp] occurs

In[ ]:=
```
Clear[relp]
sm3m1 /. eps → 1 // compr
relp[r_] = Solve[% == 0, f[r - 1, t]][[1]] /. r → r + 1 /. m[h, r + 2] → m[h, r] + 1 // Simplify
```

Out[ ]=
```
1/(4 × (1 + p)) p Phi[-3 + h, -1 + p, r, -1 + p]
 (4 ⅈ Sqrt[2 π] t Sqrt[Abs[ell]] f[-1 + r, t] Sqrt[1 + m[h, -1 + r]] tht[1 + m[h, -1 + r]] -
   tht[m[h, 1 + r]] ((3 + h + 2 p - r + 4 ell π t^2) f[1 + r, t] - 2 t f^(0,1)[1 + r, t]))
```

Out[ ]=
```
{f[r, t] → -((ⅈ ((2 + h + 2 p - r + 4 ell π t^2) f[2 + r, t] - 2 t f^(0,1)[2 + r, t])) /
     (4 Sqrt[2 π] t Sqrt[Abs[ell]] Sqrt[1 + m[h, r]]))}
```

In[ ]:=
```
Clear[relm]
sm3m1 /. eps → -1 // compr
relm[r_] = Solve[% == 0, f[r - 1, t]][[1]] /. r → r + 1 /. m[h, r + 2] → m[h, r] - 1 // Simplify
```

Out[ ]=
```
-1/(4 × (1 + p)) p Phi[-3 + h, -1 + p, r, -1 + p]
 (4 ⅈ Sqrt[2 π] t Sqrt[Abs[ell]] f[-1 + r, t] Sqrt[m[h, -1 + r]] tht[-1 + m[h, -1 + r]] +
   tht[m[h, 1 + r]] ((3 + h + 2 p - r + 4 ell π t^2) f[1 + r, t] - 2 t f^(0,1)[1 + r, t]))
```

Out[ ]=
```
{f[r, t] →
  (ⅈ ((2 + h + 2 p - r + 4 ell π t^2) f[2 + r, t] - 2 t f^(0,1)[2 + r, t])) / (4 Sqrt[2 π] t Sqrt[Abs[ell]] Sqrt[m[h, r]])}
```

Now the relation between the components runs downward

Case eps=1, r0<p; r=p

In[ ]:=
```
f2m = f[p - 2, t] /. relp[p - 2] // Simplify
eia = efeqn[h, p, p, f, ell, m[h, p], 1] /. f^(0,1)[-2 + p, t] → D[f2m, t] /. f[-2 + p, t] → f2m /.
    m[h, p - 2] → m[h, p] - 1 /. Abs[ell] → ell // Simplify
Coefficient[
 %,
 f^(0,2)[
  p,
  t]]
```

Out[ ]= $$-\frac{i\,\left(\left(4+h+p+4\,\text{ell}\,\pi\,t^2\right) f[p,\,t]-2\,t\,f^{(0,1)}[p,\,t]\right)}{4\,\sqrt{2\,\pi}\,\,t\,\sqrt{\text{Abs[ell]}}\,\sqrt{1+m[h,\,-2+p]}}$$

Out[ ]= $$\Big\{f[p,\,t]\left(4+\frac{h^2}{12}-\frac{j^2}{3}-nu^2+4\,p+\frac{h\,p}{2}+\frac{7\,p^2}{4}-4\,\text{ell}\,\pi\,t^2-2\,\text{ell}\,h\,\pi\,t^2+2\,\text{ell}\,p\,\pi\,t^2-4\,\text{ell}^2\,\pi^2\,t^4-8\,\text{ell}\,\pi\,t^2\,m[h,\,p]\right)+t\left(-\left((3+2\,p)\,f^{(0,1)}[p,\,t]\right)+t\,f^{(0,2)}[p,\,t]\right),$$
$$f[p,\,t]\left(-\frac{h^3}{9}+\frac{2\,j^3}{9}-2\,j\,nu^2+24\,p-\frac{h^2\,p}{2}-j^2\,p-3\,nu^2\,p+24\,p^2+\frac{15\,p^3}{2}-24\,\text{ell}\,p\,\pi\,t^2+12\,\text{ell}\,p^2\,\pi\,t^2-24\,\text{ell}^2\,p\,\pi^2\,t^4+h\left(\frac{j^2}{3}+nu^2-12\,\text{ell}\,p\,\pi\,t^2\right)-48\,\text{ell}\,p\,\pi\,t^2\,m[h,\,p]\right)+$$
$$6\,p\,t\left(-\left((3+2\,p)\,f^{(0,1)}[p,\,t]\right)+t\,f^{(0,2)}[p,\,t]\right)\Big\}$$

Out[ ]= $\{t^2,\ 6\,p\,t^2\}$

In[ ]:=
```
6 p eia[[1]] - eia[[2]] // Factor
```

Out[ ]= $$\frac{1}{9}\,(h-2\,j+3\,p)\,(h+j-3\,nu+3\,p)\,(h+j+3\,nu+3\,p)\,f[p,\,t]$$

This has indeed the right solutions h = -3 p + 2w'

Case eps = 1, r0 =p; r = p

No substitution needed

In[ ]:=
```
efeqn[h, p, p, f, ell, 0, 1] // Factor
```

Out[ ]= $$\Big\{\frac{1}{12}\times\left(48\,f[p,\,t]+h^2\,f[p,\,t]-4\,j^2\,f[p,\,t]-12\,nu^2\,f[p,\,t]-6\,h\,p\,f[p,\,t]+9\,p^2\,f[p,\,t]-48\,\text{ell}\,\pi\,t^2\,f[p,\,t]-24\,\text{ell}\,h\,\pi\,t^2\,f[p,\,t]-24\,\text{ell}\,p\,\pi\,t^2\,f[p,\,t]-48\,\text{ell}^2\,\pi^2\,t^4\,f[p,\,t]-36\,t\,f^{(0,1)}[p,\,t]+12\,t^2\,f^{(0,2)}[p,\,t]\right),$$
$$-\frac{1}{9}\,(h-2\,j-3\,p)\,(h+j-3\,nu-3\,p)\,(h+j+3\,nu-3\,p)\,f[p,\,t]\Big\}$$

The second coordinate gives the right factor

Case eps = -1, r0 >p, r=p

In[ ]:=
```
f2m = f[p - 2, t] /. relm[p - 2]
eib = efeqn[h, p, p, f, ell, m[h, p], -1] /. f^(0,1)[-2 + p, t] → D[f2m, t] /. f[-2 + p, t] → f2m /.
    m[h, p - 2] → m[h, p] + 1 // Simplify
Coefficient[% , f^(0,2)[p, t]]
```

Out[ ]=
$$\frac{i\left(\left(4+h+p+4\,\text{ell}\,\pi\,t^2\right)f[p,t]-2\,t\,f^{(0,1)}[p,t]\right)}{4\sqrt{2\pi}\,t\,\sqrt{\text{Abs[ell]}}\,\sqrt{m[h,-2+p]}}$$

Out[ ]=
$$\Big\{\frac{1}{12}\,f[p,t]\left(48+h^2-4\,j^2-12\,\text{nu}^2+48\,p+21\,p^2-48\,\text{ell}\,\pi\,t^2+24\,\text{ell}\,p\,\pi\,t^2-48\,\text{ell}^2\,\pi^2\,t^4+6\,h\left(p-4\,\text{ell}\,\pi\,t^2\right)-96\,\pi\,t^2\,\text{Abs[ell]}\left(1+m[h,p]\right)\right)+t\left(-\left((3+2\,p)\,f^{(0,1)}[p,t]\right)+t\,f^{(0,2)}[p,t]\right),$$
$$-\frac{1}{18}\,f[p,t]\left(2\,h^3-4\,j^3+36\,j\,\text{nu}^2-432\,p+9\,h^2\,p+18\,j^2\,p+54\,\text{nu}^2\,p-432\,p^2-135\,p^3+432\,\text{ell}\,p\,\pi\,t^2-216\,\text{ell}\,p^2\,\pi\,t^2+432\,\text{ell}^2\,p\,\pi^2\,t^4-6\,h\left(j^2+3\,\text{nu}^2-36\,\text{ell}\,p\,\pi\,t^2\right)+864\,p\,\pi\,t^2\,\text{Abs[ell]}\left(1+m[h,p]\right)\right)+6\,p\,t\left(-\left((3+2\,p)\,f^{(0,1)}[p,t]\right)+t\,f^{(0,2)}[p,t]\right)\Big\}$$

Out[ ]=
$$\{t^2,\ 6\,p\,t^2\}$$

In[ ]:=
```
6 p eib[[1]] - eib[[2]] // Factor
```

Out[ ]=
$$\frac{1}{9}\,(h-2\,j+3\,p)\,(h+j-3\,\text{nu}+3\,p)\,(h+j+3\,\text{nu}+3\,p)\,f[p,t]$$

Case eps = -1,-p< r0 <=p, r=r0

In[ ]:=
```
f2m = f[r0 - 2, t] /. relm[r0 - 2] /. m[h, r0 - 2] → 1
eic =
 efeqn[h, p, r0, f, ell, 0, -1] /. f^(0,1)[r0 - 2, t] → D[f2m, t] /. f[r0 - 2, t] → f2m // Simplify
Coefficient[%, f^(0,2)[r0, t]] // Simplify
```

Out[ ]=
$$\frac{i\left(\left(4+h+2\,p-r0+4\,\text{ell}\,\pi\,t^2\right)f[r0,t]-2\,t\,f^{(0,1)}[r0,t]\right)}{4\sqrt{2\pi}\,t\,\sqrt{\text{Abs[ell]}}}$$

Out[ ]=
$$\Big\{\left(4+\frac{h^2}{12}-\frac{j^2}{3}-\text{nu}^2+2\,p+\frac{h\,p}{2}+p^2+2\,r0+\frac{p\,r0}{2}+\frac{r0^2}{4}-4\,\text{ell}\,\pi\,t^2-2\,\text{ell}\,h\,\pi\,t^2+2\,\text{ell}\,p\,\pi\,t^2-4\,\text{ell}^2\,\pi^2\,t^4-8\,\pi\,t^2\,\text{Abs[ell]}\right)f[r0,t]+t\left(-\left((3+p+r0)\,f^{(0,1)}[r0,t]\right)+t\,f^{(0,2)}[r0,t]\right),\ -\frac{1}{9}$$
$$\left(h^3-2\,j^3+18\,j\,\text{nu}^2-9\,h^2\,r0+9\,j^2\,r0+27\,\text{nu}^2\,r0-27\,r0^3-3\,h\left(j^2+3\,\text{nu}^2-9\,r0^2\right)-216\,\text{ell}\,p\,\pi\,t^2-108\,\text{ell}\,p^2\,\pi\,t^2+216\,\text{ell}\,\pi\,r0\,t^2+108\,\text{ell}\,\pi\,r0^2\,t^2+432\,\pi\,r0\,t^2\,\text{Abs[ell]}\right)f[r0,t]+\frac{3}{4}\,(p+r0)$$
$$\left(-\left(\left(-16+h^2-8\,r0+3\,r0^2+16\,\text{ell}\,\pi\,t^2-16\,\text{ell}\,\pi\,r0\,t^2+16\,\text{ell}^2\,\pi^2\,t^4+2\,h\left(p-2\,r0+4\,\text{ell}\,\pi\,t^2\right)+p\left(-8-6\,r0+8\,\text{ell}\,\pi\,t^2\right)\right)f[r0,t]\right)-4\,t\left((3+p+r0)\,f^{(0,1)}[r0,t]-t\,f^{(0,2)}[r0,t]\right)\right)\Big\}$$

Out[ ]=
$$\{t^2,\ 3\,(p+r0)\,t^2\}$$

In[ ]:=
```
3 (p + r0) eic⟦1⟧ - eic⟦2⟧ /. Abs[ell] → -ell // Factor
```

Out[ ]= $\frac{1}{9}$ (h - 2 j + 3 p) (h + j - 3 nu + 3 p) (h + j + 3 nu + 3 p) f[r0, t]

Case eps = -1,r0=-p, r=-p

In[ ]:=
```
efeqn[h, p, -p, f, ell, 0, -1] /. Abs[ell] → -ell //
  Simplify
%⟦2⟧ // Factor
```

Out[ ]= $\left\{\frac{1}{12}\times\left(48+h^2-4\,j^2-12\,nu^2+9\,p^2+48\,ell\,\pi\,t^2+24\,ell\,p\,\pi\,t^2-48\,ell^2\,\pi^2\,t^4+6\,h\left(p-4\,ell\,\pi\,t^2\right)\right)\right.$ $f[-p,t]+t\left(-3\,f^{(0,1)}[-p,t]+t\,f^{(0,2)}[-p,t]\right),$ $\left.-\frac{1}{9}(h-2\,j+3\,p)\left(h^2+j^2-9\,nu^2+6\,j\,p+9\,p^2+2\,h\,(j+3\,p)\right)f[-p,t]\right\}$

Out[ ]= $-\frac{1}{9}$ (h - 2 j + 3 p) (h + j - 3 nu + 3 p) (h + j + 3 nu + 3 p) f[-p, t]