

## 15b. Intertwining operator $v \mapsto -v$

Check of (3.80)

```
In[ ]:= Clear[c]
```

```
c[h_, p_, r_, nu_] := Gamma[1 + (p - nu) / 2 + (h - r) / 4] Gamma[1 + (p - nu) / 2 + (r - h) / 4]
Gamma[1 + (p + nu) / 2 + (h - r) / 4] ^ (-1) Gamma[1 + (p + nu) / 2 + (r - h) / 4] ^ (-1);
```

```
In[ ]:= Clear[ii0]
```

```
ii0[t^(2 + nu_) Phi[h_, p_, r_, q_]] := c[h, p, r, nu] * phi[h, p, r, q, -nu]
ii0[ff_ gg_] := ff ii0[gg] /; FreeQ[ff, t] && FreeQ[ff, Phi]
```

Inverse

```
In[ ]:= phi[h, p, r, q, nu]
```

```
ii0[%]
```

```
ii0[%]
```

```
Out[ ]:= t^(2+nu) Phi[h, p, r, q]
```

$$\text{Out[ ]} = \left( t^{2-\text{nu}} \Gamma\left[1 + \frac{1}{2}(-\text{nu} + p) + \frac{h-r}{4}\right] \Gamma\left[1 + \frac{1}{2}(-\text{nu} + p) + \frac{1}{4}(-h+r)\right] \text{Phi}[h, p, r, q] \right) /$$

$$\left( \Gamma\left[1 + \frac{\text{nu} + p}{2} + \frac{h-r}{4}\right] \Gamma\left[1 + \frac{\text{nu} + p}{2} + \frac{1}{4}(-h+r)\right] \right)$$

```
Out[ ]:= t^(2+nu) Phi[h, p, r, q]
```

Check of intertwining property for shift operators

```
In[ ]:= Clear[h, p, r, nu]
```

```
ii0[sh[3, 1, phi[h, p, r, p, nu], subtriv]] /
sh[3, 1, ii0[phi[h, p, r, p, nu]], subtriv] // FullSimplify
```

```
Out[ ]:= 1
```

```
In[ ]:=
```

```
ii0[sh[3, -1, phi[h, p, r, p, nu], subtriv]] /
sh[3, -1, ii0[phi[h, p, r, p, nu]], subtriv] // FullSimplify
```

```
Out[ ]:= 1
```

```
In[ ]:=
```

```
ii0[sh[-3, 1, phi[h, p, r, p, nu], subtriv]] /
sh[-3, 1, ii0[phi[h, p, r, p, nu]], subtriv] // FullSimplify
```

```
Out[ ]:= 1
```

In[ \* ]:=

```
ii0[sh[-3, -1, phi[h, p, r, p, nu], subtriv]]/  
sh[-3, -1, ii0[phi[h, p, r, p, nu]], subtriv] // FullSimplify
```

Out[ \* ]= 1