22b. Case $m_0(j_1) = 0$, and $\varepsilon = 1$

In[ ]:=
```
m0[j1] == 0 /. mrsub
```
Out[ ]=
$$\frac{1}{2}\,\text{eps}\,(\text{p} - \text{r0}) == 0$$

So $r_0 = p$

In[ ]:=
```
Clear[F, Fp, Fm, f, fp, t, r0]
F = tht[0] × f[p, t] × Phi[h, p, p, p]
```
Out[ ]=
$$\text{f[p, t]} \times \text{Phi[h, p, p, p]} \times \text{tht[0]}$$

In[ ]:=
```
Fp = sh[-3, -1, F, subnab] // Simplify
```
Out[ ]=
$$-\frac{1}{4\times(1+\text{p})}\,\text{p Phi}[-3+\text{h}, -1+\text{p}, -1+\text{p}, -1+\text{p}]\times\text{tht}[0]\left(\left(4+\text{h}+\text{p}+4\,\text{ell}\,\pi\,\text{t}^2\right)\text{f[p, t]} - 2\,\text{t}\,\text{f}^{(0,1)}[\text{p, t}]\right)$$

The sole component of F^{+} has an expression in terms of f_p and on the other hand has an explicit expression in a V-Whittaker function.

With **cnz** denoting a non-zero factor, and **r0** the quantity $r_0(h)$:

In[ ]:=
```
eq = (((4 + h + p + 4 ell π t^2) f[p, t] - 2 t f^(0,1)[p, t]) == cnz t^(0 + 1)
      WhittakerV[-p + 1 - (j1 + 1)/2, nu1/2, 2 Pi Abs[ell] t^2]) /. eps → 1 /. hpsub // Simplify
```
Out[ ]=
$$\left(4+\text{h}+\text{p}+4\,\text{ell}\,\pi\,\text{t}^2\right)\text{f[p, t]} == \text{t}\left(\text{cnz WhittakerV}\left[\frac{1}{4}\times(2-\text{h}-\text{p}),\ \frac{1}{4}\,\text{Abs[h + p]},\ 2\,\pi\,\text{t}^2\,\text{Abs[ell]}\right] + 2\,\text{f}^{(0,1)}[\text{p, t}]\right)$$

Find expressions for derivatives of f_p

In[ ]:=
```
fpd = f^(0,1)[p, t] /. Solve[eq, f^(0,1)[p, t]][[1]] // Simplify
fpdd = D[fpd, t] /. Whrel // Simplify
```
Out[ ]=
$$\frac{1}{2\,\text{t}}\left(\left(4+\text{h}+\text{p}+4\,\text{ell}\,\pi\,\text{t}^2\right)\text{f[p, t]} - \text{cnz t WhittakerV}\left[\frac{1}{4}\times(2-\text{h}-\text{p}),\ \frac{1}{4}\,\text{Abs[h + p]},\ 2\,\pi\,\text{t}^2\,\text{Abs[ell]}\right]\right)$$

Out[ ]=
$$\frac{1}{16\,\text{t}^2}\Bigg(-8\times\left(4+\text{h}+\text{p}-4\,\text{ell}\,\pi\,\text{t}^2\right)\text{f[p, t]} + \text{t}\Bigg(-4\,\text{cnz}\left(-2+\text{h}+\text{p}+4\,\pi\,\text{t}^2\,\text{Abs[ell]}\right)\text{WhittakerV}\left[\frac{1}{4}\times(2-\text{h}-\text{p}),\ \frac{1}{4}\,\text{Abs[h + p]},\ 2\,\pi\,\text{t}^2\,\text{Abs[ell]}\right] - \text{cnz}\left((-4+\text{h}+\text{p})^2 - \text{Abs[h + p]}^2\right)\text{WhittakerV}\left[\frac{1}{4}\times(6-\text{h}-\text{p}),\ \frac{1}{4}\,\text{Abs[h + p]},\ 2\,\pi\,\text{t}^2\,\text{Abs[ell]}\right] + 8\times\left(4+\text{h}+\text{p}+4\,\text{ell}\,\pi\,\text{t}^2\right)\text{f}^{(0,1)}[\text{p, t}]\Bigg)\Bigg)$$

Insert these relations into the eigenfunction equations

In[ ]:=
```
ei = efeqn[h, p, p, f, ell, 0, 1] /. {nu → nu1, j → j1} /. f^(0,2)[p, t] → fpdd /. f^(0,1)[p, t] → fpd /.
    hpsub /. Abs[h + p]^2 → (h + p)^2 /. Abs[ell] → ell // Simplify
```

Out[ ]= $\left\{\frac{1}{2}\,\text{cnz}\,t\left(-\left(\left(-2+h+p+4\,\text{ell}\,\pi\,t^2\right)\text{WhittakerV}\left[\frac{1}{4}\times(2-h-p),\,\frac{1}{4}\,\text{Abs}[h+p],\,2\,\text{ell}\,\pi\,t^2\right]\right)+(-2+h+p)\,\text{WhittakerV}\left[\frac{1}{4}\times(6-h-p),\,\frac{1}{4}\,\text{Abs}[h+p],\,2\,\text{ell}\,\pi\,t^2\right]\right),\,0\right\}$

This should be zero.

Insert the asymptotic behavior.

In[ ]:=
```
Clear[tau]
ei /. t → Sqrt[tau] / Sqrt[2 Pi Abs[ell]] /. WhittakerV[kap_, s_, tau_] :→
    -E^(-Pi I kap) tau^(-kap) E^(tau / 2) /. Abs[ell] → ell // Simplify
% E^(-tau / 2) tau^(-1 - (h + p) / 4) // Simplify
Limit[%, tau → Infinity] //.
  {(1 / ell)^ee_ :→ ell^(-ee), (ell tau)^ee_ :→ ell^ee tau^ee} // Simplify
```

Out[ ]= $\left\{-\frac{1}{2\sqrt{2\pi}\,\sqrt{\text{tau}}}\,i\,\text{cnz}\,e^{\frac{1}{4}i(h\,\pi+p\,\pi-2\,i\,\text{tau})}\left(\frac{1}{\text{ell}}\right)^{\frac{1}{4}\times(2+h+p)}\sqrt{\text{ell}}\,(\text{ell tau})^{\frac{1}{4}\times(-2+h+p)}\left(-2+h+p-2\,\text{tau}+h\,\text{tau}+p\,\text{tau}+2\,\text{tau}^2\right),\,0\right\}$

Out[ ]= $\left\{-\frac{1}{2\sqrt{2\pi}}\,i\,\text{cnz}\,e^{\frac{1}{4}i(h+p)\pi}\left(\frac{1}{\text{ell}}\right)^{\frac{1}{4}\times(2+h+p)}\sqrt{\text{ell}}\,\text{tau}^{\frac{1}{4}\times(-6-h-p)}(\text{ell tau})^{\frac{1}{4}\times(-2+h+p)}\left(-2+h+p-2\,\text{tau}+h\,\text{tau}+p\,\text{tau}+2\,\text{tau}^2\right),\,0\right\}$

Out[ ]= $\left\{-\frac{i\,\text{cnz}\,e^{\frac{1}{4}i(h+p)\pi}}{\sqrt{\text{ell}}\,\sqrt{2\pi}},\,0\right\}$

This shows that **cnz** should be zero.