26c. Fourier terms

We check the transition from (5.19) to (5.20) .

We use the following spectral parameters:

In[◦]:=
```
{j, nu} = S2[S1[{-h - 3, 1}]]
```

Out[◦]= $\left\{\frac{h}{2}, -2 - \frac{h}{2}\right\}$

Fourier term of order 0 is multiple of the function in (3.77).

Take g=n(x,y,r)a(t)

In[◦]:=
```
Clear[h, x, y, r, t, k, g, F0, f0]
g = nm[x, y, r].am[t]; gs = ns[x, y, r] ** as[t]
```

Out[◦]= ns[x, y, r] ** as[t]

In[◦]:=
```
F0[actX[g, {I, 0}]] == t ^ (h / 2) f0[gs]
% /. f0[ns[x, y, r] ** as[t]] → t ^ (2 + nu) // zsub
```

Out[◦]= $\mathrm{F0}[\{2 r + i (t^2 + x^2 + y^2), i x + y\}] == t^{h/2}\, \mathrm{f0}[\mathrm{ns}[x, y, r] ** \mathrm{as}[t]]$

Out[◦]= F0[{z, u}] == 1

For the Fourier term of order (ell, c, d) we use (3.88)

In[◦]:=
```
Clear[Fl, fl]
rel = Fl[actX[g, {I, 0}]] == t ^ (h / 2) fl[gs]
```

Out[◦]= $\mathrm{Fl}[\{2 r + i (t^2 + x^2 + y^2), i x + y\}] == t^{h/2}\, \mathrm{fl}[\mathrm{ns}[x, y, r] ** \mathrm{as}[t]]$

In[◦]:=
```
rel1 =
  rel /. fl[n_ ** as[t]] → Theta[ell, c, hlm0, n] t WhittakerW[kap, nu / 2, 2 Pi Abs[ell] t ^ 2] /.
        kap → -m0 - (eps j + 1) / 2 /. Abs[ell] → ell /. eps → 1 /. m0 → 0 //. Whrel //.
    {(pp_ ^ ff_) ^ ee_ :→ pp ^ (ff ee), (pp_ qq_) ^ ee_ :→ pp ^ ee qq ^ ee} // Simplify
```

Out[◦]= $\mathrm{Fl}[\{2 r + i (t^2 + x^2 + y^2), i x + y\}] == e^{-\mathrm{ell}\, \pi\, t^2}\, \mathrm{ell}^{-\frac{1}{2} - \frac{h}{4}}\, (2\pi)^{\frac{1}{4} \times (-2-h)}\, \mathrm{Theta}[\mathrm{ell}, c, \mathrm{hlm0}, \mathrm{ns}[x, y, r]]$

In[◦]:=
```
Clear[k, sum]
rel2 = rel1 /. {Theta[ell_, c_, hlm0_, ns[x, y, r]] →
        sum[k] Exp[2 Pi I ell (r - x (c / ell + 2 k + y))] hlm0[c / (2 ell) + k + y]} /.
    {hlm0[xi_] :→ Sqrt[2] ell ^ (1 / 4) E ^ (-2 Pi ell xi ^ 2)} // Simplify
```

Out[◦]= $\sqrt{\pi}\, \mathrm{Fl}[\{2 r + i (t^2 + x^2 + y^2), i x + y\}] == e^{-\mathrm{ell}\, \pi \left(t^2 + 2\left(\frac{c}{2\,\mathrm{ell}} + k + y\right)^2 - 2 i \left(r - x\left(\frac{c}{\mathrm{ell}} + 2 k + y\right)\right)\right)}\, \mathrm{ell}^{-\frac{1}{4} - \frac{h}{4}}\, (2\pi)^{-h/4}\, \mathrm{sum}[k]$

In[ ]:=
```
Simplify[zsub[rel2]]
rel3 = % /. E^xx_ :→ E^(Expand[xx]) // Simplify
```

Out[ ]= $\sqrt{\pi}\ \mathrm{Fl}[\{z, u\}] == e^{-\frac{\pi\left(c^2+4\,c\,\mathrm{ell}\,(k+u)+2\,\mathrm{ell}^2\left(2\,k^2+4\,k\,u+u^2-i\,z\right)\right)}{2\,\mathrm{ell}}}\ \mathrm{ell}^{-\frac{1}{4}-\frac{h}{4}}\ (2\,\pi)^{-h/4}\ \mathrm{sum}[k]$

Out[ ]= $\sqrt{\pi}\ \mathrm{Fl}[\{z, u\}] == e^{-\frac{\pi\left(c^2+4\,c\,\mathrm{ell}\,(k+u)+2\,\mathrm{ell}^2\left(2\,k^2+4\,k\,u+u^2-i\,z\right)\right)}{2\,\mathrm{ell}}}\ \mathrm{ell}^{-\frac{1}{4}-\frac{h}{4}}\ (2\,\pi)^{-h/4}\ \mathrm{sum}[k]$

Comparison with (5.20)

In[ ]:=
```
rel3 /. Fl[{z, u}] → ell^(-1/4) E^(Pi I ell z) Pi^(-1/2) (2 Pi ell)^(-h/4)
    E^(Pi (ell/2) (c/ell + 2 k)^2) E^(-Pi ell (u + c/ell + 2 k)^2) sum[k] // Simplify
```

Out[ ]= True