27c. Holomorphic Fourier terms for general lattice

Computations for (5.45)

Now we have to deal with

```
In[ ]:= fl0 = Thetatau[ell, c, hlm0, ns[x, y, r]] t WhittakerW[kap, nu / 2, 2 Pi Abs[ell] t ^ 2] /.
            kap → -m0 - (eps j + 1) / 2 /. Abs[ell] → ell /. eps → 1 /. m0 → 0 //. Whrel // Simplify
```

Out[ ]= $e^{-\text{ell}\,\pi\,t^2}\,(2\,\pi)^{-\frac{1}{2}-\frac{h}{4}}\,t\,(\text{ell}\,t^2)^{-\frac{1}{2}-\frac{h}{4}}$ Thetatau[ell, c, hlm0, ns[x, y, r]]

Use of (5.38)

```
In[ ]:= Theta[ell, c, U[hlm0], Ald[tau, ns[x, y, r]]]
thetagen = % /. {Theta[ell, c, ph_, ns[x_, y_, r_]] :→
            sum[k] Exp[2 Pi I ell (r - x (c / ell + 2 k + y))] ph[c / (2 ell) + k + y]} /.
        {U[hlm0][xi_] :→ Im[tau] ^ (1 / 4) E ^ (-2 Pi I ell  xi ^ 2 Re[tau]) hlm0[xi Im[tau] ^ (1 / 2)]} /.
      {hlm0[xi_] :→ Sqrt[2] ell ^ (1 / 4) E ^ (-2 Pi ell xi ^ 2)} // Simplify
```

Out[ ]= Theta[ell, c, U[hlm0], ns[$\frac{\text{x Im[tau]} - \text{y Re[tau]}}{\sqrt{\text{Im[tau]}}}$, $\frac{\text{y}}{\sqrt{\text{Im[tau]}}}$, r]]

Out[ ]= $\sqrt{2}\, e^{-\frac{\pi\left(4\,\text{ell}^2\,(-i\,r+i\,x\,y+y^2)+4\,\text{ell}\,(c+2\,\text{ell}\,k)\,(i\,x+y)\,\sqrt{\text{Im[tau]}}+(c+2\,\text{ell}\,k)^2\,\text{Im[tau]}+i\,(c+2\,\text{ell}\,k)^2\,\text{Re[tau]}\right)}{2\,\text{ell}}}\,\text{ell}^{1/4}\,\text{Im[tau]}^{1/4}$ sum[k]

Insertion of the general theta function, and transition to coordinates on X.

```
In[ ]:= Clear[retau, imtau]
fl0 /. Thetatau[ell, c, hlm0, ns[x, y, r]] → thetagen /.
   (ell t^2) ^ ee_ :→ ell ^ ee t ^ (2 ee) // Simplify
rel = {Fl0[actX[nm[x, y, r].am[t], {I, 0}]], t ^ (h / 2) %} /. {x → Im[u], y → Re[u], r → Re[z] / 2,
        t ^ 2 → Im[z] - u Conjugate[u]} /. {Im[tau] → imtau, Re[tau] → retau} //.
    {Re[zz_] :→ (zz + Conjugate[zz]) / 2, Im[zz_] :→ (zz - Conjugate[zz]) / (2 I)} // Simplify
```

Shift of the summation variable

```
In[ ]:= Clear[al]
rel1 = rel /. {retau → Re[tau], imtau → Im[tau]} /. k → al - c / (2 ell) /.
    sum[al - c/(2 ell)] → summod[al, c / (2 ell)] /. E ^ xx_ :→ E ^ Expand[FullSimplify[xx]]
```

Out[ ]= $\{$Fl0[{z, u}], $2^{-h/4}\, e^{-\text{ell}\,\pi\,u^2+i\,\text{ell}\,\pi\,z-4\,\text{al}\,\text{ell}\,\pi\,u\,\sqrt{\text{Im[tau]}}-2\,\text{al}^2\,\text{ell}\,\pi\,\text{Im[tau]}-2\,i\,\text{al}^2\,\text{ell}\,\pi\,\text{Re[tau]}}$
$\text{ell}^{-\frac{1}{4}-\frac{h}{4}}\,\pi^{-\frac{1}{2}-\frac{h}{4}}\,\text{Im[tau]}^{1/4}$ summod[al, $\frac{c}{2\,\text{ell}}$]$\}$

Comparison

In[ ]:=
```
rel1⟦2⟧ == (Im[tau])^(1/4) Pi^(-1/2) ell^(-1/4)
    (2 Pi ell)^(-h/4) E^(-Pi ell u^2) E^(Pi I ell z) summod[al, c/(2 ell)]
    E^(-2 Pi ell al^2 Im[tau] - 2 Pi I ell Re[tau] al^2 - 4 Pi ell u al Sqrt[Im[tau]]) // Simplify
```

Out[ ]=
```
True
```