2 Symmetric space

2a. Sphere model

2b. Upper half-plane model

See §2.1.1

Action on upper half-plane model derived from action on ball model

In[ ]:=
```
Clear[BtoX, XtoB, actX]
BtoX[{w1_, w2_}] := {I (1 + w1) / (1 - w1), I w2 / (1 - w1)} // Simplify
XtoB[{z_, u_}] := {(z - I) / (z + I), 2 u / (z + I)} // Simplify
actX[g_, {z_, u_}] := Simplify[actB[g, XtoB[{z, u}]]] // BtoX // Simplify
```

Checks

In[ ]:=
```
Clear[z, u, w1, w2]
BtoX[XtoB[{z, u}]]
XtoB[BtoX[{w1, w2}]]
```

Out[ ]= {z, u}

Out[ ]= {w1, w2}

In[ ]:=
```
Clear[al, bt, zt, b, r, t]
actX[km[zt, al, bt], {z, u}] // FullSimplify
actX[km[zt, al, bt], {I, 0}]
actX[wm, {z, u}]
actX[nm[b, r].am[t], {z, u}]
```

Out[ ]=
$$\left\{-i\left(1+\frac{2\,(i+z)}{-i-z+(2\,\text{bt}\,u+\text{al}\,(-i+z))\,\text{zt}^3}\right),\right.$$
$$\left.\left(\text{zt}^3\,(-2\,i\,u\,\text{Conjugate}[\text{al}]+\text{Conjugate}[\text{bt}]+i\,z\,\text{Conjugate}[\text{bt}])\right)\Big/\left(-i-z+(2\,\text{bt}\,u+\text{al}\,(-i+z))\,\text{zt}^3\right)\right\}$$

Out[ ]= {i, 0}

Out[ ]=
$$\left\{-\frac{1}{z},\ -\frac{i\,u}{z}\right\}$$

Out[ ]= {2 r + t (2 b u + t z) + i Abs[b]^2, t u + i Conjugate[b]}

In[ ]:=