5e. Hermite functions

See §2.3.2

Definitions and some checks

The Hermite polynomials

In[ ]:=
```
Clear[Hermpol, xi]
Hermpol[0] = 1;
Hermpol[m_] := Hermpol[m] = Block[{h}, h = Hermpol[m - 1];
    2 xi h - D[h, xi] // Expand]
Hermpol[m_] := 0 /; m < 0
```

In[ ]:=
```
Do[Print[m, "   ", Hermpol[m]], {m, 0, 5}]
```

0   1

1   2 xi

2   $-2 + 4\,xi^2$

3   $-12\,xi + 8\,xi^3$

4   $12 - 48\,xi^2 + 16\,xi^4$

5   $120\,xi - 160\,xi^3 + 32\,xi^5$

Relation

In[ ]:=
```
Table[Hermpol[m + 1] == 2 xi Hermpol[m] - 2 m Hermpol[m - 1] // Simplify, {m, 0, 15}]
```

Out[ ]=
```
{True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True}
```

Normalized Hermite functions as explicit functions, in (2.59)

In[ ]:=
```
Clear[Hermn]
Hermn[ell_, m_, x_] := 2^((1 - m)/2) Abs[ell]^(1/4) Factorial[m]^(-1/2)
    (Hermpol[m] /. xi → Sqrt[4 Pi Abs[ell]] x) E^(-2 Pi Abs[ell] x^2) // Simplify
```

In[ ]:=
```
Clear[ell]
Do[Print[m, "   ", Hermn[ell, m, xi] // Simplify], {m, 0, 5}]
```

0 $\sqrt{2}\, e^{-2\pi\, xi^2\, Abs[ell]}\, Abs[ell]^{1/4}$

1 $4\, e^{-2\pi\, xi^2\, Abs[ell]}\, \sqrt{\pi}\, xi\, Abs[ell]^{3/4}$

2 $e^{-2\pi\, xi^2\, Abs[ell]}\, Abs[ell]^{1/4}\, (-1 + 8\,\pi\, xi^2\, Abs[ell])$

3 $2\, e^{-2\pi\, xi^2\, Abs[ell]}\, \sqrt{\frac{2\pi}{3}}\, xi\, Abs[ell]^{3/4}\, (-3 + 8\,\pi\, xi^2\, Abs[ell])$

4 $\frac{1}{2\sqrt{3}}\, e^{-2\pi\, xi^2\, Abs[ell]}\, Abs[ell]^{1/4}\, (3 - 48\,\pi\, xi^2\, Abs[ell] + 64\,\pi^2\, xi^4\, Abs[ell]^2)$

5 $e^{-2\pi\, xi^2\, Abs[ell]}\, \sqrt{\frac{2\pi}{15}}\, xi\, Abs[ell]^{3/4}\, (15 - 80\,\pi\, xi^2\, Abs[ell] + 64\,\pi^2\, xi^4\, Abs[ell]^2)$

See Table 2.3

In[ ]:=

Check of orthogonality

See (2.60)

In[ ]:=
```
bd = 5;
Table[Integrate[Hermn[ell, m, xi] × Hermn[ell, ma, xi], {xi, -Infinity, Infinity}],
  {m, 0, bd}, {ma, 0, bd}] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$