

8a. Conjugation by elements of K

See (3.16), (3.17)

Embedding $SU(2,1)$ in $SL(3,C)$, a check of the relations in (7.3) can be carried out in the complex Lie algebra of $SL(3,C)$.

```
In[ * ]:= z13 = Z13 // Liesub ;
z23 = Z23 // Liesub ;
z31 = Z31 // Liesub ;
z32 = Z32 // Liesub ;
k = km[zt, al, bt];
ki = km[Conjugate[zt], Conjugate[al], -bt];
sub = {Conjugate[zt] -> zt^(-1), bt Conjugate[bt] -> 1 - al Conjugate[al]};
k.ki // . sub // MatrixForm
```

Out[*]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
In[ * ]:= k.z13.ki == al zt^3 z13 - Conjugate[bt] zt^3 z23 // . sub
```

Out[*]= True

```
In[ * ]:= k.z31.ki == Conjugate[al] zt^(-3) z31 - bt zt^(-3) z32 // . sub
```

Out[*]= True

```
In[ * ]:= k.z23.ki == bt zt^3 z13 + Conjugate[al] zt^3 z23 // . sub
```

Out[*]= True

```
In[ * ]:= k.z32.ki == Conjugate[bt] zt^(-3) z31 + al zt^(-3) z32 // . sub
```

Out[*]= True

Comparison with basis functions

```
In[ * ]:= k.z13.ki == Phi[-3, 1, -1, -1, {zt, al, bt}] z13 + Phi[-3, 1, 1, -1, {zt, al, bt}] z23 // . sub
```

```
k.z31.ki == Phi[3, 1, 1, 1, {zt, al, bt}] z31 - Phi[3, 1, -1, 1, {zt, al, bt}] z32 // . sub
```

```
k.z23.ki == Phi[-3, 1, -1, 1, {zt, al, bt}] z13 + Phi[-3, 1, 1, 1, {zt, al, bt}] z23 // . sub
```

```
k.z32.ki == -Phi[3, 1, 1, -1, {zt, al, bt}] z31 + Phi[3, 1, -1, -1, {zt, al, bt}] z32 // . sub
```

Out[*]= True

Out[*]= True

Out[*]= True

Out[*]= True