8e. Some checks of the differentiation formulas

Differentiation on K

In[ ]:=
```
Clear[f, h, p, r, q]
F = f Phi[h, p, r, q]
```

Out[ ]= f Phi[h, p, r, q]

In[ ]:=
```
R[WW0, F]/F
R[WW1, F]
R[WW2, F]
R[CKi, F]/F
Rg[Z12, F]
Rg[Z21, F]
```

Out[ ]= $-i\, q$

Out[ ]= $\frac{1}{2} f\, ((p + q)\, \text{Phi}[h, p, r, -2 + q] + (-p + q)\, \text{Phi}[h, p, r, 2 + q])$

Out[ ]= $\frac{1}{2} i\, f\, ((p + q)\, \text{Phi}[h, p, r, -2 + q] + (p - q)\, \text{Phi}[h, p, r, 2 + q])$

Out[ ]= $-i\, h$

Out[ ]= f (p + q) Phi[h, p, r, -2 + q]

Out[ ]= f (-p + q) Phi[h, p, r, 2 + q]

This is in agreement with Table 2.2. Of course, in the definitions those formulas have been used.

Comparison with results of older computations gives us confidence in the routines.

Examples

In[ ]:=
```
Clear[f, h, p, r, q]
F = f Phi[h, p, r, q]
```

Out[ ]= f Phi[h, p, r, q]

```
In[ ]:= u = R[Z31, F] (8 (p + 1)) // Simplify;
        u1 = Coefficient[u, Phi[h + 3, p + 1, r + 1, q + 1]] // Simplify
        u2 = Coefficient[u, Phi[h + 3, p + 1, r - 1, q + 1]] // Simplify
        u3 = Coefficient[u, Phi[h + 3, p - 1, r + 1, q + 1]] // Simplify
        u4 = Coefficient[u, Phi[h + 3, p - 1, r - 1, q + 1]] // Simplify
        u == u1 Phi[h + 3, p + 1, r + 1, q + 1] + u2 Phi[h + 3, p + 1, r - 1, q + 1] +
            u3 Phi[h + 3, p - 1, r + 1, q + 1] + u4 Phi[h + 3, p - 1, r - 1, q + 1] // Simplify
Out[ ]= (2 + p + r) (f (h + 2 p - r) + 2 Rna[HHr, f] - 4 i Rna[XX0, f])
Out[ ]= -2 × (2 + p - r) (Rna[XX1, f] - i Rna[XX2, f])
Out[ ]= -((p - q) (f (4 - h + 2 p + r) - 2 Rna[HHr, f] + 4 i Rna[XX0, f]))
Out[ ]= 2 (p - q) (Rna[XX1, f] - i Rna[XX2, f])
Out[ ]= True

In[ ]:= u = R[Z13, F] (8 (p + 1)) // Simplify;
        u1 = Coefficient[u, Phi[h - 3, p - 1, r - 1, q - 1]] // Simplify
        u2 = Coefficient[u, Phi[h - 3, p + 1, r - 1, q - 1]] // Simplify
        u3 = Coefficient[u, Phi[h - 3, p + 1, r + 1, q - 1]] // Simplify
        u4 = Coefficient[u, Phi[h - 3, p - 1, r + 1, q - 1]] // Simplify
        u == u1 Phi[h - 3, p - 1, r - 1, q - 1] + u2 Phi[h - 3, p + 1, r - 1, q - 1] +
            u3 Phi[h - 3, p + 1, r + 1, q - 1] + u4 Phi[h - 3, p - 1, r + 1, q - 1] // Simplify
Out[ ]= -((p + q) (f (4 + h + 2 p - r) - 2 Rna[HHr, f] - 4 i Rna[XX0, f]))
Out[ ]= (2 + p - r) (f (-h + 2 p + r) + 2 Rna[HHr, f] + 4 i Rna[XX0, f])
Out[ ]= 2 × (2 + p + r) (Rna[XX1, f] + i Rna[XX2, f])
Out[ ]= -2 (p + q) (Rna[XX1, f] + i Rna[XX2, f])
Out[ ]= True
```

In[ ]:=
```
u = R[Z32, F] (8 (p + 1)) // Simplify
u1 = Coefficient [u, Phi[h + 3, p + 1, r + 1, q - 1]] // Simplify
u2 = Coefficient [u, Phi[h + 3, p + 1, r - 1, q - 1]] // Simplify
u3 = Coefficient [u, Phi[h + 3, p - 1, r + 1, q - 1]] // Simplify
u4 = Coefficient [u, Phi[h + 3, p - 1, r - 1, q - 1]] // Simplify
u == u1 Phi[h + 3, p + 1, r + 1, q - 1] + u2 Phi[h + 3, p + 1, r - 1, q - 1] +
    u3 Phi[h + 3, p - 1, r + 1, q - 1] + u4 Phi[h + 3, p - 1, r - 1, q - 1] // Simplify
```

Out[ ]= -((2 + p + r) Phi[3 + h, 1 + p, 1 + r, -1 + q] (f (h + 2 p - r) + 2 Rna[HHr, f] - 4 ⅈ Rna[XX0, f])) -
(p + q) Phi[3 + h, -1 + p, 1 + r, -1 + q] (f (4 - h + 2 p + r) - 2 Rna[HHr, f] + 4 ⅈ Rna[XX0, f]) +
2 ((p + q) Phi[3 + h, -1 + p, -1 + r, -1 + q] + (2 + p - r) Phi[3 + h, 1 + p, -1 + r, -1 + q])
(Rna[XX1, f] - ⅈ Rna[XX2, f])

Out[ ]= -((2 + p + r) (f (h + 2 p - r) + 2 Rna[HHr, f] - 4 ⅈ Rna[XX0, f]))

Out[ ]= 2 × (2 + p - r) (Rna[XX1, f] - ⅈ Rna[XX2, f])

Out[ ]= -((p + q) (f (4 - h + 2 p + r) - 2 Rna[HHr, f] + 4 ⅈ Rna[XX0, f]))

Out[ ]= 2 (p + q) (Rna[XX1, f] - ⅈ Rna[XX2, f])

Out[ ]= True

In[ ]:=
```
u = R[Z23, F] (8 (p + 1)) // Simplify ;
u1 = Coefficient [u, Phi[h - 3, p + 1, r + 1, q + 1]] // Simplify
u2 = Coefficient [u, Phi[h - 3, p + 1, r - 1, q + 1]] // Simplify
u3 = Coefficient [u, Phi[h - 3, p - 1, r + 1, q + 1]] // Simplify
u4 = Coefficient [u, Phi[h - 3, p - 1, r - 1, q + 1]] // Simplify
u == u1 Phi[h - 3, p + 1, r + 1, q + 1] + u2 Phi[h - 3, p + 1, r - 1, q + 1] +
    u3 Phi[h - 3, p - 1, r + 1, q + 1] + u4 Phi[h - 3, p - 1, r - 1, q + 1] // Simplify
```

Out[ ]= 2 × (2 + p + r) (Rna[XX1, f] + ⅈ Rna[XX2, f])

Out[ ]= (2 + p - r) (f (-h + 2 p + r) + 2 Rna[HHr, f] + 4 ⅈ Rna[XX0, f])

Out[ ]= 2 (p - q) (Rna[XX1, f] + ⅈ Rna[XX2, f])

Out[ ]= (p - q) (f (4 + h + 2 p - r) - 2 Rna[HHr, f] - 4 ⅈ Rna[XX0, f])

Out[ ]= True

In[ ]:=

Enveloping algebra

It is not very practical to repeat the application of right differentiation in this general context. (The computation takes long.)

In[ ]:=
```
R[Z23, R[Z31, F]] // Simplify
(* Not carried out for pdf version *)
```

In[ ]:=

In Section 11 we will give a routine based on knowledge of the action on **n** on functions on NA. That routine will be more practical for the action of U(**g**).