8f. Shift operators

Check of Table 3.6

The routine **sh[h,p,ff]** gives the shift operators $\left(S^h\right)_p$ with h∈{3,-3} and p∈{1,-1}, acting on **ff** $\in C^\infty(G)_K$.

```
Clear[sh, ident]
ident[x_] := x;
sh[h_, p_, ff_] := sh[h, p, ff, ident]
undetsh = {Phi[hh_, pp_, rr_, qq_] :> If[Simplify[pp - qq] == 0,
      Phi[hh, pp, rr, qq], 0, "(undetermined : " <> ToString[pp == qq] <> ")"]};
sh[3, 1, ff_, sub_] := R[Z31, ff] /. undetsh // sub // Simplify
sh[-3, 1, ff_, sub_] := R[Z23, ff] /. undetsh // sub // Simplify
sh[3, -1, ff_, sub_] := R[Z32, ff] /. undetsh // sub // Simplify
sh[-3, -1, ff_, sub_] := R[Z13, ff] /. undetsh // sub // Simplify
```

The routine is based on the action of the Lie algebra, restricted to highest weight spaces. We just eliminate terms in the result that are not in a highest weight space, with the subsitutiton **undetsh**. Applied to vectors that are not in a highest weight space an error message is given.
In the general version we can add an argument **sub**, which handles more information on the Lie algebra action **Rna**.

Shift operators

Now we can describe the general action of the shift operators in Table 3.6.

```
Clear[b, h, p, r]
F = b Phi[h, p, r, p]
fctlist = Union[fctlist, {b}]
```

```
b Phi[h, p, r, p]
```

```
{b, bt, f, g, Phi}
```

```
8 (p + 1) sh[3, 1, F]
```

```
(2 + p + r) Phi[3 + h, 1 + p, 1 + r, 1 + p] (b (h + 2 p - r) + 2 Rna[HHr, b] - 4 i Rna[XX0, b]) -
  2 × (2 + p - r) Phi[3 + h, 1 + p, -1 + r, 1 + p] × (Rna[XX1, b] - i Rna[XX2, b])
```

```
8 (p + 1) sh[-3, 1, F]
```

```
(2 + p - r) Phi[-3 + h, 1 + p, -1 + r, 1 + p] (b (-h + 2 p + r) + 2 Rna[HHr, b] + 4 i Rna[XX0, b]) +
  2 × (2 + p + r) Phi[-3 + h, 1 + p, 1 + r, 1 + p] × (Rna[XX1, b] + i Rna[XX2, b])
```

```
4 (p + 1) sh[3, -1, F] // Simplify
```

```
-p Phi[3 + h, -1 + p, 1 + r, -1 + p] (b (4 - h + 2 p + r) - 2 Rna[HHr, b] + 4 i Rna[XX0, b]) +
  2 p Phi[3 + h, -1 + p, -1 + r, -1 + p] × (Rna[XX1, b] - i Rna[XX2, b])
```

In[ ]:= **4 (p + 1) sh[-3, -1, F] // Simplify**

Out[ ]= p Phi[-3 + h, -1 + p, -1 + r, -1 + p] (-b (4 + h + 2 p - r) + 2 Rna[HHr, b] + 4 *i* Rna[XX0, b]) -
2 p Phi[-3 + h, -1 + p, 1 + r, -1 + p] × (Rna[XX1, b] + *i* Rna[XX2, b])