9c. Kernel relations

Table 3.8

Check of the kernel relations

```
In[ ]:= Clear[F, f, h, p, r, sum]
        F = chbt (f[r, t] × Phi[h, p, r, p] + f[r + 2, t] × Phi[h, p, r + 2, p]);
```

The term with Phi[3+h,1+p,1+r,1+p] has to be combined with the term
the term with Phi[3+h,1+p,-1+r,1+p] in which r is replaced by r+2.
Furthermore, the extremal values r give additional terms that should be zero.

```
In[ ]:= u = 8 (p + 1) sh[3, 1, F, subab] // Simplify
        up = Coefficient[u, Phi[3 + h, 1 + p, 1 + r, 1 + p]];
        um = Coefficient[u, Phi[3 + h, 1 + p, -1 + r, 1 + p]];
        up /. r → p // Simplify (* in the maximal weight one term *)
        umid = (um /. r → r + 2) + up // Simplify
        um /. r → -p // Simplify (* in the minimal weight one term *)
```

```
Out[ ]= chbt (f[r, t] (-4 i betac π (2 + p - r) t Phi[3 + h, 1 + p, -1 + r, 1 + p] +
            (h + 2 p - r) (2 + p + r) Phi[3 + h, 1 + p, 1 + r, 1 + p]) +
        f[2 + r, t] (-4 i betac π (p - r) t Phi[3 + h, 1 + p, 1 + r, 1 + p] +
            (-2 + h + 2 p - r) × (4 + p + r) Phi[3 + h, 1 + p, 3 + r, 1 + p]) +
        2 t ((2 + p + r) Phi[3 + h, 1 + p, 1 + r, 1 + p] f^(0,1)[r, t] +
            (4 + p + r) Phi[3 + h, 1 + p, 3 + r, 1 + p] f^(0,1)[2 + r, t]))
```

```
Out[ ]= 2 chbt (1 + p) ((h + p) f[p, t] + 2 t f^(0,1)[p, t])
```

```
Out[ ]= chbt ((h + 2 p - r) (2 + p + r) f[r, t] - 8 i betac π (p - r) t f[2 + r, t] + 2 × (2 + p + r) t f^(0,1)[r, t])
```

```
Out[ ]= -8 i betac chbt (1 + p) π t f[-p, t]
```

```
In[ ]:= Coefficient[umid, f[r, t]]
        Coefficient[umid, f^(0,1)[r, t]]
        Coefficient[umid, f[r + 2, t]]
```

```
Out[ ]= chbt (h + 2 p - r) (2 + p + r)
```

```
Out[ ]= 2 chbt (2 + p + r) t
```

```
Out[ ]= -8 i betac chbt π (p - r) t
```

```
In[ ]:= u = 8 (p + 1) sh[-3, 1, F, subab] // Simplify
        up = Coefficient[u, Phi[-3 + h, 1 + p, 1 + r, 1 + p]];
        um = Coefficient[u, Phi[-3 + h, 1 + p, -1 + r, 1 + p]];
        up /. r → p // Simplify
        umid = (um /. r → r + 2) + up // Simplify
        um /. r → -p // Simplify
```

```
Out[ ]= chbt (f[r, t] (-((h - 2 p - r) (2 + p - r) Phi[-3 + h, 1 + p, -1 + r, 1 + p]) +
            4 i beta π (2 + p + r) t Phi[-3 + h, 1 + p, 1 + r, 1 + p]) +
        f[2 + r, t] ((p - r) (2 - h + 2 p + r) Phi[-3 + h, 1 + p, 1 + r, 1 + p] +
            4 i beta π (4 + p + r) t Phi[-3 + h, 1 + p, 3 + r, 1 + p]) +
        2 t ((2 + p - r) Phi[-3 + h, 1 + p, -1 + r, 1 + p] f^(0,1)[r, t] +
            (p - r) Phi[-3 + h, 1 + p, 1 + r, 1 + p] f^(0,1)[2 + r, t]))
```

```
Out[ ]= 8 i beta chbt (1 + p) π t f[p, t]
```

```
Out[ ]= 2 chbt (2 i beta π (2 + p + r) t f[r, t] + (p - r) ((2 - h + 2 p + r) f[2 + r, t] + 2 t f^(0,1)[2 + r, t]))
```

```
Out[ ]= 2 chbt (1 + p) ((-h + p) f[-p, t] + 2 t f^(0,1)[-p, t])
```

```
In[ ]:= Coefficient[umid, f[r, t]] / 2
        Coefficient[umid, f^(0,1)[r + 2, t]] / 2
        Coefficient[umid, f[r + 2, t]] / 2 // Factor
```

```
Out[ ]= 2 i beta chbt π (2 + p + r) t
```

```
Out[ ]= chbt (2 p t - 2 r t)
```

```
Out[ ]= chbt (p - r) (2 - h + 2 p + r)
```

For the downward shift operators the extremal values of r do not give an additional condition.

```
In[ ]:= u = 8 (p + 1) sh[3, -1, F, subab] // Simplify
        up = Coefficient[u, Phi[3 + h, -1 + p, 1 + r, -1 + p]];
        um = Coefficient[u, Phi[3 + h, -1 + p, -1 + r, -1 + p]];
        umid = (um /. r → r + 2) + up // Simplify
```

```
Out[ ]= 2 chbt p (f[r, t] (4 i betac π t Phi[3 + h, -1 + p, -1 + r, -1 + p] +
            (-4 + h - 2 p - r) Phi[3 + h, -1 + p, 1 + r, -1 + p]) + f[2 + r, t]
          (4 i betac π t Phi[3 + h, -1 + p, 1 + r, -1 + p] + (-6 + h - 2 p - r) Phi[3 + h, -1 + p, 3 + r, -1 + p]) +
        2 t (Phi[3 + h, -1 + p, 1 + r, -1 + p] f^(0,1)[r, t] + Phi[3 + h, -1 + p, 3 + r, -1 + p] f^(0,1)[2 + r, t]))
```

```
Out[ ]= 2 chbt p ((-4 + h - 2 p - r) f[r, t] + 2 t (4 i betac π f[2 + r, t] + f^(0,1)[r, t]))
```

```
In[ ]:= Coefficient[umid, f[r, t]] / 2
        Coefficient[umid, f^(0,1)[r, t]] / 2
        Coefficient[umid, f[r + 2, t]] / 2
```

```
Out[ ]= chbt p (-4 + h - 2 p - r)
```

```
Out[ ]= 2 chbt p t
```

```
Out[ ]= 8 i betac chbt p π t
```

```
In[ ]:= u = 8 (p + 1) sh[-3, -1, F, subab] // Simplify
        up = Coefficient[u, Phi[-3 + h, -1 + p, 1 + r, -1 + p]];
        um = Coefficient[u, Phi[-3 + h, -1 + p, -1 + r, -1 + p]];
        umid = (um /. r → r + 2) + up // Simplify
```

```
Out[ ]= -2 chbt p (f[r, t] ((4 + h + 2 p - r) Phi[-3 + h, -1 + p, -1 + r, -1 + p] +
            4 i beta π t Phi[-3 + h, -1 + p, 1 + r, -1 + p]) + f[2 + r, t]
          ((2 + h + 2 p - r) Phi[-3 + h, -1 + p, 1 + r, -1 + p] + 4 i beta π t Phi[-3 + h, -1 + p, 3 + r, -1 + p]) -
        2 t (Phi[-3 + h, -1 + p, -1 + r, -1 + p] f^(0,1)[r, t] +
            Phi[-3 + h, -1 + p, 1 + r, -1 + p] f^(0,1)[2 + r, t]))
```

```
Out[ ]= -4 chbt p (2 i beta π t f[r, t] + (2 + h + 2 p - r) f[2 + r, t] - 2 t f^(0,1)[2 + r, t])
```

```
In[ ]:= Coefficient[umid, f[r, t]] / 4
        Coefficient[umid, f^(0,1)[r + 2, t]] / 4
        Coefficient[umid, f[r + 2, t]] / 4
```

```
Out[ ]= -2 i beta chbt p π t
```

```
Out[ ]= 2 chbt p t
```

```
Out[ ]= -chbt p (2 + h + 2 p - r)
```

```
In[ ]:=
```