A2d. Contiguous and differential relations

Relations for the new function $V_{\kappa,s}$ and the special relations

```
In[ ]:=
Clear[Whrel]
Whrel := {WhittakerV^(0,0,1)[kp_, s_, tau_] :→ (1 / 2 - kp / tau) WhittakerV[kp, s, tau] +
     ((kp + 1 / 2) ^ 2 - s ^ 2) tau ^ (- 1) WhittakerV[kp + 1, s, tau], WhittakerV^(0,0,2)[
     kp_, s_, tau_] :→ (1 / 4 - kp / tau + (s ^ 2 - 1 / 4) / tau ^ 2) WhittakerV[kp, s, tau],
   WhittakerW[kp_, s_, tau_] /; Simplify[kp == 1 / 2 + s] :→ tau ^ kp E ^ (- tau / 2),
   WhittakerV[kp_, s_, tau_] /; Simplify[kp == - 1 / 2 + s] :→ - E ^ (- Pi I kp) tau ^ (- kp) E ^ (tau / 2)}
```

Substitutions for the application of contiguous relations. These are used repeatedly in the earlier sections.

```
In[ ]:=
Clear[Whdn, Whup, Whsup, Whsdn, Whups, Whdns]
Whdn[n_] := {WhittakerM[n, mu_, x_] :→ (1 / 2 + mu + n - 1) ^ (- 1)
     ((1 / 2 + mu - n + 1) WhittakerM[n - 2, mu, x] + (2 n - 2 - x) WhittakerM[n - 1, mu, x]),
   WhittakerW[n, mu_, x_] :→ - ((n - 1 - 1 / 2) ^ 2 - mu ^ 2) WhittakerW[n - 2, mu, x] +
     (x - 2 n + 2) WhittakerW[n - 1, mu, x], WhittakerV[n, mu_, x_] :→
    (mu ^ 2 - (n - 1 / 2) ^ 2) ^ (- 1) (WhittakerV[n - 2, mu, x] + (x - 2 n + 2) WhittakerV[n - 1, mu, x])}
Whup[n_] := {WhittakerM[n, mu_, x_] :→ (1 / 2 + mu - n - 1) ^ (- 1)
      ((1 / 2 + mu + n + 1) WhittakerM[n + 2, mu, x] + (x - 2 n - 2) WhittakerM[n + 1, mu, x]),
    WhittakerW[n, mu_, x_] :→ ((n + 1 / 2) ^ 2 - mu ^ 2) ^ (- 1)
      (- WhittakerW[n + 2, mu, x] + (x - 2 n - 2) WhittakerW[n + 1, mu, x]),
    WhittakerV[n, mu_, x_] :→ (mu - n - 3 / 2) (mu + n + 3 / 2) WhittakerV[n + 2, mu, x] -
      (x - 2 n - 2) WhittakerV[n + 1, mu, x]};
Whsdn[kp_, s_] := {WhittakerW[kp, s, tau_] :→ (s - (kp - 1 / 2)) WhittakerW[kp - 1, s, tau] +
     tau ^ (1 / 2) WhittakerW[kp - 1 / 2, s - 1 / 2, tau],
   WhittakerM[kp, s, tau_] :→ (s + kp - 1 / 2) ^ (- 1) ((kp - 1 / 2 - s) WhittakerM[kp - 1, s, tau] +
       2 s tau ^ (1 / 2) WhittakerM[kp - 1 / 2, s - 1 / 2, tau]),
   WhittakerV[kp, s, tau_] :→ (kp - 1 / 2 + s) ^ (- 1)
     ( WhittakerV[kp - 1, s, tau] - I tau ^ (1 / 2) WhittakerV[kp - 1 / 2, s - 1 / 2, tau])
 }

Whsup[kp_, s_] := {WhittakerW[kp, s, tau_] :→
    - (kp - 1 / 2 + s) WhittakerW[kp - 1, s, tau] + tau ^ (1 / 2) WhittakerW[kp - 1 / 2, s + 1 / 2, tau],
   WhittakerM[kp, s, tau_] :→ WhittakerM[kp - 1, s, tau] -
     (2 s + 1) ^ (- 1) tau ^ (1 / 2) WhittakerM[kp - 1 / 2, s + 1 / 2, tau],
   WhittakerV[kp, s, tau_] :→ (kp - 1 / 2 - s) ^ (- 1)
     (WhittakerV[kp - 1, s, tau] - I tau ^ (1 / 2) WhittakerV[kp - 1 / 2, s + 1 / 2, tau])
 }
```

```
Whdns[kp_, sp_] :=
  {WhittakerW[kp, sp, tau_] :→ tau^(-1/2) (WhittakerW[kp + 1/2, sp - 1/2, tau] +
       (kp + sp - 1/2) WhittakerW[kp - 1/2, sp - 1/2, tau]), WhittakerV[kp, sp, tau_] :→
    -I (sp - 1/2 - kp) tau^(-1/2) (WhittakerV[kp + 1/2, sp - 1/2, tau] -
       (kp - sp + 1/2)^(-1) WhittakerV[kp - 1/2, sp - 1/2, tau]),
   WhittakerM[kp, sp, tau_] :→ -tau^(-1/2) (2 sp)
     (WhittakerM[kp + 1/2, sp - 1/2, tau] - WhittakerM[kp - 1/2, sp - 1/2, tau])}
Whups[kp_, sm_] := {WhittakerW[kp, sm, tau_] :→
    tau^(-1/2) (WhittakerW[kp + 1/2, sm + 1/2, tau] -
       (sm + 1/2 - kp) WhittakerW[kp - 1/2, sm + 1/2, tau]), WhittakerV[kp, sm, tau_] :→
    I tau^(-1/2) (kp + sm + 1/2) (WhittakerV[kp + 1/2, sm + 1/2, tau] -
       (sm + 1/2 + kp)^(-1) WhittakerV[kp - 1/2, sm + 1/2, tau]),
   WhittakerM[kp, sm, tau_] :→ (2 sm + 1)^(-1) tau^(-1/2)
     ((sm + 1/2 + kp) WhittakerM[kp + 1/2, sm + 1/2, tau] -
       (kp - sm - 1/2) WhittakerM[kp - 1/2, sm + 1/2, tau])
  }
```

In[ ]:=

Checks

Some of the checks take long.

In[ ]:=
```
tm[] := Block[{d}, d = DateList[];
  Print["time ", ToString[d⟦4⟧], ":", ToString[d⟦5⟧], ":", ToString[d⟦6⟧ // Floor]]]
```

Differentiation of the V-Whittaker function.

(Some of the checks take a lot of time if one puts the precision at a high value)

In[ ]:=

```
prec = 10;
tm[]
```

```
time 13:44:3
```

In[ ]:=
```
WhittakerV[kp, s, tau] /. Whsub;
u1 = D[%, tau] // Simplify;
u2 = D[WhittakerV[kp, s, tau], tau] /. Whrel /. Whsub // Simplify;
u1 == u2 // FullSimplify
```

Out[ ]=
```
True
```

In[ ]:=
```
tm[]
```

```
time 13:44:4
```

Checks for derivatives of W- and M- Whittaker functions

```
In[ ]:= u1 = D[WhittakerM[kp, s, tau], tau]
        u2 = (1/2 - kp/tau) WhittakerM[kp, s, tau] + (1/2 + kp + s) tau^(-1) WhittakerM[kp + 1, s, tau]
        u1 == u2 // Simplify
```

Out[ ]= $\left(\frac{1}{2}-\frac{kp}{tau}\right)\text{WhittakerM}[kp, s, tau]+\frac{\left(\frac{1}{2}+kp+s\right)\text{WhittakerM}[1+kp, s, tau]}{tau}$

Out[ ]= $\left(\frac{1}{2}-\frac{kp}{tau}\right)\text{WhittakerM}[kp, s, tau]+\frac{\left(\frac{1}{2}+kp+s\right)\text{WhittakerM}[1+kp, s, tau]}{tau}$

Out[ ]= True

```
In[ ]:= u1 = D[WhittakerW[kp, s, tau], tau]
        u2 = (1/2 - kp/tau) WhittakerW[kp, s, tau] - tau^(-1) WhittakerW[kp + 1, s, tau]
        u1 == u2 // Simplify
```

Out[ ]= $\left(\frac{1}{2}-\frac{kp}{tau}\right)\text{WhittakerW}[kp, s, tau]-\frac{\text{WhittakerW}[1+kp, s, tau]}{tau}$

Out[ ]= $\left(\frac{1}{2}-\frac{kp}{tau}\right)\text{WhittakerW}[kp, s, tau]-\frac{\text{WhittakerW}[1+kp, s, tau]}{tau}$

Out[ ]= True

For the M-Whittaker function we carry out a check based on the series expansion.

```
In[ ]:= Clear[whm, n, kp, s, tau, sum]
        whm[kp_, s_, tau_] = sum[n] tau^(s + 1/2) E^(-tau/2) Gamma[1/2 + s - kp + n]
            Gamma[1/2 + s - kp]^(-1) Gamma[1 + 2 s + n]^(-1) Gamma[1 + 2 s] Factorial[n]^(-1) tau^n;

In[ ]:= rel = Simplify[-D[whm[kp, s, tau], tau] + (1/2 - kp/tau) whm[kp, s, tau] +
              (1/2 + kp + s) tau^(-1) whm[kp + 1, s, tau]] E^(tau/2) tau^(1/2 - n - s) // Simplify
```

Out[ ]= $\left(\left((1+2\,kp+2\,s)\,\text{Gamma}\left[\frac{1}{2}-kp+s\right]\text{Gamma}\left[-\frac{1}{2}-kp+n+s\right]-(1+2\,kp+2\,n+2\,s-2\,tau)\,\text{Gamma}\left[-\frac{1}{2}-kp+s\right]\text{Gamma}\left[\frac{1}{2}-kp+n+s\right]\right)\text{Gamma}[1+2\,s]\,\text{sum}[n]\right)\Big/\left(2\,n!\,\text{Gamma}\left[-\frac{1}{2}-kp+s\right]\text{Gamma}\left[\frac{1}{2}-kp+s\right]\text{Gamma}[1+n+2\,s]\right)$

```
In[ ]:= rel1 = Expand[rel] /. tau ff_ :> Simplify[ff /. n -> n - 1] // Simplify ;
        (* case n≥1 *)
        % //. {} // FullSimplify
        % /. sum[n - 1] -> sum[n]
```

Out[ ]= $\frac{\text{Gamma}\left[-\frac{1}{2}-kp+n+s\right]\text{Gamma}[1+2\,s]\,(\text{sum}[-1+n]-\text{sum}[n])}{\text{Gamma}[n]\,\text{Gamma}\left[\frac{1}{2}-kp+s\right]\text{Gamma}[n+2\,s]}$

Out[ ]= 0

```
In[ ]:= (* case n=0 *)
rel1 /. sum[n - 1] → 0 /. n → 0
Out[ ]= 0
```

Check of routines for use of contigous relations
shifts of kp

```
In[ ]:= WhittakerM[kp + 1, s, tau] /. Whdn[kp + 1];
% == WhittakerM[kp + 1, s, tau] // FullSimplify
WhittakerM[kp - 1, s, tau] /. Whup[kp - 1];
% == WhittakerM[kp - 1, s, tau] // FullSimplify
Out[ ]= True
Out[ ]= True
In[ ]:= tm[]
time 13:44:4
In[ ]:= WhittakerW[kp + 1, s, tau] /. Whdn[kp + 1];
% == WhittakerW[kp + 1, s, tau] // FullSimplify
WhittakerW[kp - 1, s, tau] /. Whup[kp - 1];
% == WhittakerW[kp - 1, s, tau];
% // FullSimplify
Out[ ]= True
Out[ ]= True
In[ ]:= tm[]
time 13:44:5
In[ ]:= WhittakerV[kp + 1, s, tau] /. Whdn[kp + 1]
WhittakerV[kp + 1, s, tau] - % /. Whsub /. Whup[kp - 1] // FullSimplify
```

Out[ ]= $\frac{1}{-\left(\frac{1}{2}+kp\right)^2+s^2}$ (WhittakerV[-1 + kp, s, tau] + (2 - 2 × (1 + kp) + tau) WhittakerV[kp, s, tau])

```
Out[ ]= 0
In[ ]:= tm[]
time 13:44:5
In[ ]:= WhittakerV[kp - 1, s, tau] /. Whup[kp - 1]
WhittakerV[kp - 1, s, tau] - % /. Whsub /. Whdn[kp + 1] // FullSimplify
```

Out[ ]= -((-2 - 2 × (-1 + kp) + tau) WhittakerV[kp, s, tau]) + $\left(-\frac{1}{2}-kp+s\right)\times\left(\frac{1}{2}+kp+s\right)$ WhittakerV[1 + kp, s, tau]

```
Out[ ]= 0
```

In[ ]:= **tm[]**

```
time 13:44:6
```

Coupled shifts of kp and s

Sometimes explicit computation does not work. Then we use an asymptotic expansion instead.

In[ ]:=
```
x = (WhittakerW[kp + 1/2, s, tau] /. Whsdn[kp + 1/2, s]) -
    WhittakerW[kp + 1/2, s, tau] // FullSimplify
Series[E^(tau/2) x, {tau, Infinity, prec}] // FullSimplify
```

Out[ ]= $(-kp + s)\,\text{WhittakerW}\left[-\frac{1}{2} + kp, s, tau\right] + \sqrt{tau}\,\text{WhittakerW}\left[kp, -\frac{1}{2} + s, tau\right] - \text{WhittakerW}\left[\frac{1}{2} + kp, s, tau\right]$

Out[ ]= $tau^{kp}\,O\left[\frac{1}{tau}\right]^{21/2}$

In[ ]:= **tm[]**

```
time 13:44:6
```

In[ ]:=
```
x = (WhittakerW[kp + 1/2, s, tau] /. Whsup[kp + 1/2, s]) -
    WhittakerW[kp + 1/2, s, tau] // FullSimplify
Series[E^(tau/2) x, {tau, Infinity, prec}] // FullSimplify
```

Out[ ]= $-\left((kp + s)\,\text{WhittakerW}\left[-\frac{1}{2} + kp, s, tau\right]\right) + \sqrt{tau}\,\text{WhittakerW}\left[kp, \frac{1}{2} + s, tau\right] - \text{WhittakerW}\left[\frac{1}{2} + kp, s, tau\right]$

Out[ ]= $tau^{kp}\,O\left[\frac{1}{tau}\right]^{21/2}$

In[ ]:= **tm[]**

```
time 13:44:6
```

In[ ]:=
```
(WhittakerM[kp + 1/2, s + 1/2, tau] /. Whsdn[kp + 1/2, s + 1/2]) -
   WhittakerM[kp + 1/2, s + 1/2, tau] // FullSimplify
Series[%, {tau, 0, prec}]
```

Out[ ]= $\frac{1}{\frac{1}{2} + kp + s}\left(\left(-\frac{1}{2} + kp - s\right)\text{WhittakerM}\left[-\frac{1}{2} + kp, \frac{1}{2} + s, tau\right] + (1 + 2\,s)\,\sqrt{tau}\,\text{WhittakerM}[kp, s, tau]\right) - \text{WhittakerM}\left[\frac{1}{2} + kp, \frac{1}{2} + s, tau\right]$

Out[ ]= $tau^{s}\,O[tau]^{11}$

In[ ]:= tm[]

```
time 13:44:7
```

In[ ]:= (WhittakerM[kp + 1/2, s - 1/2, tau] /. Whsup[kp + 1/2, s - 1/2]) -
WhittakerM[kp + 1/2, s - 1/2, tau] // FullSimplify
Series[%, {tau, 0, prec}]

Out[ ]= $\text{WhittakerM}\left[-\frac{1}{2}+\text{kp}, -\frac{1}{2}+\text{s}, \text{tau}\right]-\frac{\sqrt{\text{tau}}\ \text{WhittakerM}[\text{kp}, \text{s}, \text{tau}]}{2\ \text{s}}-\text{WhittakerM}\left[\frac{1}{2}+\text{kp}, -\frac{1}{2}+\text{s}, \text{tau}\right]$

Out[ ]= $\text{tau}^{\text{s}}\ \text{O}[\text{tau}]^{11}$

In[ ]:= tm[]

```
time 13:44:7
```

In[ ]:= (WhittakerV[kp + 1/2, s, tau] /. Whsdn[kp + 1/2, s]) - WhittakerV[kp + 1/2, s, tau] /. Whsub //
FullSimplify
Series[%, {tau, 0, prec}]

Out[ ]= $i\, e^{-i \pi \text{s}}\, \pi\, \text{Csc}[2 \pi \text{s}] \left(\left((-1+2\text{s})\, \text{WhittakerM}\left[-\frac{1}{2}+\text{kp}, -\text{s}, \text{tau}\right]+\sqrt{\text{tau}}\ \text{WhittakerM}\left[\text{kp}, \frac{1}{2}-\text{s}, \text{tau}\right]+(1-2\text{s})\, \text{WhittakerM}\left[\frac{1}{2}+\text{kp}, -\text{s}, \text{tau}\right]\right) \Big/ (\text{Gamma}[2-2\text{s}]\, \text{Gamma}[1+\text{kp}+\text{s}]) - \left(e^{2 i \pi \text{s}} \left((-\text{kp}+\text{s})\, \text{WhittakerM}\left[-\frac{1}{2}+\text{kp}, \text{s}, \text{tau}\right]-2\text{s}\sqrt{\text{tau}}\ \text{WhittakerM}\left[\text{kp}, -\frac{1}{2}+\text{s}, \text{tau}\right]+(\text{kp}+\text{s})\, \text{WhittakerM}\left[\frac{1}{2}+\text{kp}, \text{s}, \text{tau}\right]\right)\right) \Big/ ((\text{kp}+\text{s})\, \text{Gamma}[1+\text{kp}-\text{s}]\, \text{Gamma}[1+2\text{s}])\right)$

Out[ ]= $\text{tau}^{-\text{s}} \left(1+\text{tau}^{2\text{s}}\right) \text{O}[\text{tau}]^{21/2}$

In[ ]:= tm[]

```
time 13:44:30
```

In[ ]:=
```
(WhittakerV[kp + 1/2, s, tau] /. Whsup[kp + 1/2, s]) - WhittakerV[kp + 1/2, s, tau] /. Whsub //
  FullSimplify
Series[%, {tau, 0, prec}]
```

Out[ ]= $i\, e^{-i \pi s}\, \pi\, \mathrm{Csc}[2 \pi s]$

$$\left(\left(-\left((kp + s)\,\mathrm{WhittakerM}\left[-\frac{1}{2} + kp, -s, tau\right]\right) + 2 s \sqrt{tau}\,\mathrm{WhittakerM}\left[kp, -\frac{1}{2} - s, tau\right] + (kp - s)\,\mathrm{WhittakerM}\left[\frac{1}{2} + kp, -s, tau\right]\right) \Big/ ((kp - s)\,\mathrm{Gamma}[1 - 2 s]\,\mathrm{Gamma}[1 + kp + s]) + \left(e^{2 i \pi s}\left((1 + 2 s)\,\mathrm{WhittakerM}\left[-\frac{1}{2} + kp, s, tau\right] - \sqrt{tau}\,\mathrm{WhittakerM}\left[kp, \frac{1}{2} + s, tau\right] - (1 + 2 s)\,\mathrm{WhittakerM}\left[\frac{1}{2} + kp, s, tau\right]\right)\right) \Big/ (\mathrm{Gamma}[1 + kp - s]\,\mathrm{Gamma}[2 + 2 s])\right)$$

Out[ ]= $tau^{-s}\left(1 + tau^{2 s}\right) O[tau]^{21/2}$

In[ ]:= **tm[]**

```
time 13:44:40
```

In[ ]:=
```
D[WhittakerM[kp, s, tau], tau] == (1/2 - kp / tau) WhittakerM[kp, s, tau] +
   (1/2 + kp + s) tau ^ (-1) WhittakerM[kp + 1, s, tau] // FullSimplify
```

Out[ ]= True

In[ ]:= **tm[]**

```
time 13:44:40
```

In[ ]:=
```
(tau - 2 kp) WhittakerM[kp, s, tau] == -(1/2 + kp + s) WhittakerM[kp + 1, s, tau] +
    (1/2 - kp + s) WhittakerM[kp - 1, s, tau] /. Whup[kp - 1] // Simplify
```

Out[ ]= True

In[ ]:= **tm[]**

```
time 13:44:40
```

In[ ]:=
```
( WhittakerW[kp, s, tau] /. Whups[kp, s]) - WhittakerW[kp, s, tau] // FullSimplify
Series[E ^ (tau / 2) %, {tau, Infinity, prec}] // FullSimplify
```

Out[ ]= $-\mathrm{WhittakerW}[kp, s, tau] + \frac{1}{\sqrt{tau}}$

$$\left(\left(-\frac{1}{2} + kp - s\right)\mathrm{WhittakerW}\left[-\frac{1}{2} + kp, \frac{1}{2} + s, tau\right] + \mathrm{WhittakerW}\left[\frac{1}{2} + kp, \frac{1}{2} + s, tau\right]\right)$$

Out[ ]= $tau^{kp}\, O\left[\frac{1}{tau}\right]^{11}$

In[ ]:= **tm[]**

```
time 13:44:40
```

In[ ]:= (WhittakerM[kp, s, tau] /. Whups[kp, s]) - WhittakerM[kp, s, tau] // FullSimplify
Series[%, {tau, 0, prec}] // Simplify

Out[ ]= $-\text{WhittakerM}[kp, s, tau] + \frac{1}{(1+2\,s)\sqrt{tau}} \left(-\left(\left(-\frac{1}{2}+kp-s\right)\text{WhittakerM}\left[-\frac{1}{2}+kp, \frac{1}{2}+s, tau\right]\right)+\left(\frac{1}{2}+kp+s\right)\text{WhittakerM}\left[\frac{1}{2}+kp, \frac{1}{2}+s, tau\right]\right)$

Out[ ]= $tau^{s}\, O[tau]^{21/2}$

In[ ]:= tm[]

time 13:44:41

In[ ]:= (WhittakerV[kp, s, tau] /. Whups[kp, s]) - WhittakerV[kp, s, tau] //. Whsub // FullSimplify
Series[%, {tau, 0, prec}] // FullSimplify

Out[ ]= $\frac{1}{2\sqrt{tau}} i\, e^{-i\pi s}\,\pi\,\text{Csc}[2\pi s]\left(\left(4\,s\,\text{WhittakerM}\left[-\frac{1}{2}+kp, -\frac{1}{2}-s, tau\right]+2\sqrt{tau}\,\text{WhittakerM}[kp, -s, tau]-4\,s\,\text{WhittakerM}\left[\frac{1}{2}+kp, -\frac{1}{2}-s, tau\right]\right)\Big/\left(\text{Gamma}[1-2\,s]\,\text{Gamma}\left[\frac{1}{2}+kp+s\right]\right)+\left(e^{2i\pi s}\left((1-2\,kp+2\,s)\,\text{WhittakerM}\left[-\frac{1}{2}+kp, \frac{1}{2}+s, tau\right]-2\times(1+2\,s)\sqrt{tau}\,\text{WhittakerM}[kp, s, tau]+2\times\left(\frac{1}{2}+kp+s\right)\text{WhittakerM}\left[\frac{1}{2}+kp, \frac{1}{2}+s, tau\right]\right)\right)\Big/\left(\text{Gamma}\left[\frac{1}{2}+kp-s\right]\text{Gamma}[2+2\,s]\right)\right)$

Out[ ]= $tau^{-s}\left(1+tau^{2\,s}\right) O[tau]^{21/2}$

In[ ]:= tm[]

time 13:44:51

In[ ]:= (WhittakerW[kp, s, tau] /. Whdns[kp, s]) - WhittakerW[kp, s, tau] // FullSimplify
Series[E^(tau/2) %, {tau, Infinity, prec}] // FullSimplify

Out[ ]= $-\text{WhittakerW}[kp, s, tau] + \frac{1}{\sqrt{tau}}\left(\left(-\frac{1}{2}+kp+s\right)\text{WhittakerW}\left[-\frac{1}{2}+kp, -\frac{1}{2}+s, tau\right]+\text{WhittakerW}\left[\frac{1}{2}+kp, -\frac{1}{2}+s, tau\right]\right)$

Out[ ]= $tau^{kp}\, O\left[\frac{1}{tau}\right]^{11}$

In[ ]:= tm[]

time 13:44:51

In[ ]:=
```
(WhittakerM[kp, s, tau] /. Whdns[kp, s]) - WhittakerM[kp, s, tau] // FullSimplify
Series[%, {tau, 0, prec}] // Simplify
```

Out[ ]=

$$-\text{WhittakerM}[kp, s, tau] + \frac{1}{\sqrt{tau}} 2\, s \left(\text{WhittakerM}\left[-\frac{1}{2} + kp, -\frac{1}{2} + s, tau\right] - \text{WhittakerM}\left[\frac{1}{2} + kp, -\frac{1}{2} + s, tau\right]\right)$$

Out[ ]=

$$tau^{s}\, O[tau]^{21/2}$$

In[ ]:=
```
tm[]
```

time 13:44:52

In[ ]:=
```
(WhittakerV[kp, s, tau] /. Whdns[kp, s]) - WhittakerV[kp, s, tau] //. Whsub // FullSimplify
Series[%, {tau, 0, prec}] // Simplify
```

Out[ ]=

$$\left(i\, e^{-i \pi s} \pi\, \text{Csc}[2 \pi s] \left((-1 + 2\, kp + 2\, s)\, \text{WhittakerM}\left[-\frac{1}{2} + kp, \frac{1}{2} - s, tau\right] + 2 \times (1 - 2\, s) \sqrt{tau}\; \text{WhittakerM}[kp, -s, tau] + (-1 - 2\, kp + 2\, s)\, \text{WhittakerM}\left[\frac{1}{2} + kp, \frac{1}{2} - s, tau\right]\right)\right) \Big/ \left(2 \sqrt{tau}\; \text{Gamma}[2 - 2\, s]\, \text{Gamma}\left[\frac{1}{2} + kp + s\right]\right)$$

Out[ ]=

$$tau^{-s}\, O[tau]^{21/2}$$

In[ ]:=
```
tm[]
```

time 13:44:54