Analysis in one complex variable
Clarification: $\int_\gamma |dz|$

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What is dz and how do I work with it?

For a loop γ we have

$$\int_\gamma dz = 0,$$

$$\int_\gamma |dz| = \text{length}(\gamma).$$

What is dz and how do I work with it?

- dz is a 1-form.

  $$z = x + iy \Rightarrow dz = dx + idy$$

  ???
  dz is no different than dx or dt.
- dz is something you integrate over paths.
  Given a path $z = \gamma(t)$, the chain rule gives us

  $$dz = \gamma'(t)dt$$

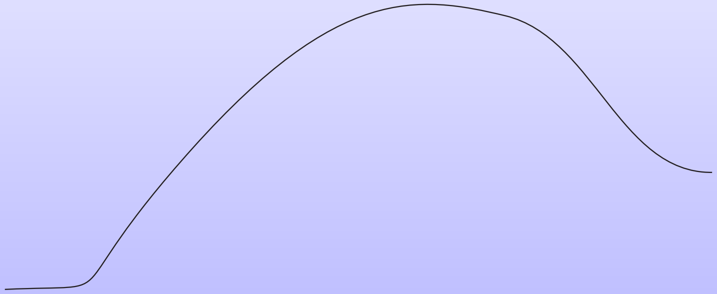
What is *dz* and how do I work with it?

$$\int_\gamma f(z)dz = \int_0^1 f(\gamma(t))\gamma'(t)dt,$$

$$\int_\gamma |dz| = \int_0^1 |\gamma'(t)||dt| = \text{length}(\gamma).$$

Why is this the length?

It comes from the definition of integral as a limit and Pythagoras theorem.



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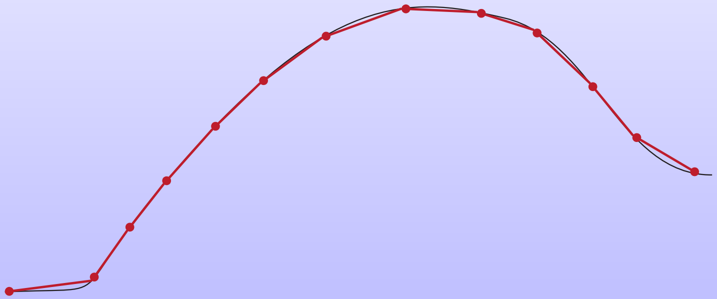

Why is this the length?

$$\begin{aligned}\text{length}(\gamma) &= \lim_{\Delta t \to 0} \sum |\gamma(j\Delta t) - \gamma((j-1)\Delta t)| \\ &= \lim_{\Delta t \to 0} \sum \left| \frac{\gamma(j\Delta t) - \gamma((j-1)\Delta t)}{\Delta t} \Delta t \right| \\ &= \int_0^1 |\gamma'(t)| dt\end{aligned}$$