Analysis in one complex variable
Lecture 15 – Level sets

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The real and imaginary parts of a holomorphic function

Remark

If $f = u + iv$ is a holomorphic function, the gradients of u and v are perpendicular to each other.

Proof.

The Cauchy–Riemann relations $u_x = v_y$ and $u_y = -v_x$ yield

$$\langle \nabla u, \nabla v \rangle = u_x v_x + u_y v_y = u_x v_x - v_x u_x = 0.$$

$\square$

The real and imaginary parts of a holomorphic function

Corollary

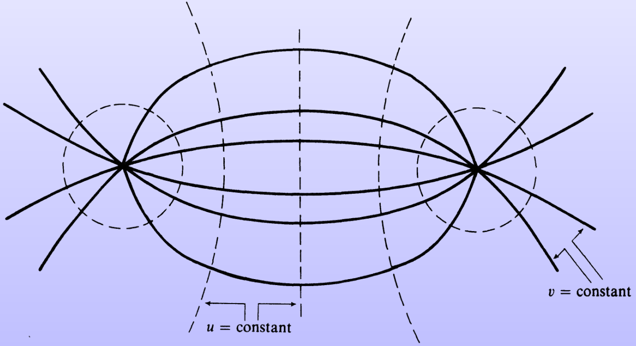
If $f = u + iv$ is a holomorphic function, u is constant along the flow lines of ∇v and vice versa.

Proof.

$$\mathcal{L}_{\nabla v} u = \langle \nabla u, \nabla v \rangle = 0.$$

□

The real and imaginary parts of a holomorphic function



The real and imaginary parts of a holomorphic function

Exercise

Draw the level sets of the real and imaginary parts of the function $f\colon \mathbb{C}\backslash\{0\} \to \mathbb{C}, f(z) = \frac{1}{z}$.