

Analysis in one complex variable  
Lecture 16 – Mock exam Q1

Gil Cavalcanti

Utrecht University

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Utrecht

## Exercise (1)

Let  $a_1, \dots, a_n \in \mathbb{C}$  be a collection of complex numbers of norm 1. Show that there is a point inside the unit disc such that

$$\prod_{i=1}^n |z - a_n| > 1.$$

## Proof.

Define the polynomial

$$p(z) = \prod_{i=1}^n (z - a_n).$$

Since  $p$  is not constant, by the maximum modulus theorem the maximum of  $|p|$  on the unit disc happens at the boundary. In particular, there is  $z$  with  $|z| = 1$  such that

$$\prod_{i=1}^n |z - a_n| = |p(z)| > |p(0)| = \prod_{i=1}^n |a_n| = 1$$

