Differentiable manifolds – homework 3

1)Let $f : S^2 \longrightarrow \mathbb{R}$ be the function $f(x, y, z) = z$. Compute the expressions for df obtained from stereographic projection.

2) Show that $f : \mathbb{R} \longrightarrow S^1$ given by $f(t) = e^{2\pi it}$ is smooth. Argue that the inverse map $\theta : S^1 \longrightarrow \mathbb{R}$, $\theta(e^{2\pi it}) = t$ is not well defined, but $d\theta$ is a well defined 1-form in the circle.

3) Expanding on Exercise 2, let $(U_\alpha : \alpha \in A)$ be an open cover of a manifold M and let $f_\alpha : U_\alpha \longrightarrow \mathbb{R}$ be a family of smooth functions such that in $U_\alpha \cap U_\beta$, $f_\alpha - f_\beta$ is constant, for all $\alpha, \beta \in A$. Show that df_α is a globally defined 1-form.

4) Show that for $n > 1$ not every 1-form defined on a manifold (or even in an open ball in $\mathbb{R}^n$) is of the form df for some smooth function f. Show, however that every 1-form φ defined on $\mathbb{R}$ is of the form df. Is the same true for 1-forms defined on the circle?