Differentiable manifolds – hand-in sheet 3

Hand in by 13/Dec

The tautological bundle

Exercise 1. Let $E_1 \subset \mathbb{R}^2 \times \mathbb{R}P^1$ be the set

$$E_1 = \{(x, l) \in \mathbb{R}^2 \times \mathbb{R}P^1 | x \in l\}.$$

- Find a natural set of coordinates for E which make it into a smooth manifold.
- Show that the following map is smooth and find its critical points

$$\pi_2 : E_1 \longrightarrow \mathbb{R}P^1, \qquad \pi_2(x, l) = l.$$

- Show that $\pi_2 : E_1 \longrightarrow \mathbb{R}P^1$ is indeed a line bundle over $\mathbb{R}P^1$. Is this bundle trivial?

Exercise 2. Let $E \subset \mathbb{R}^{n+1} \times \mathbb{R}P^n$ be the set

$$E = \{(x, l) \in \mathbb{R}^{n+1} \times \mathbb{R}P^n | x \in l\}.$$

- Find a natural set of coordinates for E which make it into a smooth manifold.
- Show that the following map is smooth

$$\pi_2 : E \longrightarrow \mathbb{R}P^n, \qquad \pi_2(x, l) = l.$$

- Show that $\pi_2 : E \longrightarrow \mathbb{R}P^n$ is indeed a line bundle over $\mathbb{R}P^n$.
- Consider the map

$$\mathbb{R}P^1 \hookrightarrow \mathbb{R}P^n, [x_0, x_1] \mapsto [x_0, x_1, 0, \cdots, 0].$$

  Show that the pull back of E to $\mathbb{R}P^1$ is E_1. Is E isomorphic to the trivial bundle?