Sheet 4

Solve exercises 7 and 8 from chapter 1.1.

Exercise 1. Show that if $p : \tilde{X} \longrightarrow X$ is a covering space of X then the following homotopy lifting property holds:

c) If $F : Y \times I \longrightarrow X$ is continuous and $\tilde{F} : Y \times \{0\} \longrightarrow \tilde{X}$ is a lift of $F|_{Y\times\{0\}}$, i.e., the following diagram commutes

$$\begin{array}{ccc} Y \times \{0\} & \xrightarrow{\tilde{F}} & \tilde{X} \\ \downarrow{\scriptstyle \mathrm{Id}} & & \downarrow{\scriptstyle p} \\ Y \times \{0\} & \xrightarrow{F|_{Y\times\{0\}}} & X \end{array}$$

then there is a a unique extension of $\tilde{F}$ to $Y \times I$ for which the diagram below commutes

$$\begin{array}{ccc} Y \times I & \xrightarrow{\tilde{F}} & \tilde{X} \\ \downarrow{\scriptstyle \mathrm{Id}} & & \downarrow{\scriptstyle p} \\ Y \times I & \xrightarrow{F} & X \end{array}$$

Exercise 2. Let $p_1, \ldots, p_m$ be points in the n-dimensional sphere. Show that

$$S^n \backslash \{p_1, \ldots, p_m\} \simeq (m-1) \vee S^{n-1}.$$

Exercise 3. Let p be a point in the 2-torus T^2. Show that $T^2 \backslash \{p\}$ deform retracts to $S^1 \vee S^1$. Conclude that $T^2 \backslash \{p\} \simeq S^2 \backslash \{p_1, p_2, p_3\}$.