

Group theory – Sheet 6

The exercises from the book are 10.1, 10.2, 10.3, 10.7, 10.8, 10.10, 10.11, 11.2, 11.3, 11.4, 11.9, 11.11.

1) Let $\varphi : G \rightarrow H$ be an isomorphism. Show that

- G and H have the same number of elements;
- If G is Abelian then H is Abelian;
- φ maps the center of G bijectively into the center of H ;
- φ sends elements of order k to elements of order k ;
- If g_1 and g_2 are conjugate in G , then $\varphi(g_1)$ and $\varphi(g_2)$ are conjugated in H , hence φ sends conjugacy classes to conjugacy classes.

2) Show that the center of $G \times H$ is the product of the centers of G and H .

3) Decide which of the following groups are isomorphic

- D_6 and \mathbb{Z}_{12} ;
- $D_{10} \times D_{10}$ and D_{100} ;
- $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_3$, $\mathbb{Z}_4 \times \mathbb{Z}_9$ and $\mathbb{Z}_6 \times \mathbb{Z}_6$;
- $D_4 \times D_4$ and D_{32} ;
- $D_5 \times D_5$ and D_{50} ;
- D_3 and S_3 ;
- D_{12} , S_4 and $\mathbb{H} \times \mathbb{Z}_3$, where \mathbb{H} is the quaternion group with 8 elements;
- $D_5 \times \mathbb{Z}_2$ and D_{10} .