

HOMEWORK 1; GIVEN ON SEPT 7, 2026

Exercise 1. Let $M = S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ endowed with the Euclidean topology.

- (1) Let $\mathcal{S}$ be the collection consisting of two charts (U_N, σ_N) , (U_S, σ_S) of S^1 , given by

$$U_N = S^1 \setminus \{(0, 1)\}, \quad \sigma_N(x, y) = \frac{x}{1 - y},$$

$$U_S = S^1 \setminus \{(0, -1)\}, \quad \sigma_S(x, y) = \frac{x}{1 + y}.$$

Describe explicitly the change of coordinates map, including the precise description of the domain and codomain. Deduce that $\mathcal{S}$ is a smooth atlas.

- (2) Let $\mathcal{P}$ be the collection consisting of four charts of S^1 ,

$$(U_n, \text{pr}_n), (U_s, \text{pr}_s), (U_e, \text{pr}_e), (U_v, \text{pr}_v),$$

given by

$$U_n = \{(x, y) \in S^1 : y > 0\}, \quad \text{pr}_n(x, y) = x,$$

$$U_s = \{(x, y) \in S^1 : y < 0\}, \quad \text{pr}_s(x, y) = x,$$

$$U_e = \{(x, y) \in S^1 : x > 0\}, \quad \text{pr}_e(x, y) = y,$$

$$U_v = \{(x, y) \in S^1 : x < 0\}, \quad \text{pr}_v(x, y) = y.$$

Prove that $\mathcal{P}$ is a smooth atlas.

- (3) Prove that $\mathcal{S}$ and $\mathcal{P}$ induce the same smooth structure¹ on S^1 :

$$\mathcal{S}^{\max} = \mathcal{P}^{\max}.$$

¹recall that any smooth atlas $\mathcal{A}$ on a space M induces a smooth structure on M , denoted $\mathcal{A}^{\max}$, defined as the collection of all the topological charts of M that are smoothly compatible with all the charts from $\mathcal{A}$ (see Proposition 2.9 in the notes).