

HOMEWORK 2; GIVEN ON SEPT 14, 2026

Exercise 1. Let $M = S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ endowed with the smooth atlases $\mathcal{S}$ and $\mathcal{P}$ from the previous homework. We know that they induce the same smooth structure on S^1 , and we consider S^1 as a manifold endowed with that smooth structure. We now consider

$$F : S^1 \rightarrow S^1, f(x, y) = (x^2 - y^2, 2xy).$$

- (1) Prove that the map F is smooth making use of the atlas $\mathcal{S}$;
(how many checks do you have to do?)
- (2) Prove that the map F is smooth making use of the atlas $\mathcal{S}$ on the domain of F and of $\mathcal{P}$ on its codomain;
(how many checks do you have to do?)
- (3) Describe two smooth charts of S^1 , call them χ and χ' , such that the representation of F is simply the map $t \mapsto 2t$.
- (4) Use the map F to produce a bijection $\overline{F} : \mathbb{P}^1 \rightarrow S^1$, then prove that that map is actually a diffeomorphism;
(the map $\overline{F}$ also appeared in Inleiding Topologie).