

HOMEWORK 5 (OCTOBER 5, 2026)

Exercise 1.

- a) Show that the following is a submanifold of $\mathbb{R}^3$:

$$M = \{(x, y, z) \in \mathbb{R}^3 : y = xz\}.$$

- b) Show that the following defines a vector field on M

$$V := 2z \frac{\partial}{\partial x} + (x + 2z^2) \frac{\partial}{\partial y} + \frac{\partial}{\partial z}.$$

- c) Show that the following is an integral curve of V :

$$\gamma : \mathbb{R} \rightarrow \mathbb{R}^3, \quad \gamma(t) = (t^2, t^3, t).$$

- d) Compute the flow of V , describing explicitly all the diffeomorphisms ϕ_V^t .

- e) And now the other way around: find a vector field W on M with flow

$$\phi_W^t(x, y, z) = (x + t, y + tz, z).$$

- f) Find another vector field on M for which γ is an integral curve and explain how you found it.