

Hand in exercise 2

(March 2nd, to be handed in on March 16, at the start of the lecture)

EXERCISE 0.1. Show that, if (X, d) is a metric space and A is a subspace of X , then the function

$$d_A : X \longrightarrow \mathbb{R}, \quad d_A(x) = d(x, A)$$

is continuous.

Deduce that, for any closed subset A of a metric space X , there exists a continuous function $f : X \longrightarrow [0, 1]$ such that $A = f^{-1}(0)$. What happens if A is not closed in X ? Find an example in which A is countable and the function d_A is identical zero ($d_A(x) = 0$ for all $x \in A$).