Exercise: Let X be a topological space and $\{\eta_i\}_{i=1}^p$ a partition of unity subordinated to the finite open cover $\{U_i\}_{i=1}^p$ of X. Assume that there is an $n \in \mathbb{N}$ such that for $0 \leq i \leq p$ there are homeomorphisms

$$\chi_i : U_i \to \mathbb{R}^n.$$

Show that the map

$$\begin{aligned} \chi : X &\to \mathbb{R}^p \times \underbrace{\mathbb{R}^n \times \ldots \times \mathbb{R}^n}_{p \text{ times}} = \mathbb{R}^{p(n+1)} \\ \chi(x) &= (\eta_1(x), \ldots, \eta_p(x), \eta_1(x)\chi_1(x), \ldots, \eta_p(x)\chi_p(x)) \end{aligned}$$

is well-defined, continuous and injective. Deduce that any compact topological manifold can be embedded in some $\mathbb{R}^N$ (for N large enough).