

Homework (given on May 22, related to partitions of unity)

Exercise 1. Let X be a Hausdorff, locally compact and 2^{nd} countable space. Let

$$r : X \rightarrow \mathbb{R}$$

be a map which is not necessarily continuous but which has the property that: for any $x \in X$, there exists a neighborhood V_x of x in X such that

$$\inf\{r(y) : y \in V_x\} > 0.$$

Show that there exists a continuous function $f : X \rightarrow \mathbb{R}$ such that

$$0 < f(x) < r(x) \quad \text{for all } x \in X.$$