True or False?
(test 8, 13/01/2014)

Naam:________________________________
Studentnr.:____________________________

Note: all the exercises below are over $\mathbb{R}$ (hence all the vector spaces and algebras are over $\mathbb{R}$ and by $\mathcal{C}(X)$ we denote the algebra of $\mathbb{R}$-valued continuous functions on $X$).

| True | | False |
|---|---|---|
| ☐ | 1. In every normed vector space, any Cauchy sequence is convergent. | ☐ |
| ☐ | 2. The space $M_{n\times n}(\mathbb{R})$ of $n$ by $n$ matrices is an algebra over $\mathbb{R}$.<br>(the operations are the usual addition, multiplication etc of matrices) | ☐ |
| ☐ | 3. In any algebra $A$ one has $(a+b)(a-b) = a^2 - b^2$ for all $a, b \in A$. | ☐ |
| ☐ | 4. The space of polynomials in one variable $X$, which are divisible by $X-2$, form a subalgebra of $\mathbb{R}[X]$.<br>(the operations are the usual addition, multiplication etc of polynomials). | ☐ |
| ☐ | 5. The space $GL_n(\mathbb{R})$ of invertible $n$ by $n$ matrices is a subalgebra of $M_{n\times n}(\mathbb{R})$.<br>(the operations are the usual addition, multiplication etc of matrices) | ☐ |
| ☐ | 6. For every space $X$, $\mathcal{A} := \mathcal{C}(X)$ is point-separating. ☐ | |
| ☐ | 7. In any algebra $A$ there exists $b \in A \setminus \{0\}$ such that $(a+b)(a-b) = a^2 - b^2$ for all $a \in A$. | ☐ |
| ☐ | 8. The space of continuous functions on $\mathbb{R}$ with the property that $f(x) = f(x+2\pi)$ for all $x$.<br>is a sub-algebra of the algebra of all continuous functions. | ☐ |
| ☐ | 9. For any normal space $X$, the algebra $\mathcal{C}(X)$ of all continuous functions on $X$ is point-separating. | ☐ |
| ☐ | 10. Let $\mathcal{S}$ be a set of continuous functions $f : \mathbb{R} \to \mathbb{R}$ with the property that<br>every $f \in \mathcal{S}$ is periodic with some period $t_f \in [1, 2014]$.<br>Then $\mathcal{S}$ cannot be dense in $\mathcal{C}(\mathbb{R})$. | ☐ |

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