Quizz 1, Nov 17, 2026

Q 1 In a metric space X, for arbitrary points $x, y, z \in X$ which of the following statements are true:

a). the norm satisfies the triangle inequality $||x + y|| \leq ||x|| + ||y||$;

b). the distance between x and y is the norm of their difference: $d(x, y) = ||x - y||$;

c). the distance satisfies the triangle inequality $d(x, y) \leq d(x, z) + d(z, y)$;

d). if x and y are positive then so is their sum $x + y$;

e). if $x \leq y$ and $y \leq x$ then $x = y$.

Q 2 If R is an equivalence relation on a set X, and we denote by X/R the quotient of X modulo R and by $R(x) \in X/R$ the equivalence class of an element $x \in X$, which of the following are true:

a). if $(x, y) \in R$ and $(y, x) \in R$ then $x = y$;

b). if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$;

c). the function $f : X/R \to X$ that associates to a class $R(x)$ the element x is surjective.

d). $(x, y) \in R$ for all $y \in R(x)$;

e). $y \in R(x)$ for all y such that $(x, y) \in R$.

Q 3 For functions $f : X \to Y$ between two sets, which of the following hold true:

a). if X is finite and f is surjective then Y is finite.

b). if Y is finite and f is injective then X is finite.

c). for any two subsets $A, B \subset X$ one has:

$$f(A \cup B) = f(A) \cup f(B), \quad f(A \cap B) = f(A) \cap f(B).$$

d). for any two subsets $C, D \subset Y$ one has:

$$f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D), \quad f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D).$$

e). f is injective if and only iff $f(A \cap B) = f(A) \cap f(B)$ for any two subsets $A, B \subset X$.