

PROBLEM 2.11. Increased CO₂-concentrations will lead to stratospheric cooling.

Demonstrate that the stratosphere in radiative equilibrium cools due an increase of the CO₂-concentration only if Solar radiation is absorbed in the stratosphere (e.g. by O₃). Approximate the stratosphere as one optically thin layer.

Stratospheric cooling due to enhanced emission

Why does the stratosphere cool due to an increase in greenhouse gas content?

Carbon dioxide is the most important absorber of long wave radiation in the stratosphere, while ozone is the most important absorber of short wave radiation. Most of the ozone heating occurs in the upper 10 hPa of the atmosphere. The value of the long wave optical path, δ_{strat} , of the upper 10 hPa (1000 Pa) of the atmosphere due to this effect can be estimated as follows.

$$\delta_{strat} = \frac{R_d q_{CO_2} \sigma_{CO_2} \Delta p}{g R_{CO_2}} \approx \frac{287 \times 0.0004 \times 0.3 \times 1000}{9.8 \times 189} \approx 0.018.$$

For the absorption cross section of carbon dioxide we have taken the value given in eq. 2.53c. Therefore

$$\delta_{strat} \ll 1,$$

which implies that the stratosphere is optically thin, implying also that its net emission coefficient,

$$\varepsilon_{strat} = 1 - \exp(-\delta_{strat}) \ll 1.$$

Let us write down the *approximate* radiation balance of the stratosphere, assuming that this is the uppermost layer of the atmosphere and assuming furthermore that the long-wave absorption/emission coefficient, ε_{strat} , of this layer is much smaller than 1. The **short wave irradiance absorbed by the atmosphere and the Earth's surface** is

$$(1 - \alpha_p)(Q - S_{O_3}) + S_{O_3} = (1 - \alpha_p)Q + \alpha_p S_{O_3} \equiv \sigma T_e^4,$$

where $Q = S_0/4$, α_p is the planetary albedo (determined by cloud cover and ground albedo), S_{O_3} is the Solar irradiance that is absorbed by ozone in the stratosphere and T_e is the emission temperature, defined according to the above formula.

The stratosphere will absorb only a very small fraction of the **upwelling long-wave irradiance** coming from the atmosphere and the Earth's surface below. We may therefore assume that this upwelling radiation is approximately equal to σT_e^4 and that a fraction, ε_{strat} , of this upwelling radiation is absorbed by the stratosphere.

In equilibrium, therefore, the **stratosphere absorbs** S_{O_3} from above, $\varepsilon_{strat} \sigma T_e^4$ from below, and emits $\varepsilon_{strat} \sigma T_{strat}^4$ downwards and upwards. This is consistent with the following equation.

$$S_{O_3} + \varepsilon_{strat} \sigma T_e^4 = 2 \varepsilon_{strat} \sigma T_{strat}^4,$$

or

$$T_{strat} = \sqrt[4]{\frac{(\varepsilon_{strat})^{-1} S_{O_3} + \sigma T_e^4}{2\sigma}}.$$

In the case of no stratospheric heating by absorption due to ozone, we recover the expression (Box 2.3) for the skin temperature, which does not depend on the emission coefficient, ε_{strat} . Otherwise we confirm that the temperature of the stratosphere decreases with increasing value of ε_{strat} . Note that T_e does not depend on ε_{strat} !