Physical interpretation of wave drag in the model and in reality

- Model wave drag “mimics” the effect of potential vorticity mixing on $[u]$. 

- PV-mixing reduces the meridional PV-gradient, making it more homogeneous, which in particular reduces the amplitude of the Polar Cap PV-anomaly.

- PV-mixing is manifest as meridional isentropic fluxes of both PVS and of mass.

- An equator-ward PVS-flux is equivalent to a westward force.

- Apply PV-inversion to understand the effect of PV-mixing on the zonal mean zonal wind.
# Schedule of the BLT&M-2 2019

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
<th>Study Material</th>
<th>Hand-in/Deadline</th>
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</table>
| 26 April   | Introduction to radiative transfer; “grey gas”; radiative equilibrium | *Study sections 12.1 and 12.2 (Introduction to the course), sections 2.1-2.4 and boxes 2.1-2.4*  
(1) problem 12.1 (response time)  
*Hand-in before 3 May* |                                                           |
| 3 May      | Radiatively determined state                                            | *Study sections 2.5-2.9 and 12.3-12.4*  
(2) problem 12.3 (radiative equilibrium/determined isentropic density)  
*Hand-in before 10 May* |                                                           |
| 10 May     | Potential vorticity inversion.                                          | *Study chapter 7*  
(3) In couples choose one project from the following list.  
*List of projects:* Problems 7.1(p.71), 7.3(p.89), 8.1(p.99), 10.2, 10.3(p.138), 11.1(p.161), 11.5, 11.6, 11.7(p.178), 12.5(p.242), 12.13(p.243)  
*Present on 14 June* |                                                           |
| 17 May     | Radiative-dynamical interaction, with and without the water cycle      | *Study section 12.2-12.4 (again!!) and sections 12.5-12.7*  
(4) Essay about controversies in climate science: Problem 12.15  
*Hand-in before before 25 June* |                                                           |
| 24 May     | Role of eddy-heat (mass) and –momentum (vorticity) fluxes in the general circulation | *Study chapters 10 and 11 (also box 1.12, Dynamical Meteorology)*  
(5) problem 12.6 & 7 (what-if?-thought-experiment)  
*Hand-in before 3 June, 17:00* |                                                           |
| 28 May     | Radiative-dynamical interaction, incl. water cycle and “wave drag.”    | *Study section 12.8-12-9* |                                                           |
| 7 June     | “Wave drag” as a manifestation of the consequences of meridional PV-mixing and maintenance of thermal wind balance. | *Study box 12.1* |                                                           |
| 14 June    | Presentations of the project-results (listed under (3)).               |                                                                  |                                                           |
|           | Hand in essay on Monday 24 June at the latest.                         |                                                                  |                                                           |
Main model result:
illustrated in the following 3 slides

Details in given lecture 6
Observed

- January (CIRA)
- stratospheric winter jet
- westward wind
- upward bulge of 370 K isentrope
- subtropical jets
- dynamical tropopause

Pressure:
- 3 hPa
- 5 hPa
- 10 hPa
- 30 hPa
- 50 hPa
- 100 hPa
- 300 hPa
- 500 hPa
- 1000 hPa

Latitude:
- 90°N
- 60°N
- 30°N
- 30°S
- 60°S
- 90°S

Equator
January, year 3, Run 2a

Modelled
No wave drag

Two nearly identical hemispheric-wide eastward jets

downward bulge of 370 K isentrope

dynamical tropopause

Correct results for the wrong reasons?

Why can we mimic the effect of eddies in the two-dimensional model by introducing a zonal force in the model? *Are we getting approximately correct results for the wrong reason?* Have we constructed a model that looks correct, in the sense that it resembles Roger Shepard’s elephant (below), which looks like an elephant, but, unfortunately, its four legs are not connected correctly to the body?
Interpretation of “planetary wave drag”

Why can we mimic the effect of eddies in the two-dimensional model by introducing a zonal force in the model?

Are we getting approximately correct results for the wrong reason?

Today we will see that what appears to be “wave drag” is in fact a manifestation of the effect of meridional isentropic potential vorticity-mixing and adjustment to balance.

Potential vorticity at 350 K, 1 December 2006 - 28 Feb 2007 (ERA-Interim)

see lecture 6
PV-mixing is manifest as meridional isentropic PVS-fluxes and mass-fluxes.

Flux of “PVS”: \[ [v \xi_a] = [v][\xi_a] + [v^* \xi^*] \]

Flux of mass: \[ [v \sigma] = [v][\sigma] + [v^* \sigma^*] \]

mean flux  eddy-flux
Thought Experiment

Start with a thought experiment:
Dynamical consequences of a pole-ward eddy mass in a balanced atmosphere

Blue arrows: pole-ward eddy transport of mass: \[ v \sigma > 0 \]
Mean PVS-flux is anti-correlated with eddy mass flux

This is expected in a balanced atmosphere: a negative PV-anomaly is manifest both as a positive isentropic density anomaly and a negative vorticity anomaly!

\[ [v^* \sigma^*] > 0 \]

\[ [v][\xi_a] < 0 \]

“Balanced response” to pole-ward eddy mass flux is an equator-ward PVS flux
The circulation, $\Gamma$, around a circle of constant latitude divided by the area, $S$, enclosed by this circle, is equal to the average absolute vorticity in this area. Therefore, if we denote the circumference of the parallel by $l$, then

$$\Gamma = l[u_a]$$

The subscript, $a$, stands for “absolute”. The circulation theorem, therefore, states that the area-average absolute vorticity,

$$\bar{\xi}_a \equiv \frac{\Gamma}{S} = \frac{l[u_a]}{S}$$

The change of the area-average absolute vorticity, due to PVS-fluxes across the boundary of a control volume, bounded by two isentropic surfaces and a parallel, is determined by isentropic PVS-fluxes across this parallel (remember that isentropic surfaces are impermeable to PVS).
Relation meridional PVS-flux and average vorticity

**Divergence theorem:** \[ \int_{V} \nabla \cdot \mathbf{J} \, dV = \int_{A} \mathbf{J} \cdot \hat{n} \, dA \]

\[ \mathbf{J}: \text{PVS-flux} \ [\text{m s}^{-2}] \]

**Vorticity equation in isentropic coordinates:**

\[ \frac{\partial \xi_a}{\partial t} = -\nabla \cdot \mathbf{J} \quad (1.229) \]

Therefore:

\[ \int_{V} \frac{\partial \xi_a}{\partial t} \, dV = \frac{\partial}{\partial t} \left( \int_{V} \xi_a \, dV \right) = -\int_{V} \nabla \cdot \mathbf{J} \, dV = \int_{A} \mathbf{J} \cdot \hat{n} \, dA \]

**Volume, V**  
**Average vorticity**  
**A: area bounding V**
Relation meridional PVS-flux and average vorticity

previous slide:

\[
\frac{\partial}{\partial t} \left( \int_V \zeta_a dV \right) = \int_A \mathbf{J} \cdot \hat{n} dA
\]

Time-integrated isentropic PVS-flux is denoted by:

\[
\Delta F(t) = \int_{t_0}^{t} \left[ v \zeta_a \right] dt \quad \left[ \text{m s}^{-1} \right]
\] (1.235)

Applying the these equations to the volume, \( V = S \Delta \theta \):

\[
S \Delta \theta \frac{\Delta \zeta_a}{\Delta t} = l \Delta \theta \frac{\Delta F}{\Delta t}
\]

"area"

\[
S \Delta \zeta_a = l \Delta F
\]
Meridional PVS-flux determines change in $[u]$

Previous two slides:

\[
\left\{ \begin{array}{l}
\int_{[u_a]} \frac{l}{S} = \bar{\zeta}_a \\
S \Delta \bar{\zeta}_a = C \Delta F \\
\end{array} \right. 
\]

\[\Delta F = \Delta [u]\]

Equator-ward PVS flux ($\Delta F < 0$) decelerates the zonal mean flow ($\Delta [u] < 0$)!

An equatorward vorticity-flux appears as a westward zonal force.

\[ [v \ast \sigma \ast] > 0 \]

Response to eddy mass flux decelerates or reduces $[u]$:

\[ [v] [\zeta_a] < 0 \]

Can we call this “wave drag”???

Not really !!! Why not?
Meridional PVS-flux: mean and eddy contributions

Running daily mean poleward isentropic PVS-flux (red lines), at $\theta=350$ K and at $\phi=60^\circ$N, between 1 December 2006 and 18 February 2007. **Thick red line:** mean PVS-flux. **Thin red line:** eddy PVS-flux. **Black line:** time-integrated net flux of PVS, $\Delta F$. **Blue line:** zonal mean zonal wind, $[u]$. 

**Note:**
1. $[u]$ and are highly correlated.
2. Eddy flux of PVS is anti-correlated with the mean flux of PVS.

$$\Delta F(t) = \int_{t_0}^{t} [v\zeta_a] \, dt \quad [\text{m s}^{-1}]$$

FIGURE 1.59
Meridional PVS-flux: mean and eddy contributions

\[ \Delta F(t) = \int_{t_0}^{t} \left[ v \left[ \zeta \right] + \left[ v^* \right] \zeta^* \right] dt \quad \left[ \text{m s}^{-1} \right] \]

Eddies (waves) may both **accelerate** and **decelerate** the zonal mean the zonal flow !!!
Seasonal cycle of isentropic meridional PVS-flux

At 600 K and 60°N

Yearly average cycle vorticity-flux at 600 K, 1979-2015, 60N

\[ \bar{J} \equiv \left( u \zeta_{abs} + \dot{\theta} \frac{\partial v}{\partial \theta}, \nu \zeta_{abs} - \theta \frac{\partial u}{\partial \theta}, 0 \right) \]

Zonal mean of this component

Total (adiabatic)

Eddy vorticity flux \([10^{-1} \text{ m s}^{-1} \text{ day}^{-1}]\)
Mean vorticity flux \([10^{-1} \text{ m s}^{-1} \text{ day}^{-1}]\)
Total vorticity flux \([10^{-1} \text{ m s}^{-1} \text{ day}^{-1}]\)

(zonal mean, zonal wind \([\text{m/s}]\))

Mean vorticity flux \([10^{-1} \text{ m s}^{-1} \text{ day}^{-1}]\)
Total vorticity flux \([10^{-1} \text{ m s}^{-1} \text{ day}^{-1}]\)

(thick: Savistzky-Golay filter (30 days))

[Graph showing seasonal cycle of vorticity-flux at 600 K and 60°N, with various components labeled.]
Seasonal cycle of isentropic meridional PVS-flux

At 600 K and 60°N

Yearly average cycle vorticity-flux at 600 K, 1979-2015, 60N

Long term *average eddy PVS flux is always pole-ward*, and thus accelerates the zonal mean zonal wind!

- **[u] (m/s)**
- Eddy vorticity flux \([10^{-1} \text{ m s}^{-1} \text{ day}^{-1}]\)
- Mean vorticity flux \([10^{-1} \text{ m s}^{-1} \text{ day}^{-1}]\)
- Total vorticity flux \([10^{-1} \text{ m s}^{-1} \text{ day}^{-1}]\)
  (thick: Savitzky-Golay filter (30 days))
Yes, we are getting approximately correct results for the wrong reason. Our model is like Roger Shepard’s elephant.
Is the caloric model of “heat” correct? **No**, but we still speak of “heat transport”, as if heat is a substance, while, in the context of the atmosphere, it is more correct to state that mass with a relatively high potential temperature is transported poleward, while mass with a relatively low potential temperature is transported equatorward.
“Wave drag” mimics the effect of meridional eddy-PV-mixing in a balanced atmosphere

What appears to be “wave drag” is in fact a manifestation of the effect of meridional isentropic potential vorticity-mixing, which reduces the meridional isentropic PV-gradient.

Strong (complete) mixing reduces the meridional PV-gradient to zero.

With respect to the motionless reference state, this yields a zonal mean PV-distribution with a negative PV anomaly on the pole-ward side of a positive PV-anomaly (illustrated in the next slide).

Adjustment to zonal mean thermal wind balance requires a poleward mass-flux as well as an equatorward vorticity-flux in the mixing zone.

The imposed zonal drag in the 2D GCM represents the effect on the zonal mean zonal wind of isentropic PV-mixing in an atmosphere in thermal wind balance.
Analysed [PV] anomaly

January n.h.

\[ Z = Z_{\text{ref}} + Z' \]

\[ Z^* = \frac{Z'}{Z_{\text{ref}}} \]

(chapter 7 and lecture 3)

Figure on the right:
analysed positive and negative PV-anomalies
unit: [PV]: non-dimensional

Stratospheric winter positive PV-anomaly

Surf zone

Ex-UTLS positive PV-anomaly
Can we understand the zonal mean “Balanced response” from PV-inversion?

PV-inversion equation:

\[ \frac{\partial^2 u}{\partial y^2} + \frac{Z}{g} \frac{\partial}{\partial \theta} \left( \rho \theta \frac{\partial u}{\partial \theta} \right) = \frac{df}{dy} - \sigma \frac{\partial Z}{\partial y} \]

(All variables are zonal means)

Reference state + anomaly:

\[ [Z] = Z_{\text{ref}} + Z' \]

\[ Z_{\text{ref}} = \frac{f}{\sigma_{\text{ref}}} \]

The Reference state is associated with the state of rest

The PV-anomaly induces the zonal mean zonal wind

* Isentropic density in the reference state \((\sigma_{\text{ref}})\) depends only on the vertical coordinate, \(\theta\).
In the surf zone potential vorticity is independent of latitude, as a result of meridional potential vorticity mixing.

In this example the reference potential vorticity, which is associated with the rest state, is “mixed completely”.

Complete mixing is assumed to occur between 35°N and the Pole in the layer, 400-550 K.

This creates a negative PV-anomaly, approximately between 51°N and the Pole, and a positive PV-anomaly between 35°N and approximately 51°N.
Balanced zonal wind associated with this surf-zone

Mixing creates a negative PV-anomaly, approximately between 50°N and the Pole, and a positive PV-anomaly between 35°N and approximately 50°N.

According to the solution of the PV-inversion equation this distribution of PV is associated with a westward wind between 40°N (deceleration) and the Pole and an eastward wind (acceleration) southward of 40°N.

Black: change zonal mean zonal wind due to PV-mixing (contour interval: 2 m s\(^{-1}\))

Dashed: pressure before mixing
Solid: pressure after mixing
PV-mixing requires poleward meridional isentropic mass flux

In a balanced stat, a negative PV-anomaly is associated with a positive isentropic density anomaly.

The high latitude negative PV-anomaly created by meridional PV-mixing, therefore, requires a poleward mass flux in a balanced atmosphere.

This pole-ward mass flux is shown in this figure (deduced from the pressure-change after complete mixing)

Red shading: pole-ward mass-flux (contour interval: 5x10³ kg m⁻¹ K⁻¹)

Black: change zonal mean zonal wind due to PV-mixing (contour interval: 2 m s⁻¹)

Dashed: pressure before mixing
Solid: pressure after mixing
PV-mixing is manifest as poleward meridional mass-flux (in a balanced atmosphere)

Running daily mean poleward isentropic mass-flux (red lines), at $\theta=350$ K and at $\phi=60^\circ$N, between 1 December 2006 and 18 February 2007.

**Thick red line:** mean mass-flux.  
**Thin red line:** eddy mass-flux.  
**Black line:** time integrated net flux of mass, $\Delta M$.  
**Blue line:** zonal mean zonal wind, $[u]$.  

Note:  
1. Integrated mass flux is poleward!  
2. Eddy flux of mass is “somewhat” anti-correlated with the mean flux of mass.

\[
[v\sigma] = [v][\sigma] + [v*\sigma^*]
\]

\[
\Delta M(t) = \int_{t_0}^{t} [v\sigma] dt \quad \text{[kg m}^{-1} \text{K}^{-1}]
\]

**FIGURE 1.55**
PV-anomaly are formed by concentration and dilution of PVS

PVS cannot cross isentropic surfaces!!

Mass can cross isentropic surfaces!!

Divergence of cross-isentropic mass transport due to diabatic processes will dilute/concentrate PVS.

\[ PV \text{ is the mixing ratio of PVS} \]

Therefore, divergence of cross-isentropic mass transport will concentrate or dilute \( PV \) and so create \( PV \)-anomalies in an isentropic layer, for example in the Middleworld, as is shown on the following slides.
Cross-isentropic mass flux divergence in the Middleworld

- **Potential temperature, $\Theta [K]$**
- **Latitude, $\phi$**
- **Middleworld**
- **Earth’s surface**

January

ERA40 & CIRA

- 50 hPa
- 100 hPa
- 200 hPa
- 500 hPa
- 1000 hPa
Cross-isentropic mass flux divergence in the Middleworld

\[ I_d \equiv \frac{d\theta}{d\tau} \sigma \]

\[ \sigma \approx 10 \text{ kg m}^{-2}\text{K}^{-1} \]

\[ \sigma \approx 100 \text{ kg m}^{-2}\text{K}^{-1} \]

\[ \frac{\partial I_d}{\partial \theta} < 0 \]
**Cross-isentropic mass flux divergence in the Middleworld**

- **Potential temperature, $\Theta$ [K]**
  - 500 hPa
  - 450 hPa
  - 400 hPa
  - 350 hPa
  - 300 hPa
  - 250 hPa

- **Latitude, $\phi$**
  - 90°N
  - 60°N
  - 30°N
  - 30°S
  - 60°S
  - 90°S

- **Mass out**
  - Concentration of PVS
  - Dilution of PVS
Cross-isentropic mass flux divergence in the Middleworld

Potential temperature, $\Theta [K]$ vs. latitude, $\phi$:

- Concentration of PVS
- Dilution of PVS
- Concentration of PVS

Earth’s surface

January, ERA40 & CIRA
This course is about the role of transport (radiation, water, mass, vorticity substance) and mixing (potential vorticity) in shaping the large scale thermal and dynamical structure of the atmosphere.

On the large scale the atmosphere is in thermal wind balance.

This represents a strong and useful constraint on the thermo-dynamical structure of the atmosphere, which can be deduced from the so-called PV-inversion equation.

The PV-inversion equation is an expression of thermal wind balance. Its solution indicates how temperature, pressure and wind are related to potential vorticity.
Assuming radiative equilibrium we find a nearly exponential decay of isentropic density with decreasing pressure, which implies a stong increase of potential vorticity with height.

Deviations from radiative equilibrium due to the seasonal cycle of insolation and thermal inertia lead to a simple pattern of cross-isentropic flow with diabatic upwelling in the summer hemisphere and diabatic downwelling in the winter hemisphere, establishing a PV-distribution with a positive PV-anomaly over the winter pole at all heights.

Due to latent heat release in the ITCZ and “planetary wave drag” this pattern of cross-isentropic flow is altered such that cross-isentropic upwelling occurs in the tropics, while cross-isentropic downwelling occurs in the extratropics in both hemispheres. This alters the radiatively determined isentropic density distribution.
Cross-isentropic mass convergence in the tropical Middleworld is mainly due to latent heat release in the ITCZ.

Cross-isentropic mass divergence at high latitudes in the Middleworld is due to radiative cooling.

These processes lead to the formation a PV-anomaly pattern in the Middleworld, which induces the subtropical jets.

Potential vorticity mixing by breaking planetary waves establishes an “inverse PV-anomaly pattern”, which reduces the zonal wind (interpreted as “wave drag”). This is the surf zone.

Above 530 K planetary wave drag is not dominant. Here radiative cooling to space establishes a polar night positive PV-anomaly over the winter Pole, which induces the polar night stratospheric jet.
After the break
What if questions
(a) double Coriolis
(b) double CO$_2$

Friday 14/6, 2019
9:30-12:45
Room BBG 115
10 presentations
10+5 minutes each