

Boundary Layers, Transport and Mixing - part 2

Lecture 2, Aarnout van Delden
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Radiative transfer in the atmosphere

The radiatively determined atmosphere: how is the temperature distribution of the atmosphere established by radiation *only*?

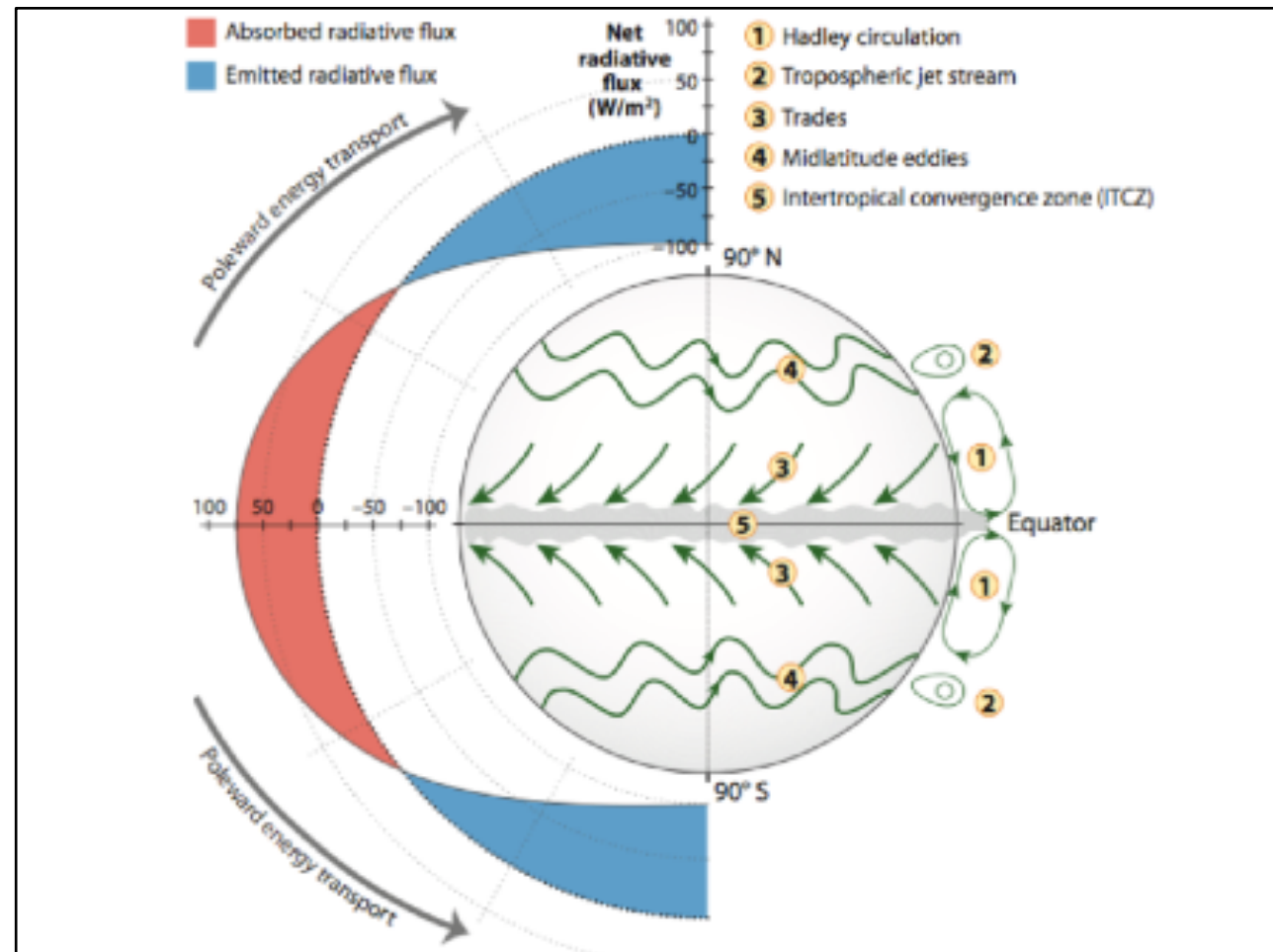
How different is the radiatively determined atmosphere from the radiative equilibrium atmosphere?

The General Circulation of the Atmosphere

“Low latitudes experience a net gain of radiative energy, while high latitudes experience a net deficit; as a result of this imbalance, the oceans and the atmosphere transport heat poleward”

Assignment 1:

Is this true ?



The General Circulation of the Atmosphere

“Low latitudes experience a net gain of radiative energy, while high latitudes experience a net deficit; as a result of this imbalance, the oceans and the atmosphere transport heat poleward”

Assignment 1:

Is this true ?

No, this is not true

Peter: “Transport drives imbalance”.

Raeven, Marleen and Daan: “ If there were no transport, the equator could reach radiative balance by itself”.

Emma: “The actual temperature experiences a lag compared to thermal equilibrium”.

Ruoke, Yujie, Frank: “Thanks to the transport, a steady state temperature is attained”.

Famke, Nicoleta: “To understand the effect of radiation on poleward heat transport, we need look at a model that consists only of the radiative component (no dynamics)”.

Johanna, Anna: “Radiative imbalance is caused by transport”.

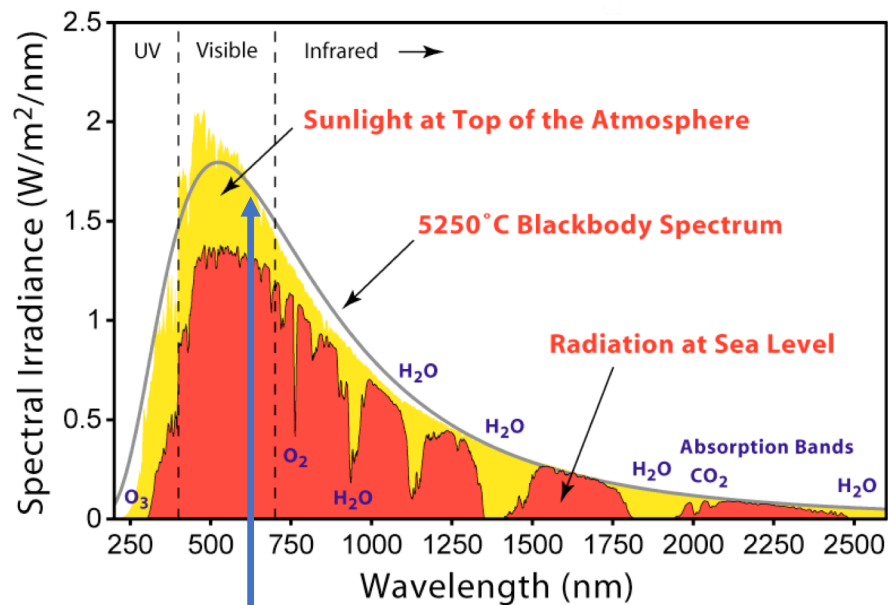
Liv: “Poleward movement is due to the rising air in the tropics and not due to imbalance”.

Marte, Jonna: “thermal inertia has an impact on the heat distribution and therefore reduces the impact of the incoming radiation differences”.

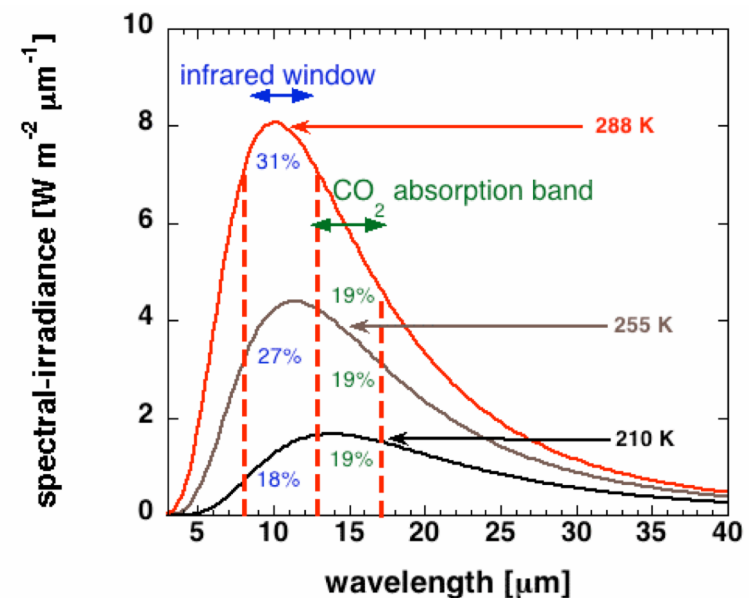
Sarah, Gijs: “Without transport the system would be in radiative equilibrium at all latitudes”.

Emission of radiation: Planck, Stefan, Boltzmann

Solar emission spectrum



Terrestrial emission spectrum



Terrestrial and solar emission in different wavelength bands

Integral of this curve is proportional to σT^4

One layer semi-gray radiation model

Energy “balance”:

$$C \frac{dT_g}{dt} = (1-\alpha) \frac{S_0}{4} + B - U = 0$$

$$\frac{c_p p_s}{g} \frac{dT_a}{dt} = \epsilon U - 2B = 0$$

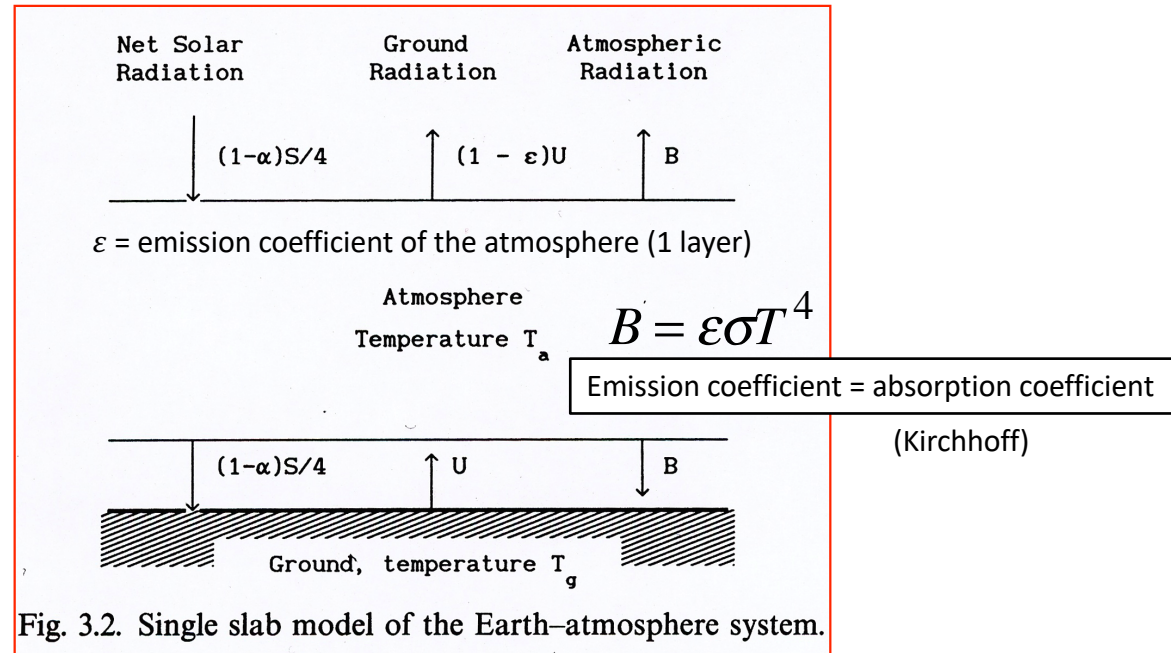
(C is heat capacity of ground)

Emission temperature:

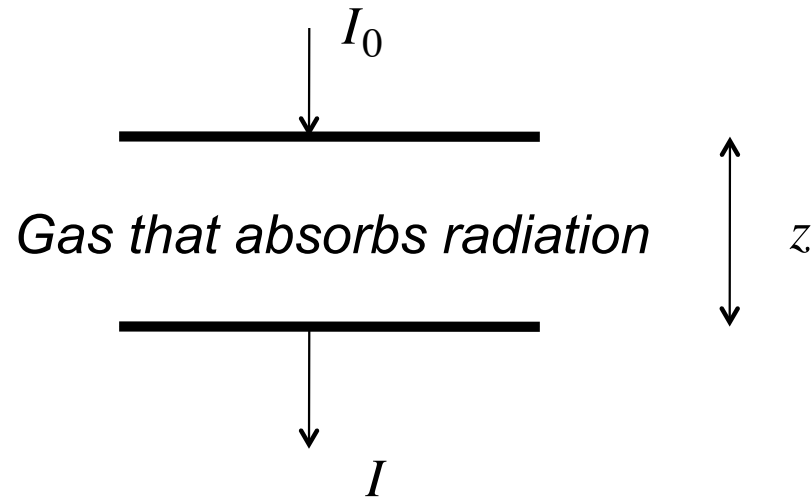
$$T_e = \sqrt[4]{\frac{S_0 / 4 (1 - \alpha_p)}{\sigma}} = 255 \text{ K}$$

Equilibrium:

$$T_g = \left\{ \frac{2}{2 - \epsilon} \right\}^{1/4} T_e \text{ and } T_a = \left\{ \frac{\epsilon}{2 - \epsilon} \right\}^{1/4} T_e$$



Absorption of radiation: Bouguer, Lambert, Beer

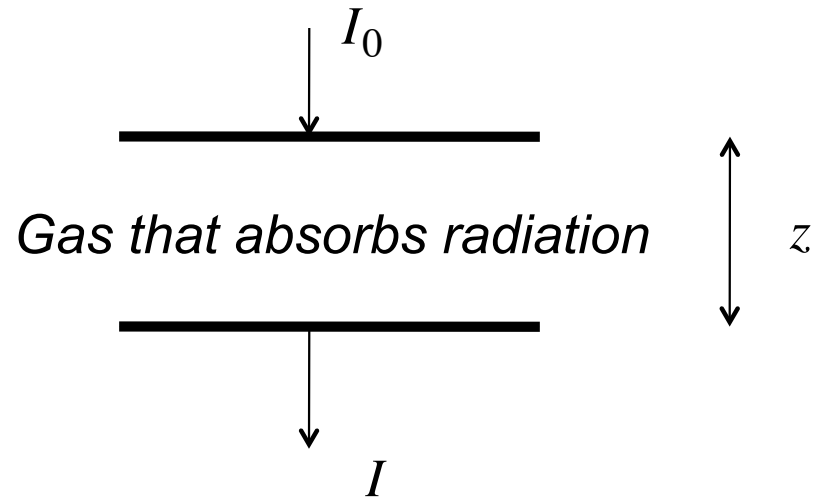


Bouguer-Lambert-Beer Law: $dI = -\kappa I dz$

κ is the absorption coefficient

What are the units of κ ?

Absorption of radiation: Bougeur, Lambert, Beer



Bouguer-Lambert-Beer Law: $dI = -\kappa I dz$ $\rightarrow$ $I = I_0 \exp(-\kappa z)$

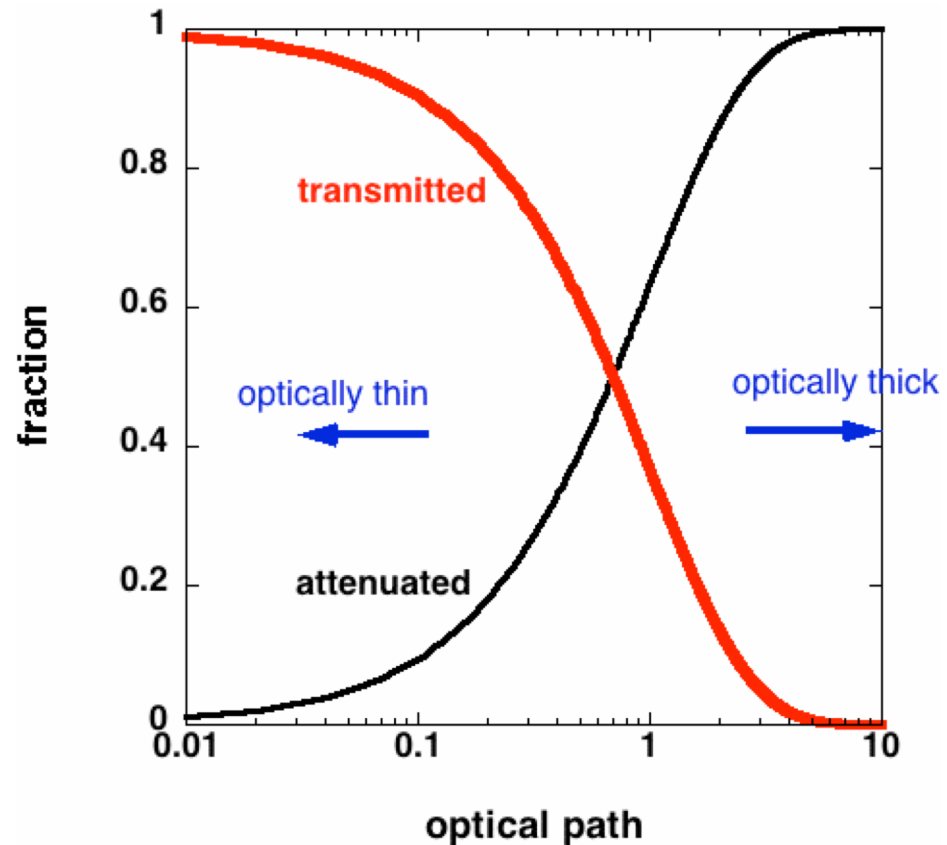
κ is the absorption coefficient

κ^{-1} is the absorption length (unit : m)

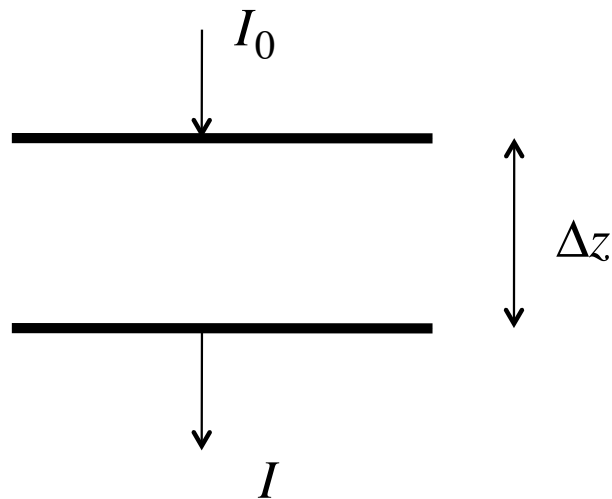
Absorption of radiation: optical path

Fraction of the irradiance that is **transmitted (red)** or **absorbed (black)** as a function of optical path.

κz is the optical path



Absorption of radiation: optical path



Optical path: $\delta = -\kappa \Delta z = \frac{\kappa \Delta p}{\rho g}$

From the Top Of the Atmosphere (TOA) : $\delta = \frac{\kappa p}{\rho g}$

Absorption coefficient: $\kappa = \sigma_a \rho_a$

density of the absorber [kg m^{-3}]

absorption cross-section [$\text{m}^2 \text{kg}^{-1}$]

$$\delta = \frac{\sigma_a \rho_a p}{\rho g}$$

and

$$\delta_s = \frac{\sigma_a \rho_a p_s}{\rho g}$$

(optical path at the surface)

Absorption cross sections

Absorption cross sections of the strongest absorbing atmospheric constituents, such as water vapour, carbon dioxide and ozone.

Ozone absorbs UV solar radiation

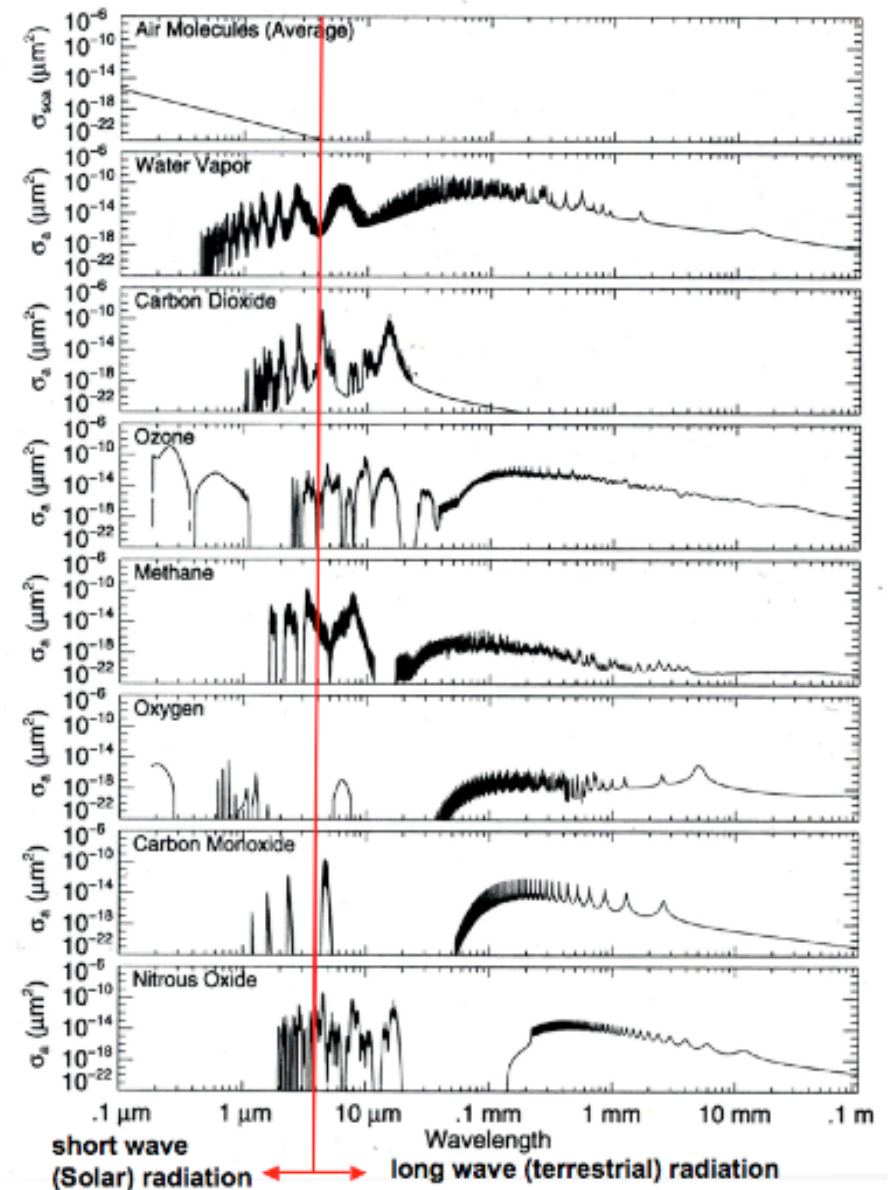
Water vapour and CO₂ absorb mainly long wave terrestrial radiation.

$$dI = -\kappa I dz$$

$$\Delta I \equiv I_0 - I = \sigma_a n_a I_0 \Delta z$$

absorption cross
section in m² per
molecule

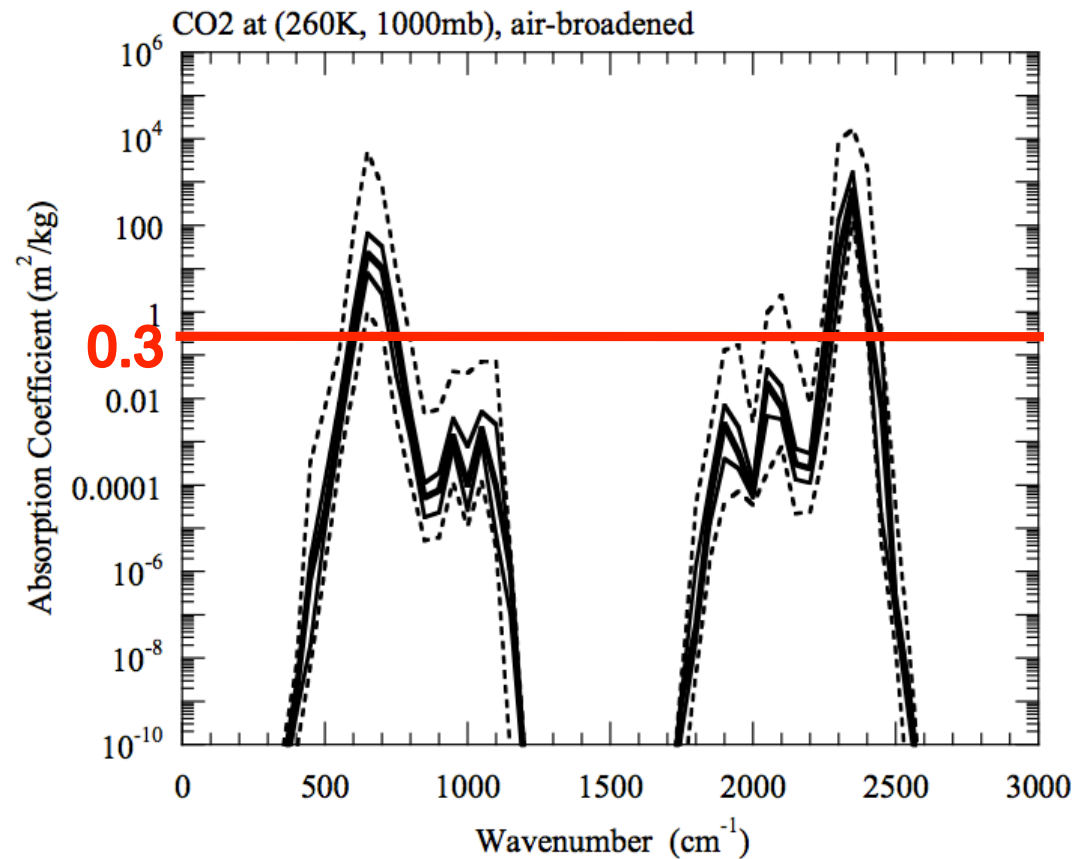
molecule number
density of
absorbing gas



Dependence of absorption by CO₂ in long-wave wavenumber band and grey gas assumption

(wavenumber is inverse of wavelength)

grey gas assumption



Some approximations to radiative transfer theory:

Semi-grey radiative transfer and Kirchhoff's law

Semi-grey:

Absorption coefficient in the short wave band is constant and absorption coefficient in the long wave band does not depend on wavelength.

Kirchhoff:

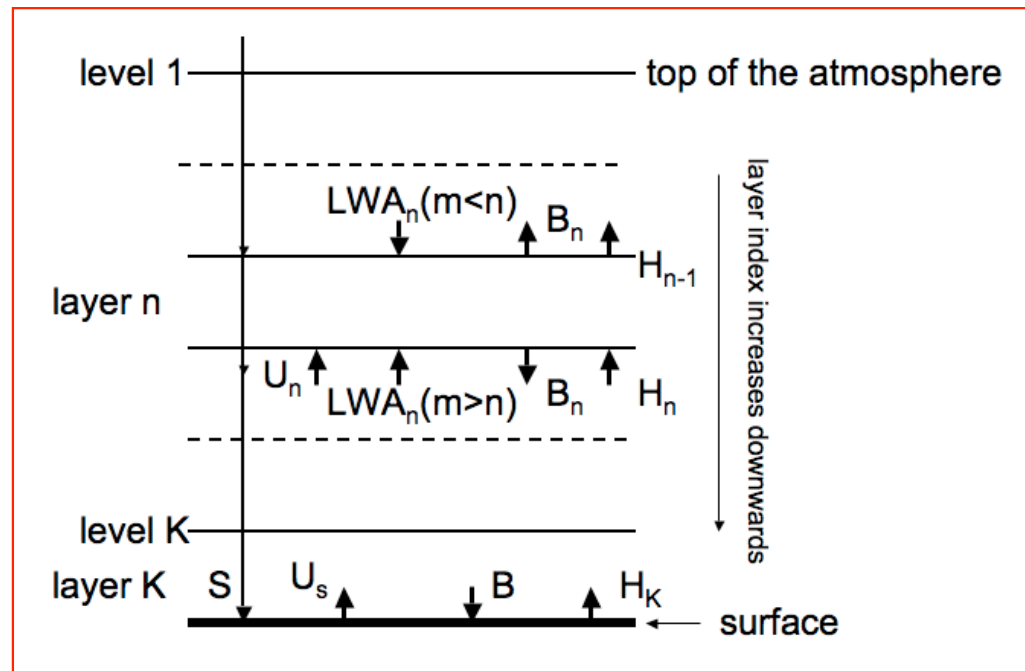
absorption coefficient = emission coefficient

K-layer semi-grey radiation model

Divide the atmosphere into K layers of equal mass. Write down the radiation budget for each layer and integrate these equations in time numerically until a steady state is reached.

LWA_n : longwave radiation flux from atmosphere reaching layer n
 B_n : longwave radiation flux emitted by layer n
 U_n : longwave radiation flux emitted by Earth's surface
 H : convective heat flux
 S : solar radiation reaching Earth's surface

$$S \equiv \frac{S_0}{4} \quad \leftarrow \text{Solar constant}$$



Radiation model equations

Model consists of equations of the type:

$$C \frac{dT_g}{dt} = (1 - \alpha) \frac{S_0}{4} + B - U$$

C = heat capacity earth's surface

$$Q \equiv (1 - \alpha) \frac{S_0}{4}$$

↑
albedo

$$\frac{c_p \Delta p}{g} \frac{dT_n}{dt} = \underbrace{U_n}_{\text{terrestrial}} + \underbrace{S_n}_{\text{Solar}} - \underbrace{2B_n}_{\text{emission}} + \sum_{m=1}^{n-1} \underbrace{LWA_n(m)}_{\text{upwelling}} + \sum_{m=n+1}^K \underbrace{LWA_n(m)}_{\text{downwelling}} + H_n - H_{n-1}$$



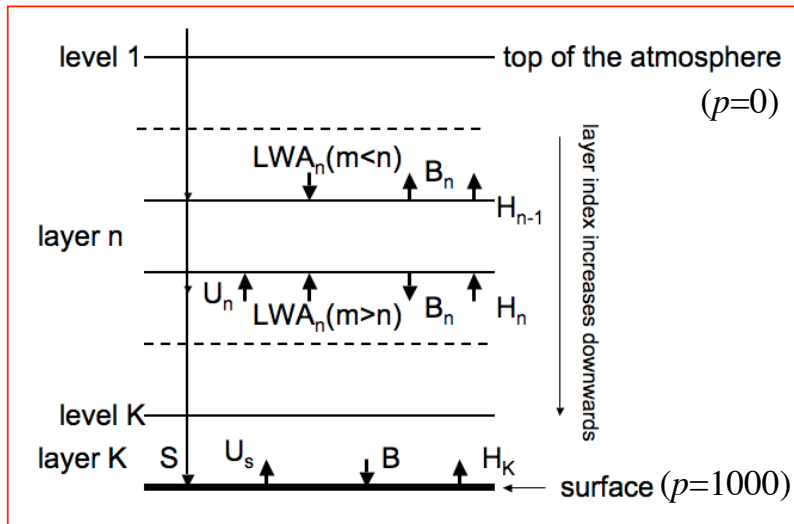
Parametrisation of convective heat transport



Sensible heat fluxes due to convection:

$$H_n = k_H (\theta_{n+1} - \theta_n) \text{ if } \theta_{n+1} > \theta_n$$

$$H_n = 0 \text{ if } \theta_{n+1} \leq \theta_n$$



Numerical integration of the model equations

To solve a system of first order coupled differential equations,

$$\frac{dX}{dt} = F(X,Y); \quad \frac{dY}{dt} = G(X,Y)$$

Choose a time increment Δt , and then find $X(t + \Delta t)$ by the following recipe (due to Runge and Kutta):

$$\begin{aligned} X_0 &= X(t), \\ X_1 &= X_0 + F(X_0, Y_0) \Delta t / 2, \\ X_2 &= X_0 + F(X_1, Y_1) \Delta t / 2, \\ X_3 &= X_0 + F(X_2, Y_2) \Delta t, \\ X_4 &= X_0 - F(X_3, Y_3) \Delta t / 2, \\ X(t + \Delta t) &= (X_1 + 2X_2 + X_3 - X_4) / 3, \end{aligned}$$

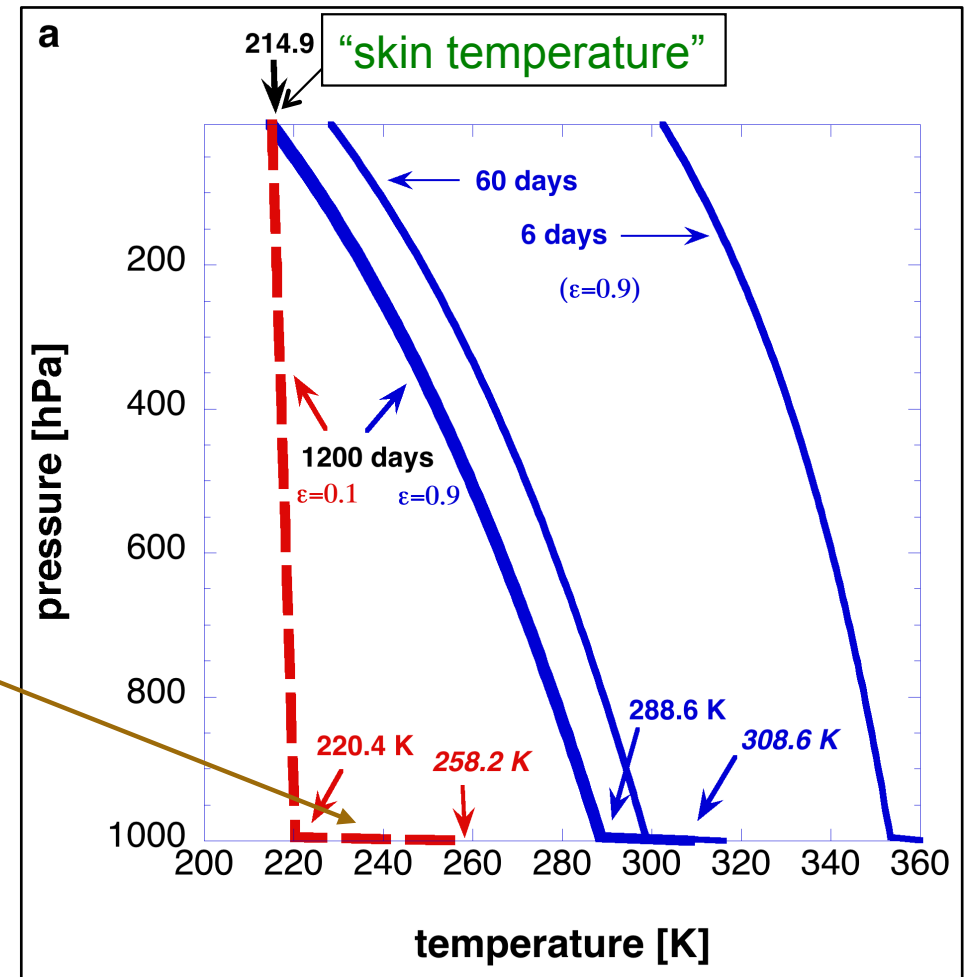
Solution for a homogeneous atmosphere, transparent to Solar radiation

Adjustment to radiative equilibrium: thermal inertia

Initially temperature is
360 K everywhere

Response time in the case of $\varepsilon = 0.9$
is in the order of a **few months** !

Discontinuity at Earth's surface!
-> “convective” instability



Solution for a homogeneous atmosphere, transparent to Solar radiation

Radiative equilibrium for $\varepsilon=0.9$

Analytical solution:

$$B \equiv \sigma T^4 = \frac{1}{2} Q(\delta + 1) \quad (\text{Box 1.4})$$

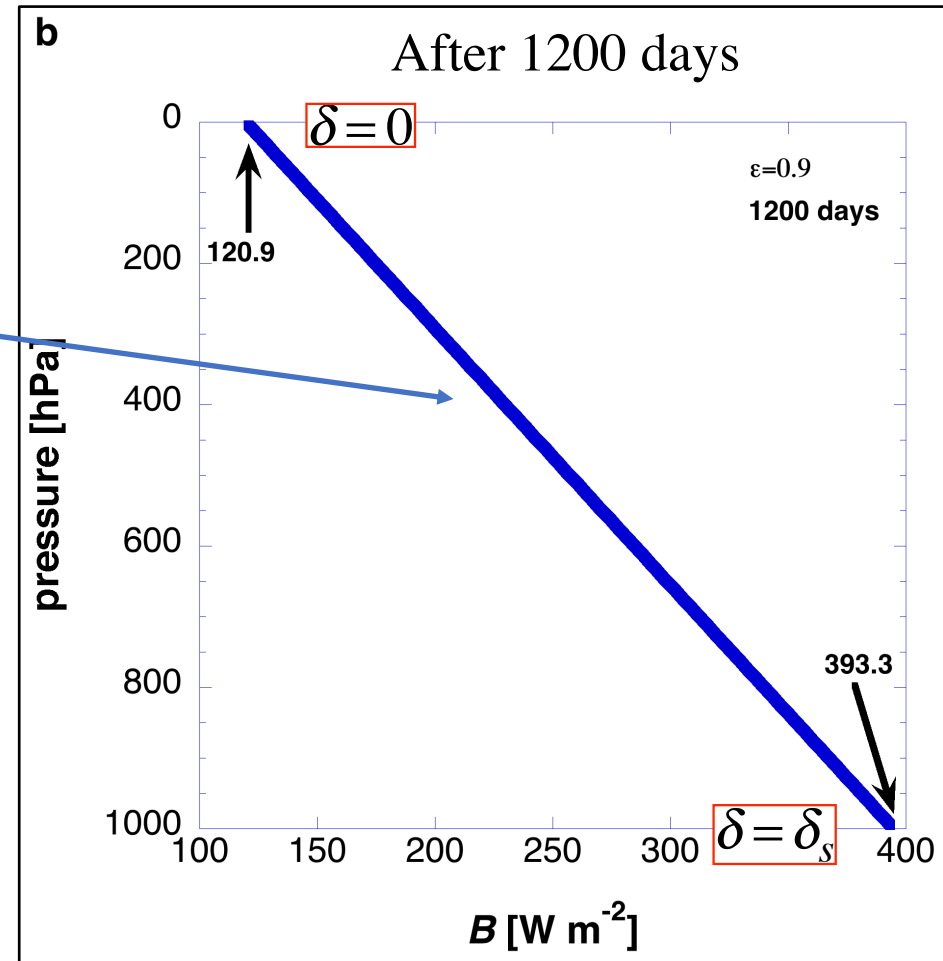
Relation between ε and δ :

$$\varepsilon = 1 - \exp(-\delta_s)$$

At the surface:

$$B \equiv \sigma T^4 = \frac{1}{2} Q(\delta + 2)$$

Static instability!!



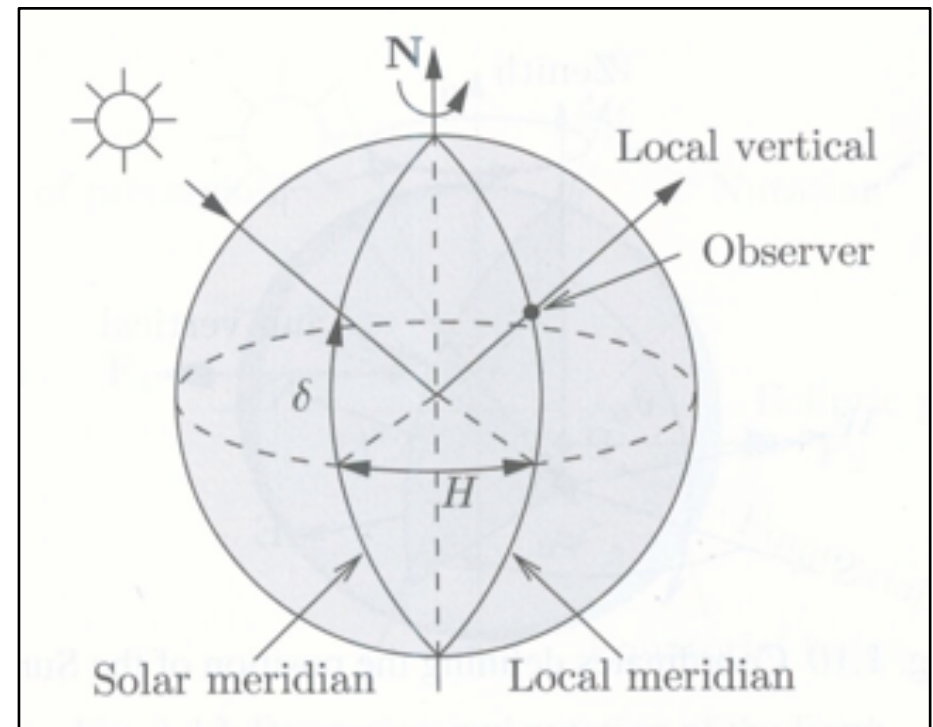
Calculating daily average Solar irradiance

The **daily average Solar irradiance**, Q (expressed in W m^{-2}), for latitude, ϕ , is calculated according to

$$Q = \frac{S_0}{\pi} \left(\frac{d_m}{d} \right)^2 (\Delta H \sin \phi \sin \delta + \cos \phi \cos \delta \sin \Delta H)$$

Orbital parameters

S_0 represents the **Solar constant (yearly average solar irradiance at AU)** ($1361\text{-}1366 \text{ W m}^{-2}$)*, d_m is the average distance between the Sun and the Earth, d represents the 'actual' distance between the Sun and the Earth, δ is the **declination** angle of the sun and ΔH is the **length of the day** in radians



Orbital parameters

Distance between the Sun and the Earth: $d(t) = d_m \left(1 + e \sin \left\{ (2\pi / 365) \cdot (272 + t) \right\} \right)$

Eccentricity: $e = 0.017$

$$d_{\min} = d_m (1 - e); \quad d_{\max} = d_m (1 + e)$$

Declination angle: $\delta(t) = (\pi \delta_{\max} / 180) \sin \left\{ (2\pi / 365) \cdot (285 + t) \right\}$

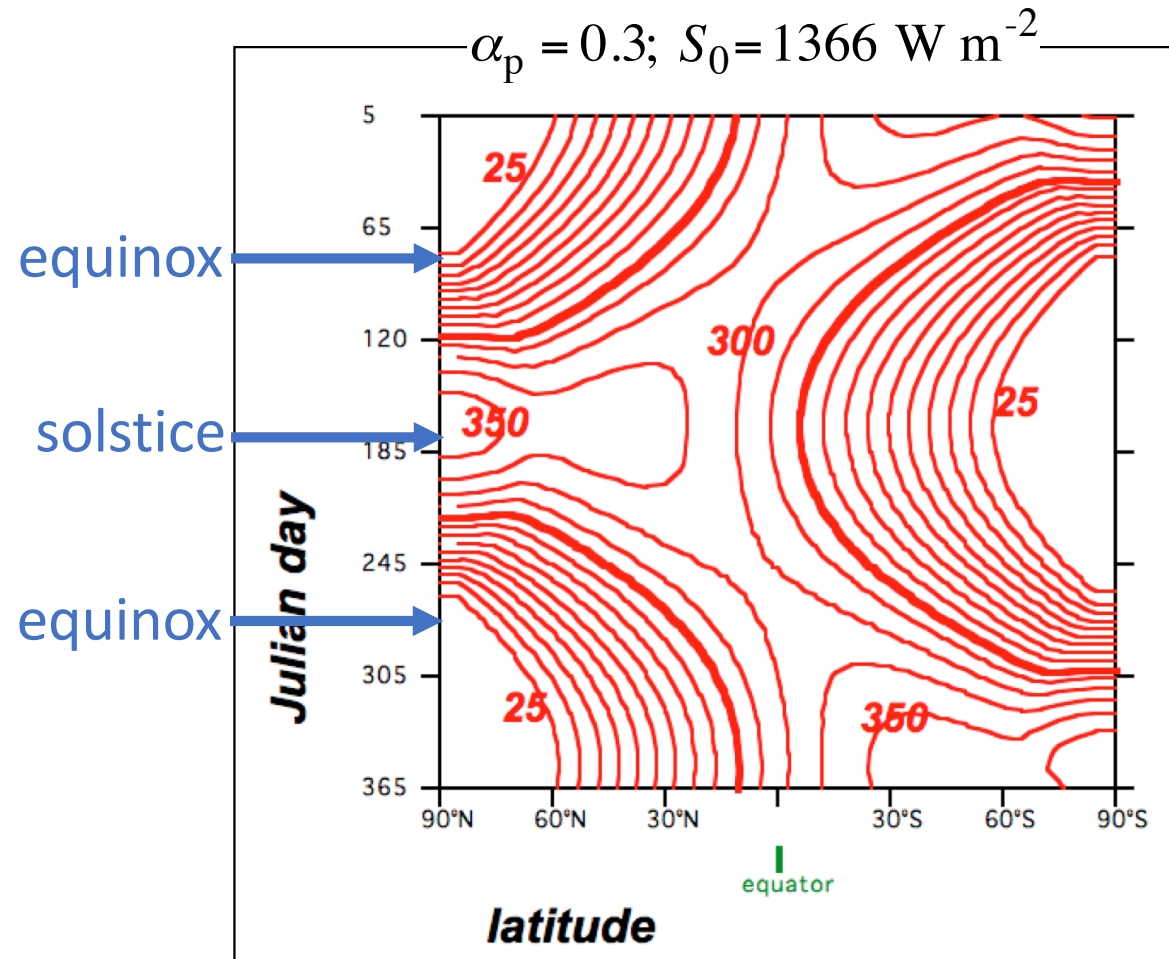

axial tilt / obliquity

Length of the day (radians): $\Delta H = \arccos(-\tan \phi \tan \delta)$

Absorbed Solar radiation

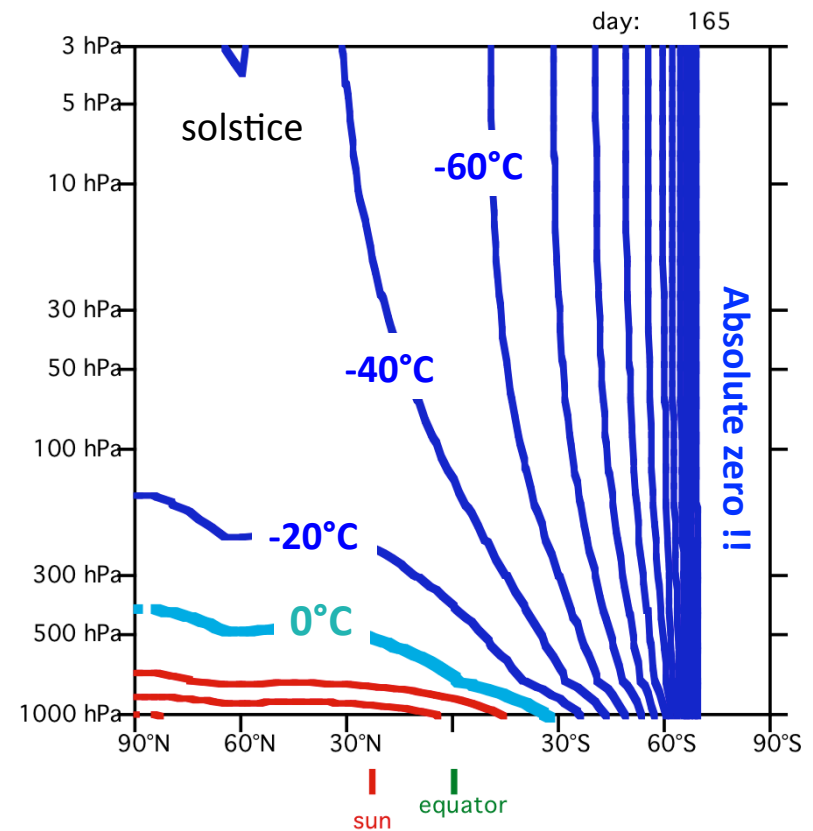
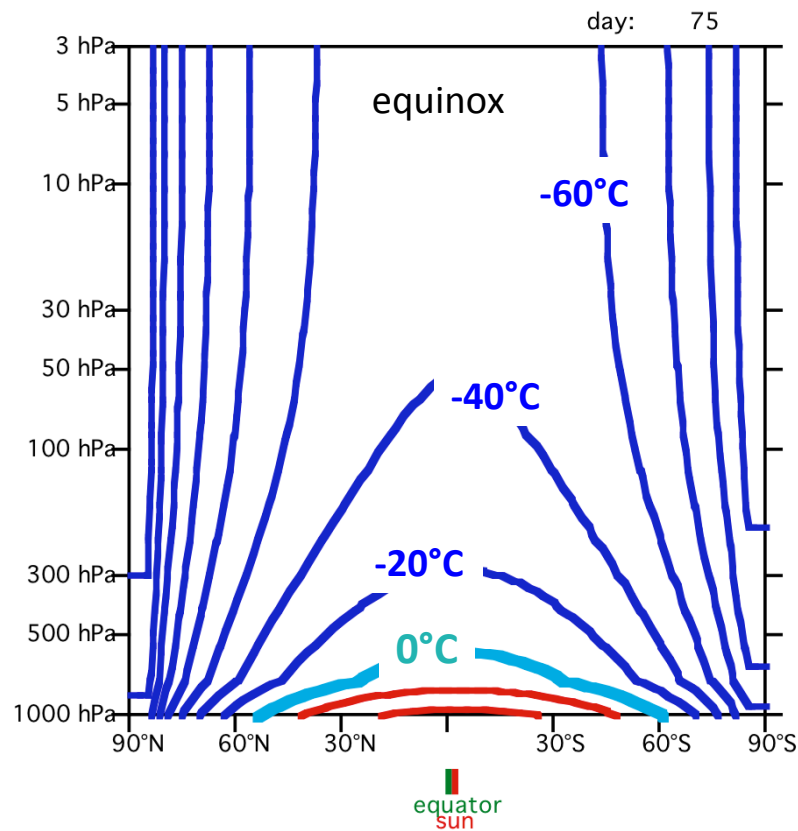
Absorbed Solar radiation (ASR)

(indicated in W m^{-2}) as a function of latitude and time for a planet with a **uniform albedo, α_p , equal to 0.3**. Contour interval is 25 W m^{-2} . **Thick contour** corresponds to 250 W m^{-2} .



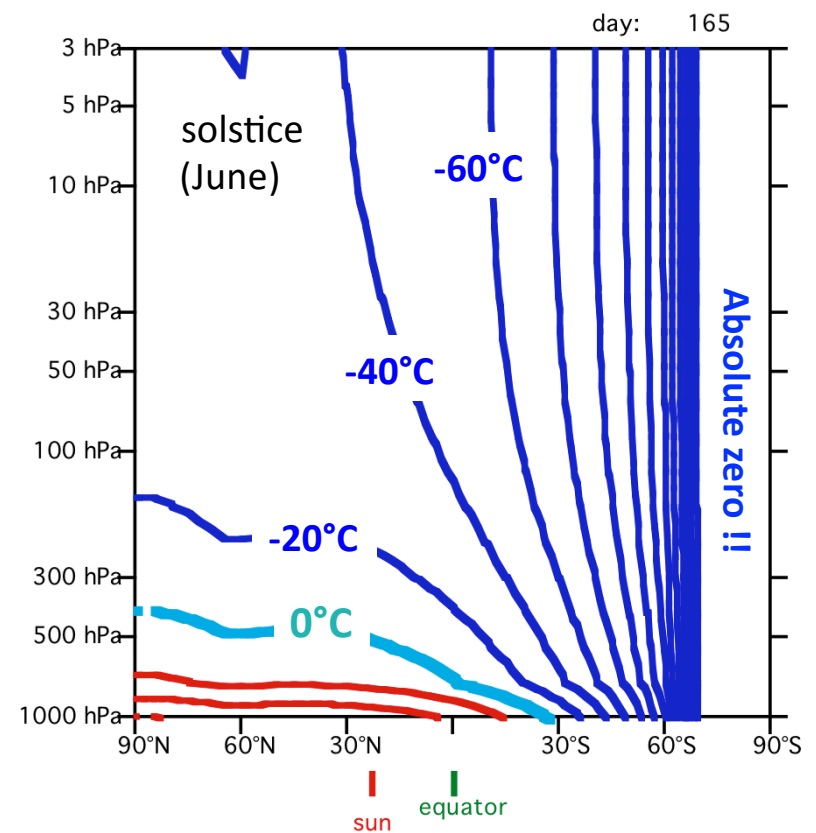
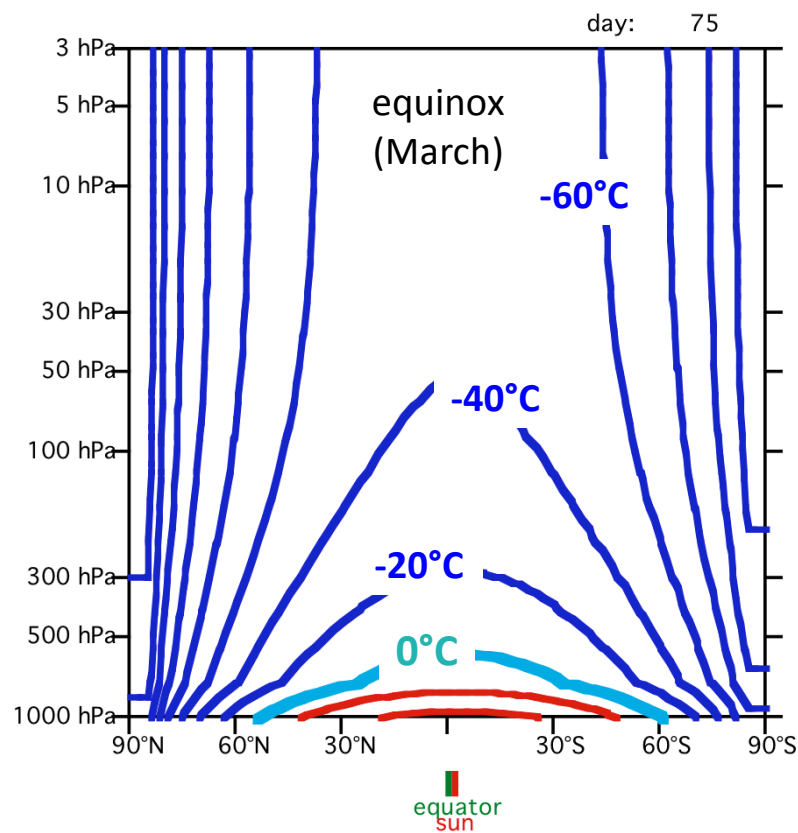
Radiative equilibrium temperature: analytical solution

Atmosphere, which is transparent to Solar radiation and contains one well-mixed greenhouse gas. Contour interval is 20°C. **The cyan contour represents the 0°C isotherm.** Temperatures below 0°C are indicated by blue contours. **Temperatures above freezing (0°C) are indicated by red contours.** In the Polar night, poleward of 66°S (right panel), the radiative equilibrium temperature is absolute zero Kelvin!



Radiative equilibrium temperature: analytical solution

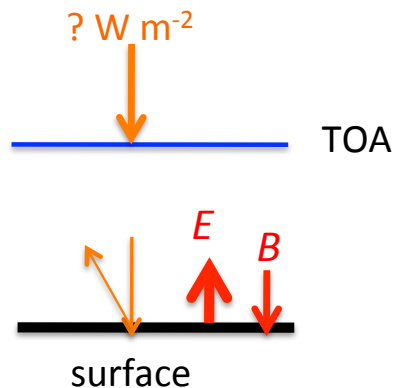
Summer pole warmer than tropics
No tropical “cold trap”
Polar night temperature: absolute zero K!



Thermal inertia: radiatively determined state

Compute the temperature as function of latitude, pressure and date, taking heat capacity into account

This leads to the so-called “**radiative determined state**”



surface temperature

absorbed Solar radiation

$$C \frac{dT_s}{dt} = Q + B - \sigma_B T_s^4$$

heat capacity of the earth's surface

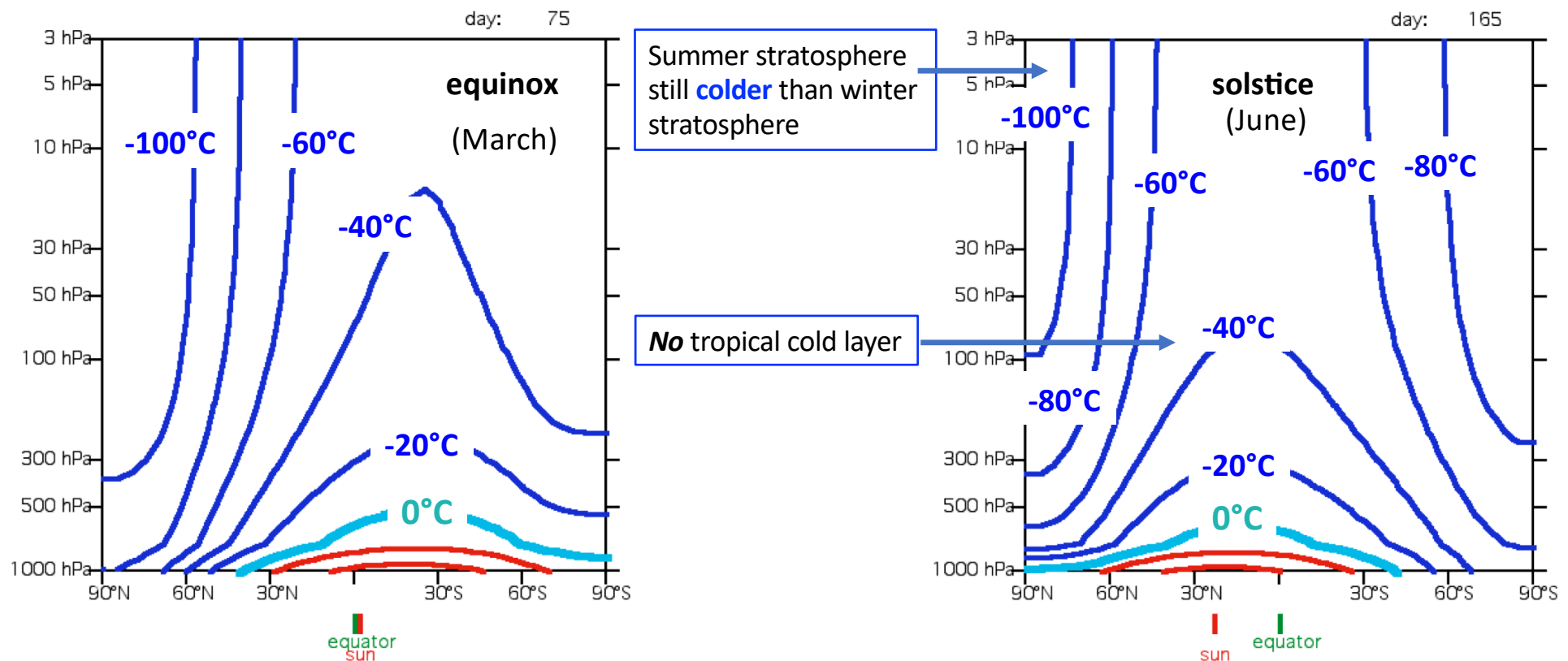
Emitted (E) terrestrial radiation (black body!)
“back-radiation” from atmosphere

C is also called the “**thermal inertia coefficient**”

Large $C \Rightarrow$ slow response to radiative imbalance: $Q + B - \sigma_B T_s^4 \neq 0$

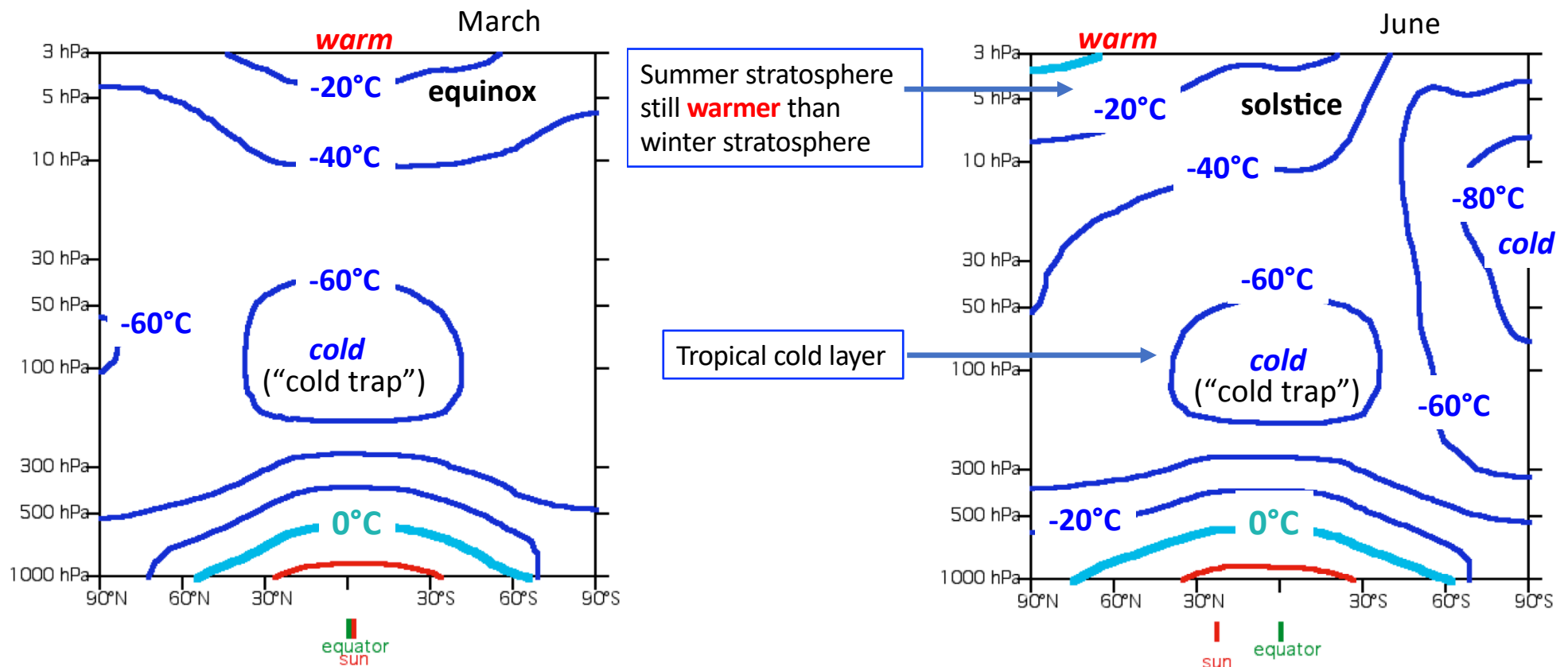
Radiatively determined temperature: numerical solution

Radiatively determined temperature [°C] as a function of latitude and pressure for day 75 (left) and for day 165 (right) in an atmosphere, which is transparent to Solar radiation and contains one well-mixed greenhouse gas. The **cyan contour represents the 0°C isotherm**. Temperatures below 0°C are indicated by **blue contours**. Temperatures above freezing (0°C) are indicated by **red contours**. The **heat capacity of the Earth's surface, $C=10^7 \text{ J K}^{-1} \text{ m}^{-2}$** .



Observed zonal-mean temperature distribution

Observed (COSPAR International Reference Atmosphere) monthly average zonal average temperature [°C] as a function of latitude and pressure for March (left panel) and for June (right panel). Contour interval is 20°C. **The cyan contour represents the 0°C isotherm.** Temperatures below 0°C are indicated by **blue** contours. Temperatures above freezing (0°C) are indicated by **red** contours.



Assignment 2: PROBLEM 12.3

Radiatively determined mass distribution (read sections 12.3-4)

(Seasonal growth and decay of the Underworld)

Compute and plot the area-weighted average mass per unit area in the **Underworld ($\theta < 300$ K) in the Northern Hemisphere** in the “radiatively determined atmosphere” as function of time. The temperature as a function of pressure in the “radiatively determined atmosphere” is computed by running the **200-layer** radiation model (described in **section 2.7 and this lecture**) for fixed latitudes for two years with seasonally varying insolation (**Box 2.1 and this lecture**), starting on 1 January with an atmosphere with constant temperature of 300 K. The surface pressure is 1000 hPa at all latitudes at all times. The “radiatively determined atmosphere” reacts to seasonal insolation variations with a certain (thermal) inertia, which depends on the heat capacity of the earth’s surface (C) and the greenhouse gas content of the atmosphere. **Output of two model runs is provided.** Both runs assume that the well-mixed greenhouse gas (proxy for CO_2) concentration is 400 ppmv. In run 1, $C = 10^7 \text{ J kg}^{-1} \text{ m}^{-2}$. In run 2, $C = 10^6 \text{ J kg}^{-1} \text{ m}^{-2}$. You will get the results of these runs. Compare the outcome with the dark blue curve in the left panel of **figure 1.66 (next slide)** (discuss the differences).

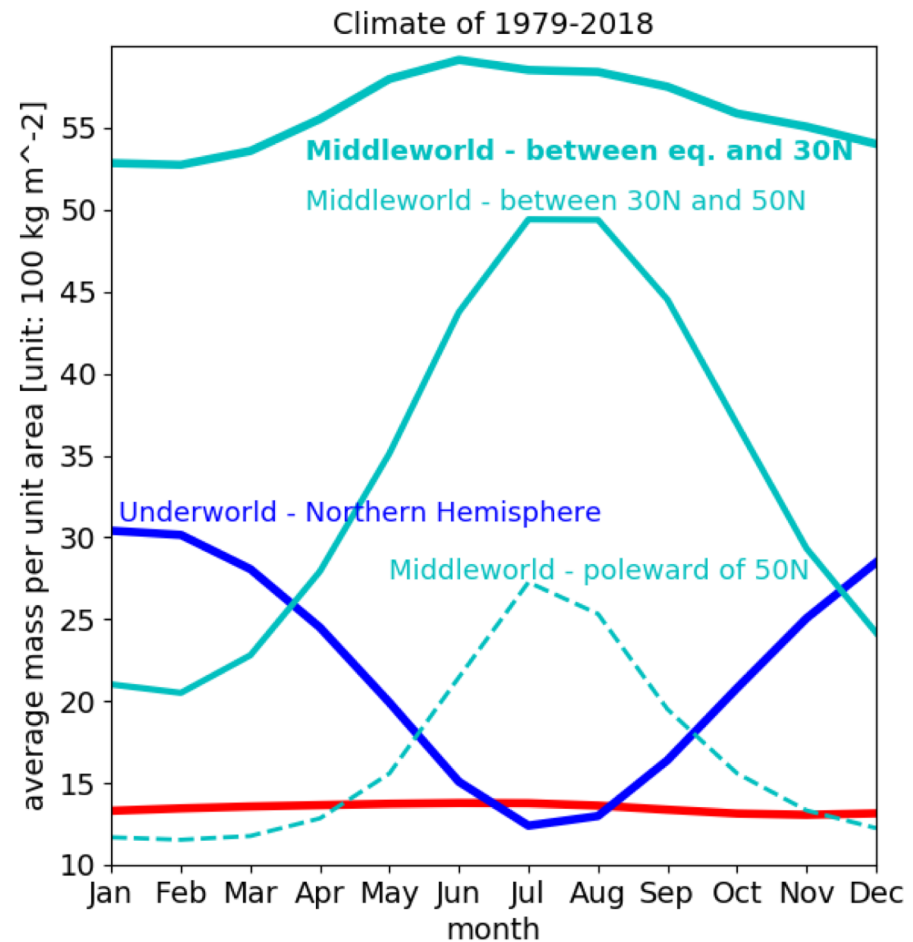
Deadline: end of Fri. 28 May 2021

22.5%

TimeevolutionRDtempNH_case1
TimeevolutionRDtempNH_case2
(see Blackboard)

Seasonal growth and decay of the Underworld

FIGURE 1.66. Ensemble mean (1979-2018) yearly cycle of the monthly mean mass in the **Underworld ($\theta < 300$ K)** in the Northern Hemisphere (**blue line**), in the **Middleworld ($300 < \theta \leq 370$ K)** poleward of 50°N (**dashed, cyan**), in the **Middleworld** between 30°N and 50°N (**thin cyan**), in the tropical (0°N - 30°N) Middleworld (**thick cyan**) and in the **Overworld ($\theta > 370$ K)** in the Northern Hemisphere (**red**). In the left panel the unit is kg m^{-2} . In right panel the unit is kg. The seasonal cycle of the total mass in the Underworld is due mainly to exchange with the Middleworld. The mass budget in January is in an approximate stationary state. Based on the monthly mean ERA-Interim reanalysis.



Data file of assignment 2: rad. det. temperature

(structure of the files)

Radiative equilibrium temperature, case 2 (analytical solution)

Level → (200 levels)

time[days]	1	2	3	4	>>	199	200	TgEq[K]	albedo	OLRTOA[Wm ⁻²]	Insolation[Wm ⁻²]	HeatFlux_ground[Wm ⁻²]	Latitude
↓ 1.000	223.23	223.75	224.26	224.77	>>	289.60	289.84	312.53	0.30	0.00	400.45	0.0	0.0
2.000	223.31	223.82	224.34	224.84	>>	289.70	289.93	312.63	0.30	0.00	400.99	0.0	0.0

Radiatively determined temperature, case 2 (numerical solution, taking thermal inertia into account)

Level → (200 levels)

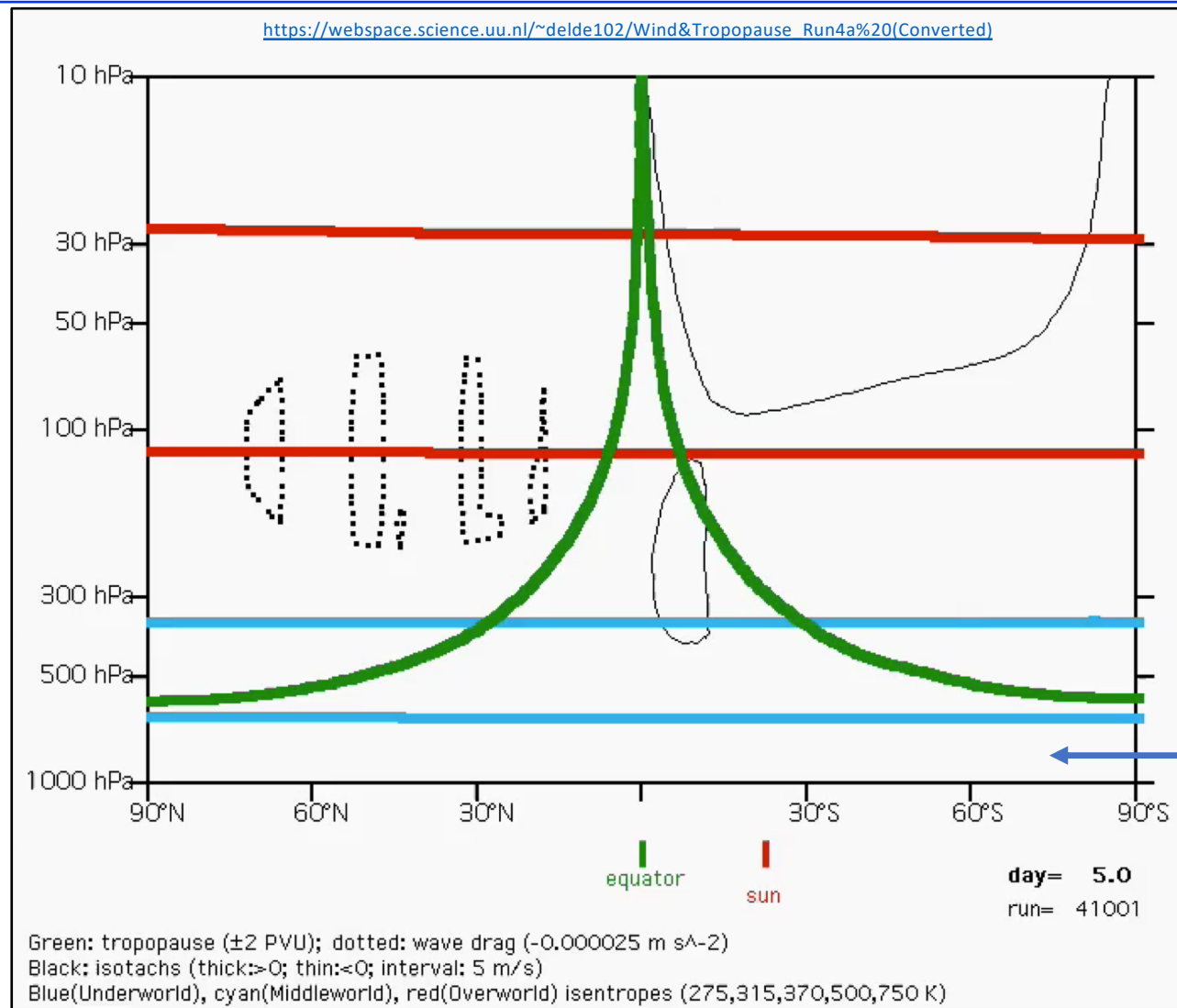
time[days]	1	2	3	4	>>	199	200	Tg[K]	albedo	OLRTOA[Wm ⁻²]	ASR[Wm ⁻²]	HeatFlux_ground[Wm ⁻²]	Latitude
↓ 1.000	293.82	293.88	293.95	294.01	>>	301.86	301.90	329.27	0.30	477.02	280.32	0.0	0.0
2.000	288.46	288.58	288.70	288.81	>>	303.33	303.40	329.09	0.30	464.30	280.69	0.0	0.0

Level 1 is at 2.5 hPa, **level 2** is at 7.5 hPa,..., **level 200** is at 997.5 hPa.

($\Delta p = 5$ hPa)

=0.7x400.45

Simulation of the General Circulation with a 2D-(SD)-GCM



“Statistical-Dynamical”

Watch the seasonal growth and decay of the lower Underworld ($\theta < 275 \text{ K}$)

Next: Lecture 3 BLT&M-2 on Friday 28 May 2021, 10:00

No lecture by Michiel van den Broeke on Wednesday, 26 May, 2021

Topic of lecture 3 BLT&M-2 (28 May, 10:00-11:45), BBG 083
Radiatively determined isentropic density and potential vorticity
Introduction to the “2D-(SD)-GCM”
The (energetics of the) water cycle (in the model)

Hand in answer to assignment 2 of BLT&M-2 on or before 28 May 2021

(problem 12.3)