Boundary Layers, Transport and Mixing - part 2

Lecture 5, Aarnout van Delden
a.j.vandelden@uu.nl

The role of eddies/waves in shaping the General Circulation, part 2

Some properties of the General Circulation

An **abrupt meridional gradient of potential vorticity (PV)** exists in the sub-tropical Middleworld.
This meridional PV-gradient **acts as a planetary wave guide**.
The abrupt meridional gradient of potential vorticity **coincides with the subtropical jet**.
The atmosphere is always **close to zonal-mean geostrophic balance and so also to thermal wind balance**

The influence of the eddies on these properties

The **action of the eddies is to mix potential vorticity** in the meridional direction
The **action of the eddies is to reduce the meridional gradient of potential vorticity**.
This process **reduces the zonal-mean zonal wind**: can we say that this **is a manifestation of wave drag**?
What is the cause of the **abrupt meridional gradient of potential vorticity?**

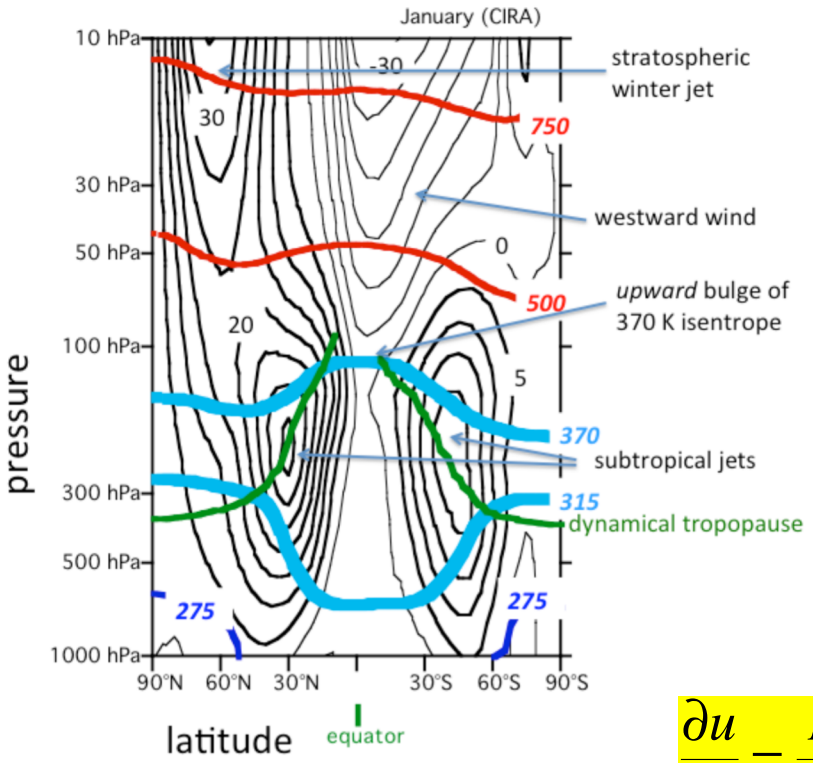
Next lecture (lecture 6): The *eight ingredients of eddy – zonal-mean flow interaction* **...**

Recap

Recap

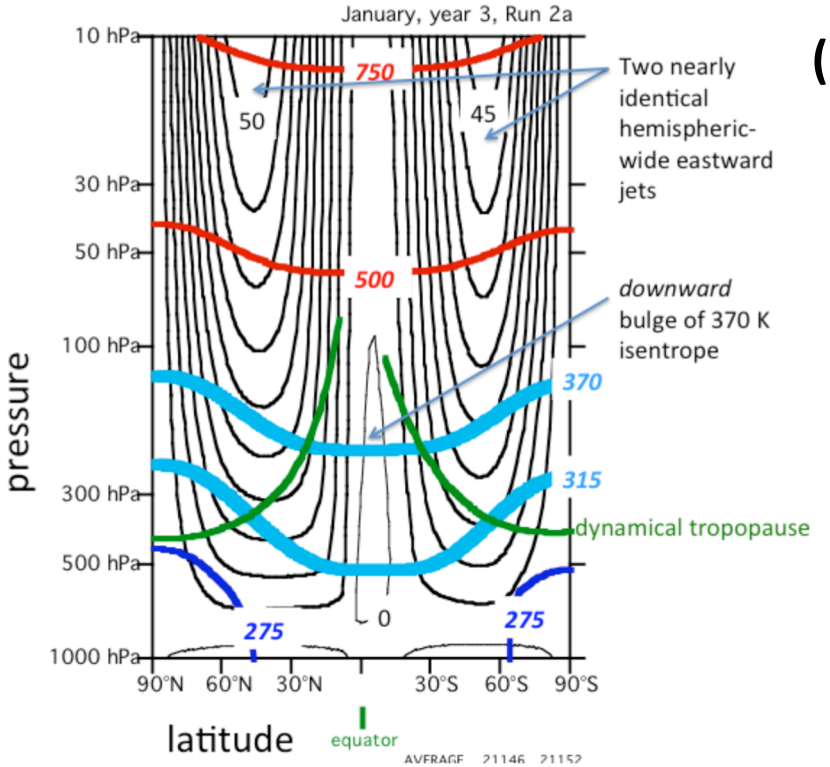
2D-General Circulation Model without “eddies”

Observed zonal mean state



Modelled zonal mean state (**no eddies/waves**)

(run 2a)



$$\frac{\partial u}{\partial p} = \frac{R}{fp}\frac{\partial T}{\partial y}$$

(in pressure coordinates)

Thermal wind balance is fulfilled qualitatively

Recap

(Section 12.8)

Recap

Planetary wave drag parametrisation in the 2D General Circulation Model (GCM)

Zonal wind equation:

Wave drag

$$\frac{\partial p_s u}{\partial t} = -\frac{\partial p_s uv}{\partial y} - \frac{\partial}{\partial \sigma}\left(p_s u \frac{d\sigma}{dt}\right) + \left(f + \frac{u\tan\phi}{a}\right)p_s v + p_s D + p_s F_x$$

$$D = D_0 B(\phi) Z(\Phi)$$

"constant"

latitude

geopotential

$$B(\phi) = \sin(2|\phi|)$$

$$Z(\Phi) = \sin\left[\pi\left(\frac{z - z_0}{z_1 - z_0}\right)\right] \text{ if } z_0 < z < z_1$$

$$Z(\Phi) = 0 \text{ if } z \le z_0 \text{ or } z \ge z_1$$

$$z_0 = 10 \text{ km and } z_1 = 25 \text{ km}$$

MINUS: DRAG

$D_0(z) = -0.0001 \text{ m s}^{-2}$ if $u>0$ at all levels below z.

$D_0(z) = 0$ if $u<0$ at all levels below z.

Recap

The best simulation with the 2D-SD-GCM (run 6 in the lecture notes; equivalent to run 4a – next slide)

Recap

Model state after 530 days with water cycle *and* "eddies"

Stippled contours: **wave drag**

in units of 10^{-6} m s^{-2}

Zonal wind is westward in the summer stratosphere

Zonal wind is eastward in the winter stratosphere

Subtropical jet is separated from the polar stratospheric jet

Upward bilge of the 370 K isentrope

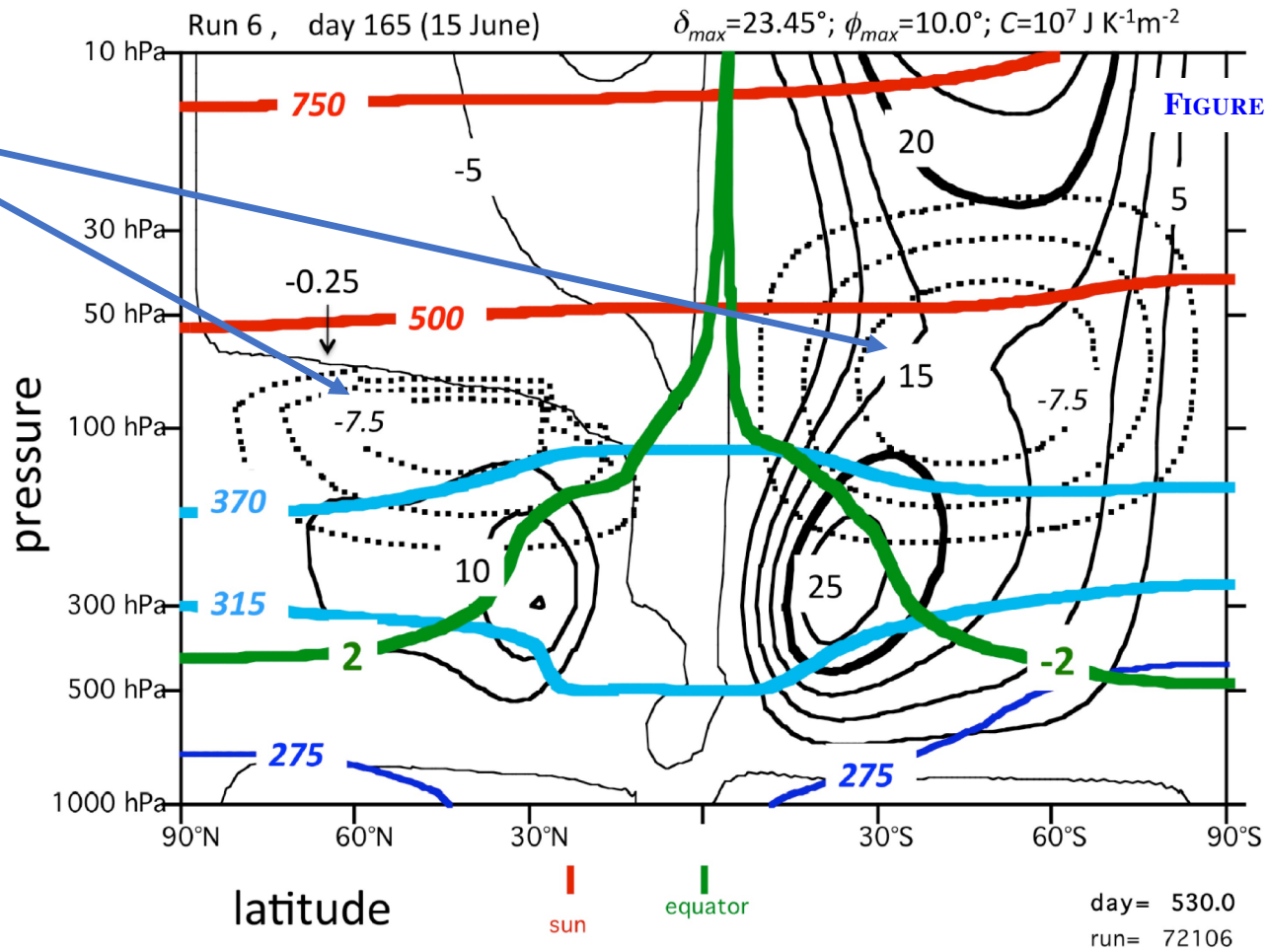


FIGURE 12.26

Best simulation with the 2D-SD-GCM

Animation

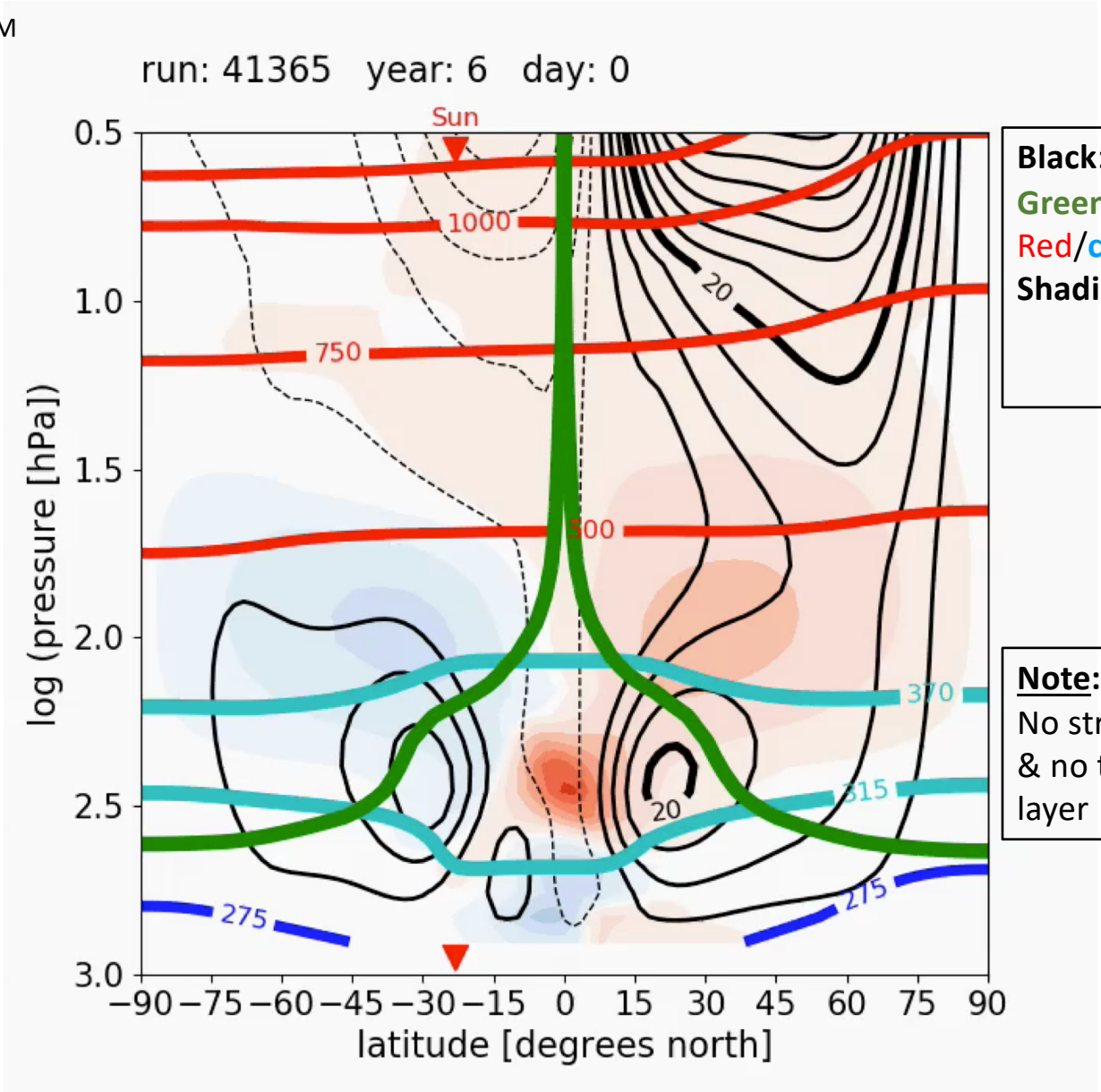
(year 6)

Recap

Run 4a

Water cycle: yes
Ozone: yes
Wave drag: yes

Latitude of the ITCZ varies between 10°S and 10°N, in phase with the overhead sun.



Black: [*u*] (interval=5 m/s)
Green: dynamical tropopause
Red/**cyan**/**blue**: isentropes
Shading: [*v*]
red: northward
blue: southward

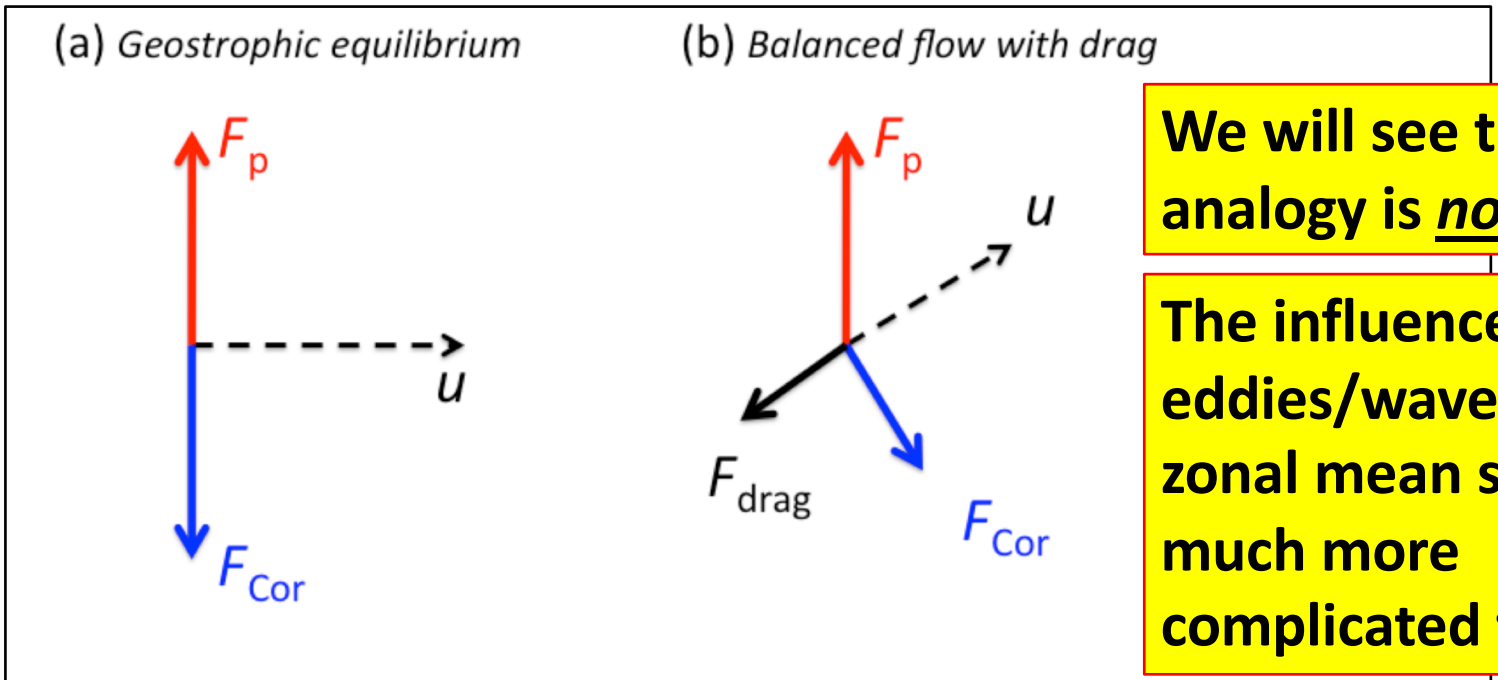
Note:
No stratospheric wind- reversals & no tropical tropopause cold layer

Recap

Recap

Intuitive understanding through analogy and metaphor:

Analogy to "Ekman pumping"



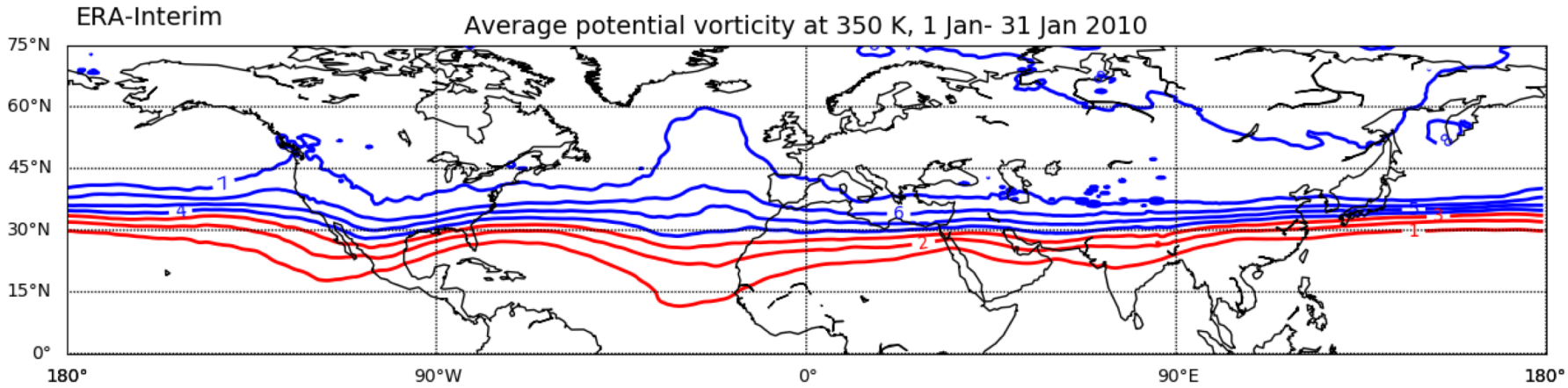
We will see that this analogy is *not* true

The influence of eddies/waves on the zonal mean state is much more complicated than this

[redacted]. Force balance for a steady geostrophic mean wind during northern hemisphere winter in the absence of wave drag (a) and with wave drag (b), where the drag force F_{drag}, opposes the wind and the Coriolis force, F_{Cor}, is directed at right angles to the wind direction. The pressure gradient force, F_p, is directed towards the north pole.

Abrupt meridional PV-gradient in Middleworld !

<u>Monthly mean</u> potential vorticity at θ=350 K



Blue: PV>4 PVU; **Red: PV<4 PVU**

Contour interval = 1 PVU

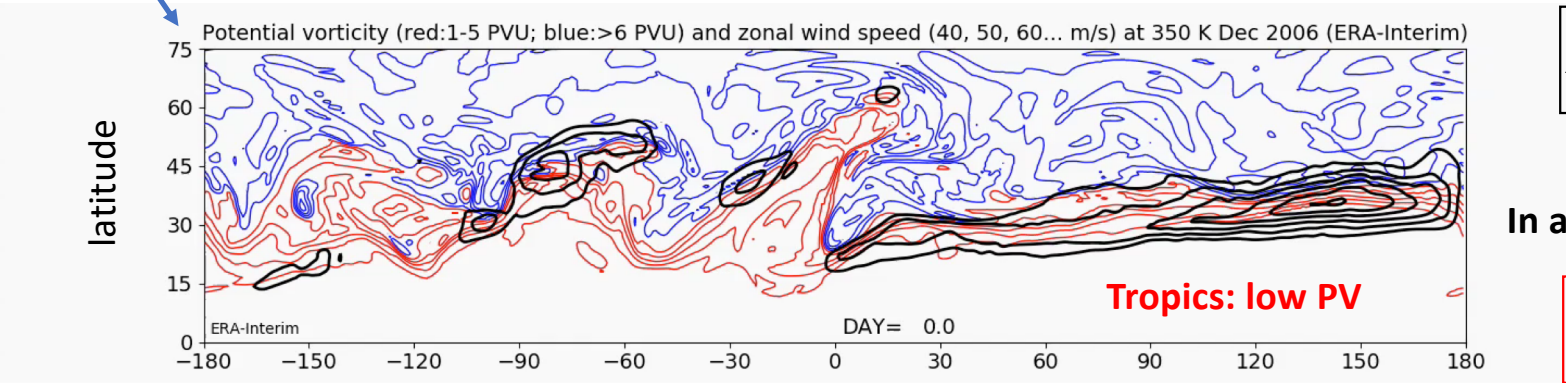
January 2010

See **figure 1.58**

This meridional PV-gradient acts as a planetary wave guide

Animation of Z_θ (PV) and zonal wind speed, u, at θ=350 K in December 2006
(contours starting at 30 m/s, interval 10 m/s)

Preferred regions of wave-breaking: east Pacific and east Atlantic



Blue: PV>4 PVU; **Red: PV<4 PVU**

Potential vorticity:

$$Z_\theta \equiv \frac{\zeta_\theta + f}{\sigma}$$

In adiabatic conditions:

$$\frac{dZ_\theta}{dt} = 0$$

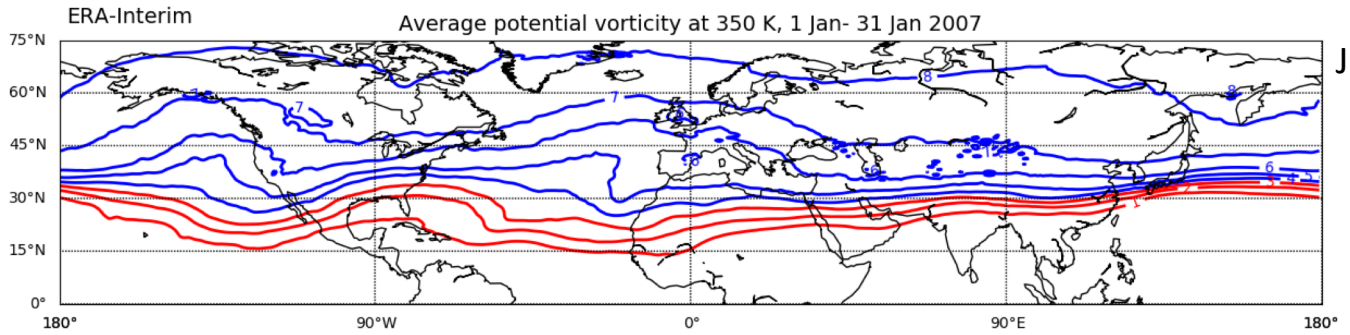
(section 1.25)

(ANIMATION OF FIGURE 1.57) Movie-file: PV_350_Dec2006.mov

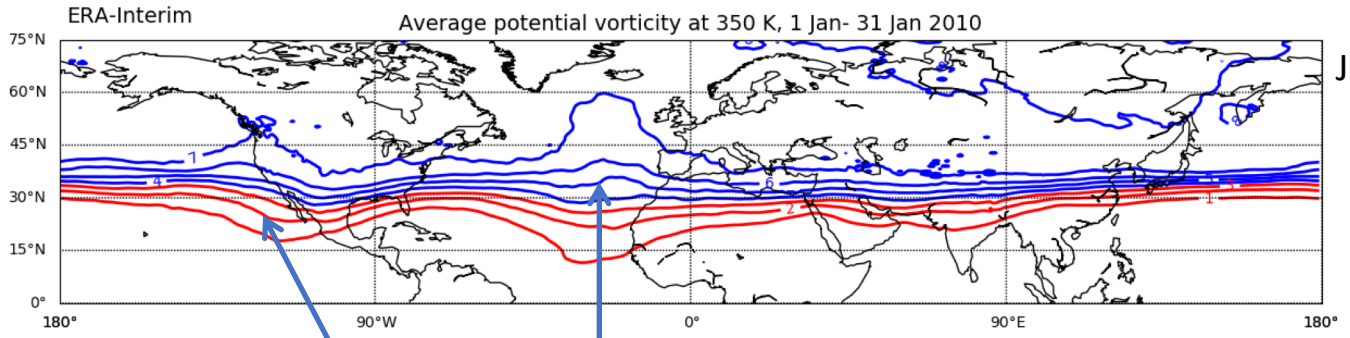
January mean potential vorticity at θ=350 K

Blue: PV>4 PVU; **Red: PV<4 PVU**

Contour interval = 1 PVU



January 2007

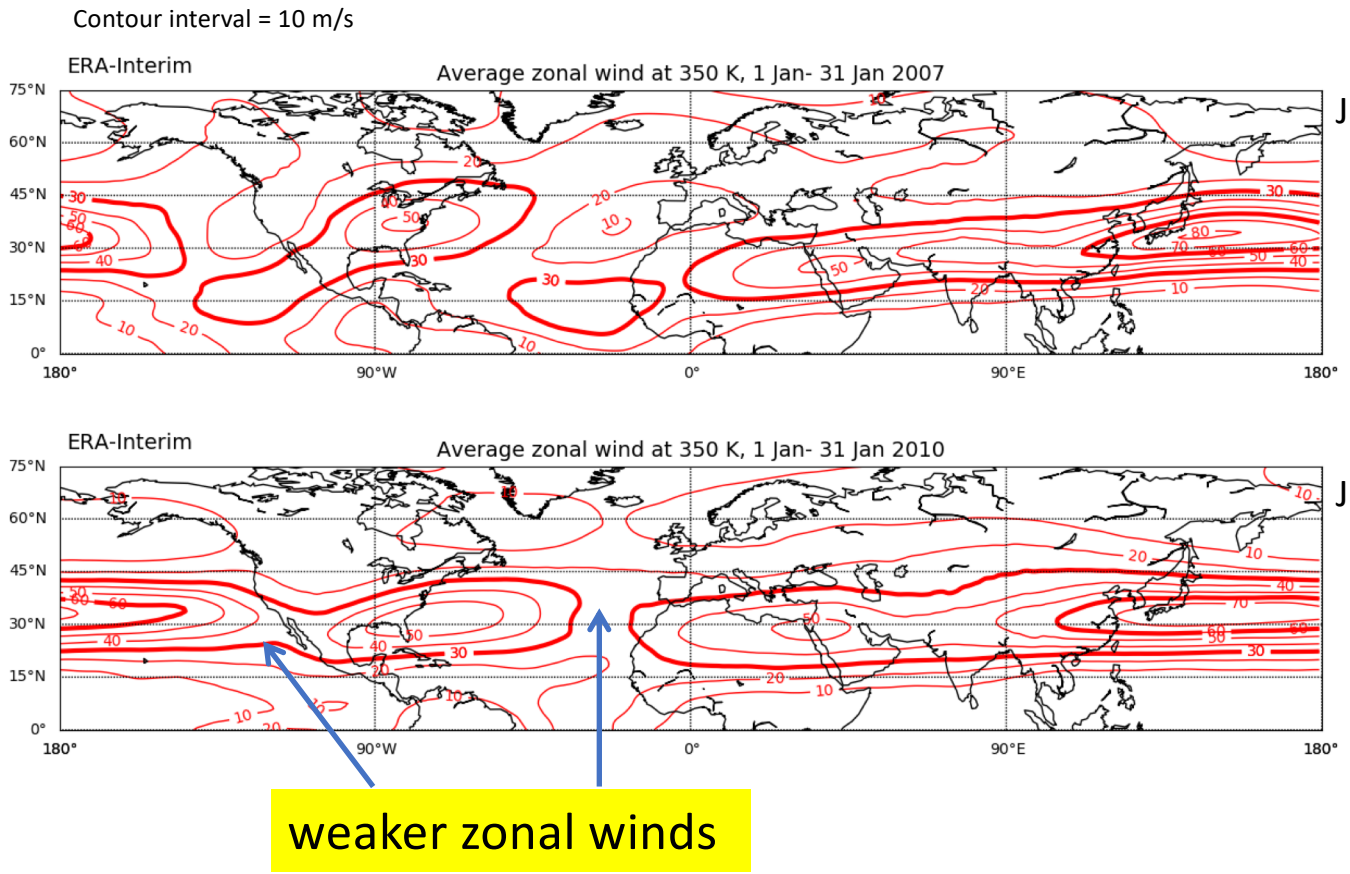


January 2010

Reduced meridional gradient of PV due strong eddy PV-mixing

See **figure 1.58**

Monthly mean zonal wind velocity at θ=350 K



See **figure 1.58**

Potential vorticity mixing reduces zonal wind velocity !?

Previous two slides suggest the following cause and effect relation:

Reduced meridional gradient of PV due strong PV-mixing due to eddies

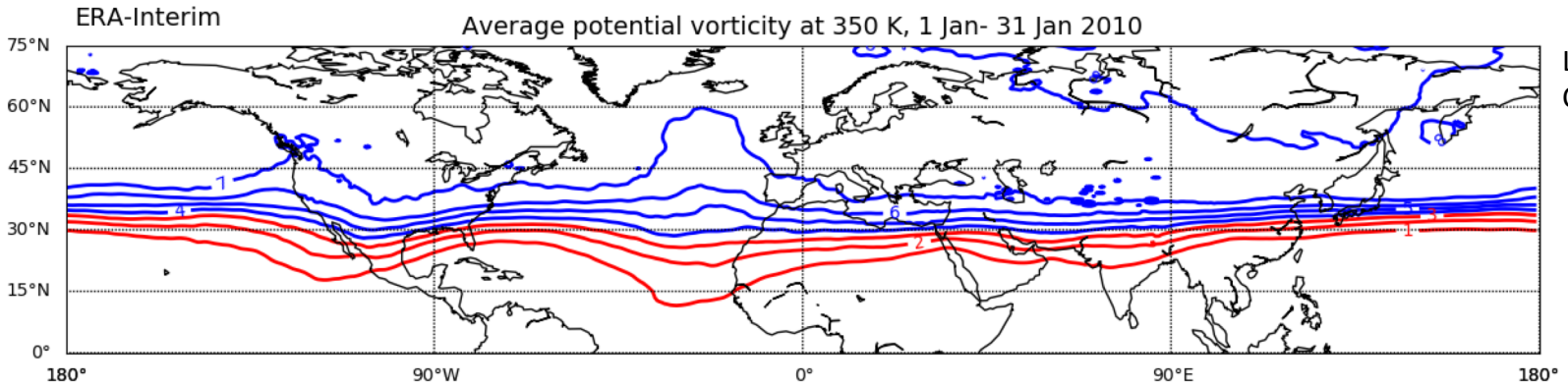


weaker zonal winds

This is qualitatively in accord with the parametrization of the effect of eddies on the zonal mean winds in the 2D-GCM: **by reducing the PV-gradient, the eddies/waves act as if a zonal force is reducing the zonal velocity**.

Why is the meridional PV gradient so sharp?

And why does the PV-gradient remain sharp in face of erosion due to eddy PV-mixing?



Labels in PVU
Contour interval = 1 PVU

The 350 K isentrope is located in the Middleworld

Sharp poleward increase of PV at about 30-35°N !

Is this due to an increase in vorticity or a decrease in isentropic density?

Remember: $Z_\theta \equiv \frac{\zeta_\theta + f}{\sigma}$ ← isentropic density

Mass-distribution: per unit area (left) and absolute (right)

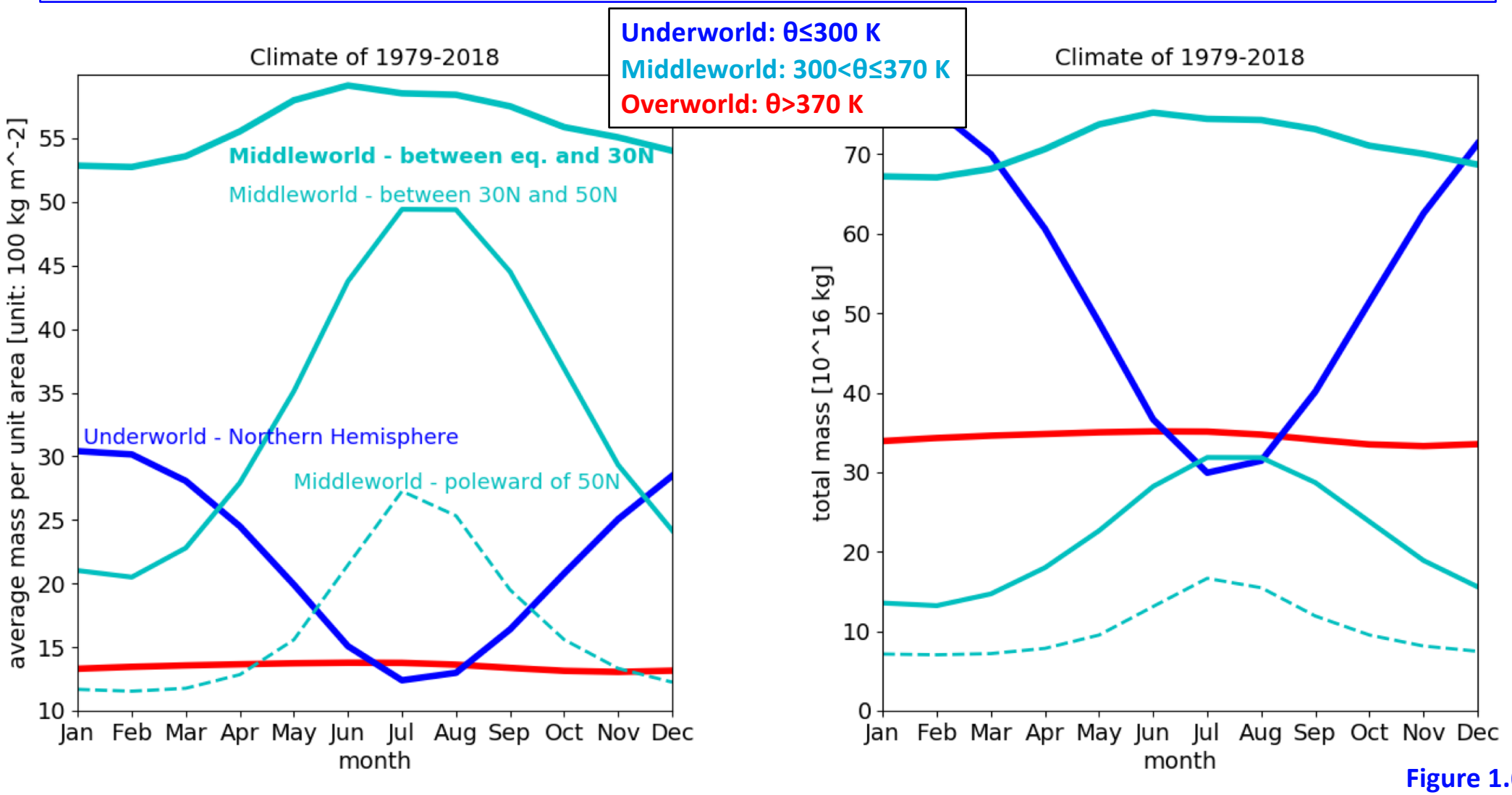
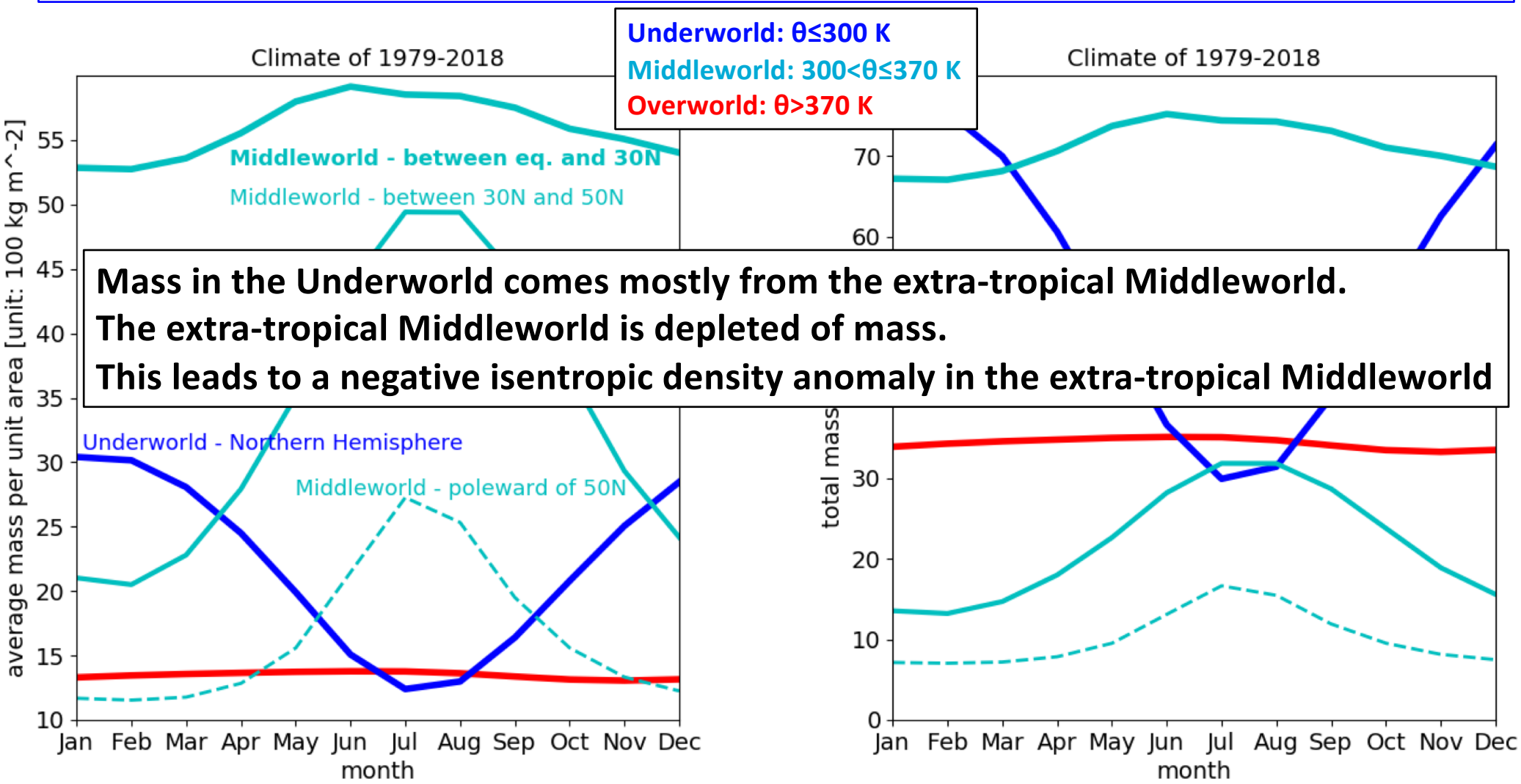


Figure 1.66

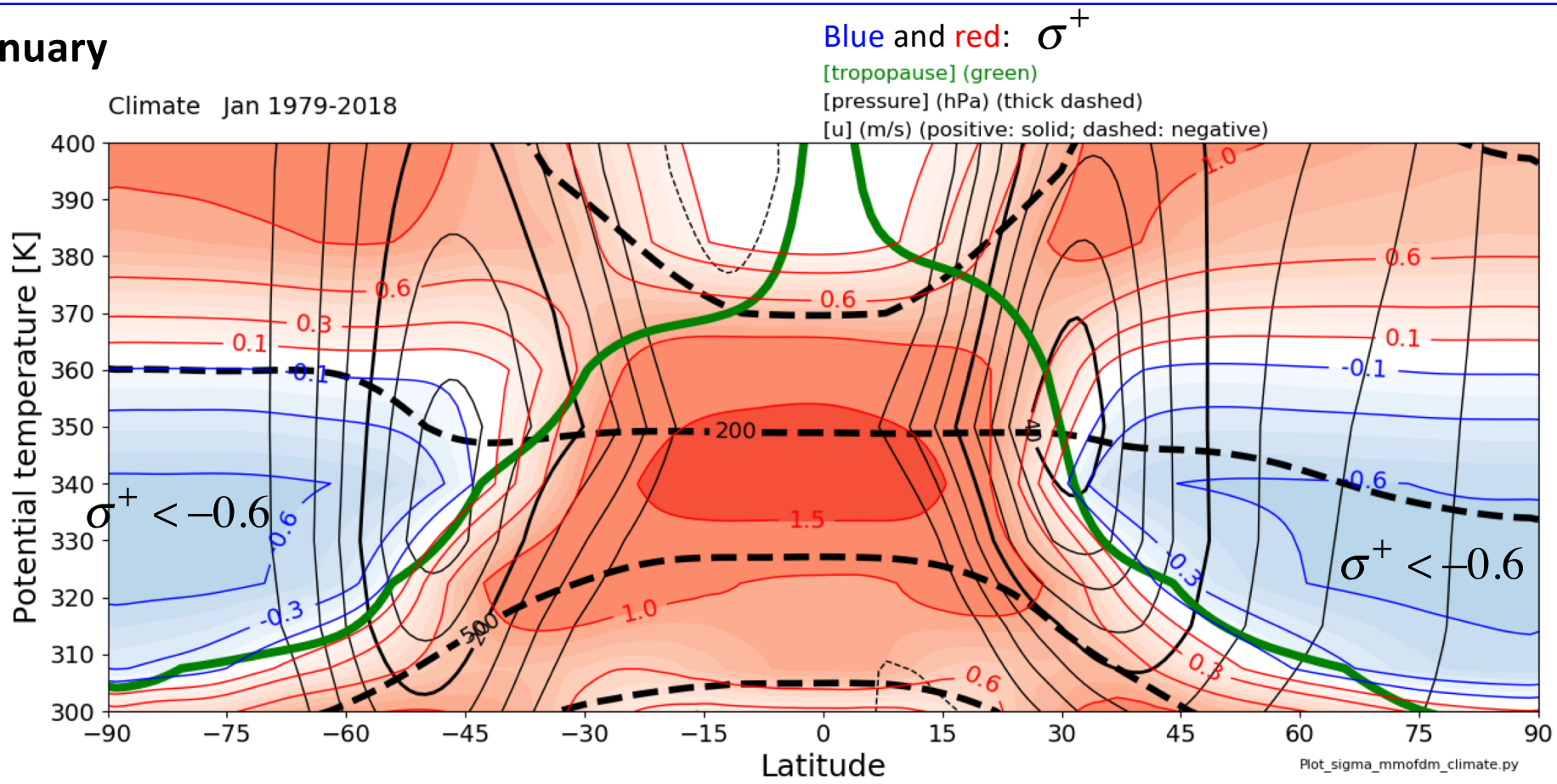
Mass-distribution: per unit area (left) and absolute (right)



Mass in the Underworld comes mostly from the extra-tropical Middleworld.
The extra-tropical Middleworld is depleted of mass.
This leads to a negative isentropic density anomaly in the extra-tropical Middleworld

Extratropical negative sigma-anomaly in the Middleworld

January



$$\sigma_{\mathrm{ref}}(\theta) = \frac{\int \sigma \cos\phi d\phi}{\int \cos\phi d\phi}$$

$$\sigma' = \sigma - \sigma_{ref}$$

$$\sigma^+ = \frac{[\sigma']}{\sigma_{ref}}$$

Figure 1.67

Climate_sigma_u_pres_tropopauseJan1979-2018.png

Extratropical negative sigma-anomaly in the Middleworld

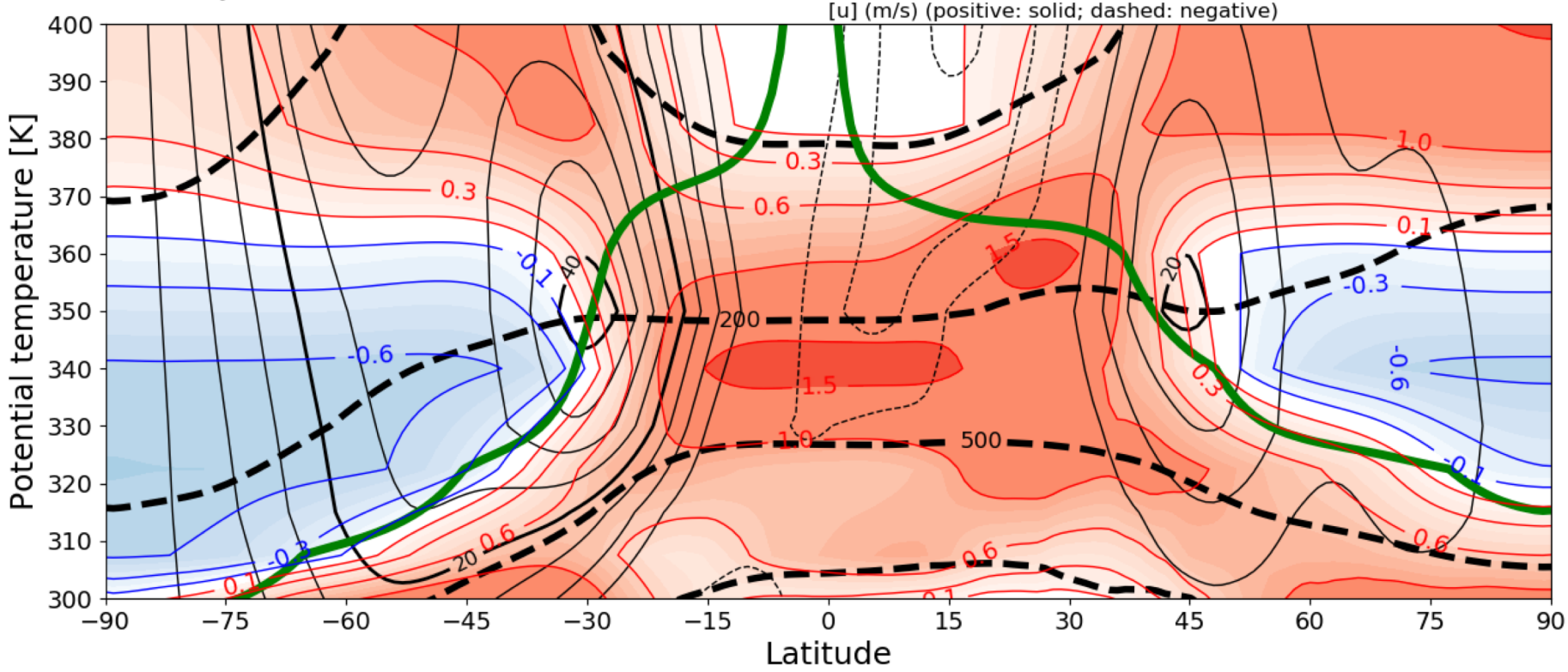
July

Blue and red: σ^+

[tropopause] (green)

[pressure] (hPa) (thick dashed)

[u] (m/s) (positive: solid; dashed: negative)



$$\sigma_{\mathrm{ref}}(\theta) = \frac{\int \sigma \cos\phi d\phi}{\int \cos\phi d\phi}$$

$$\sigma' = \sigma - \sigma_{ref}$$

$$\sigma^+ = \frac{[\sigma']}{\sigma_{ref}}$$

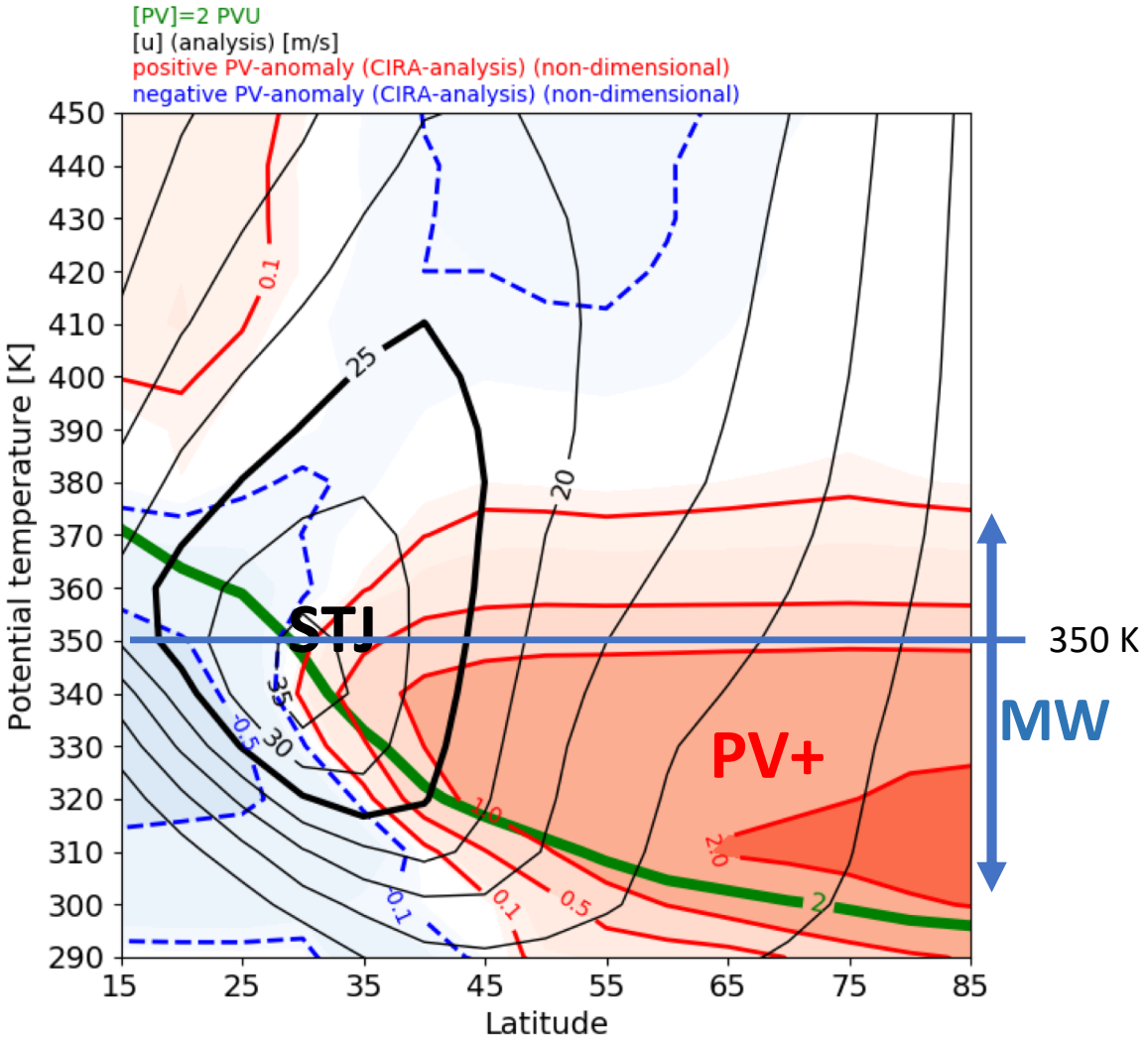
Figure 1.67

Climate_sigma_u_pres_tropopauseJul1979-2018.png

Extratropical positive PV-anomaly in the Middleworld

January

What is the relation between PV+ and the STJ ?



ReadPlotSol_PVinvCh7.py plot1b

STJ = Sub-Tropical Jet
PV+ = Positive PV-anomaly
MW = Middleworld

PV+ and the STJ are related because atmosphere is very close to balance

Zonal-mean geostrophic wind *balance*

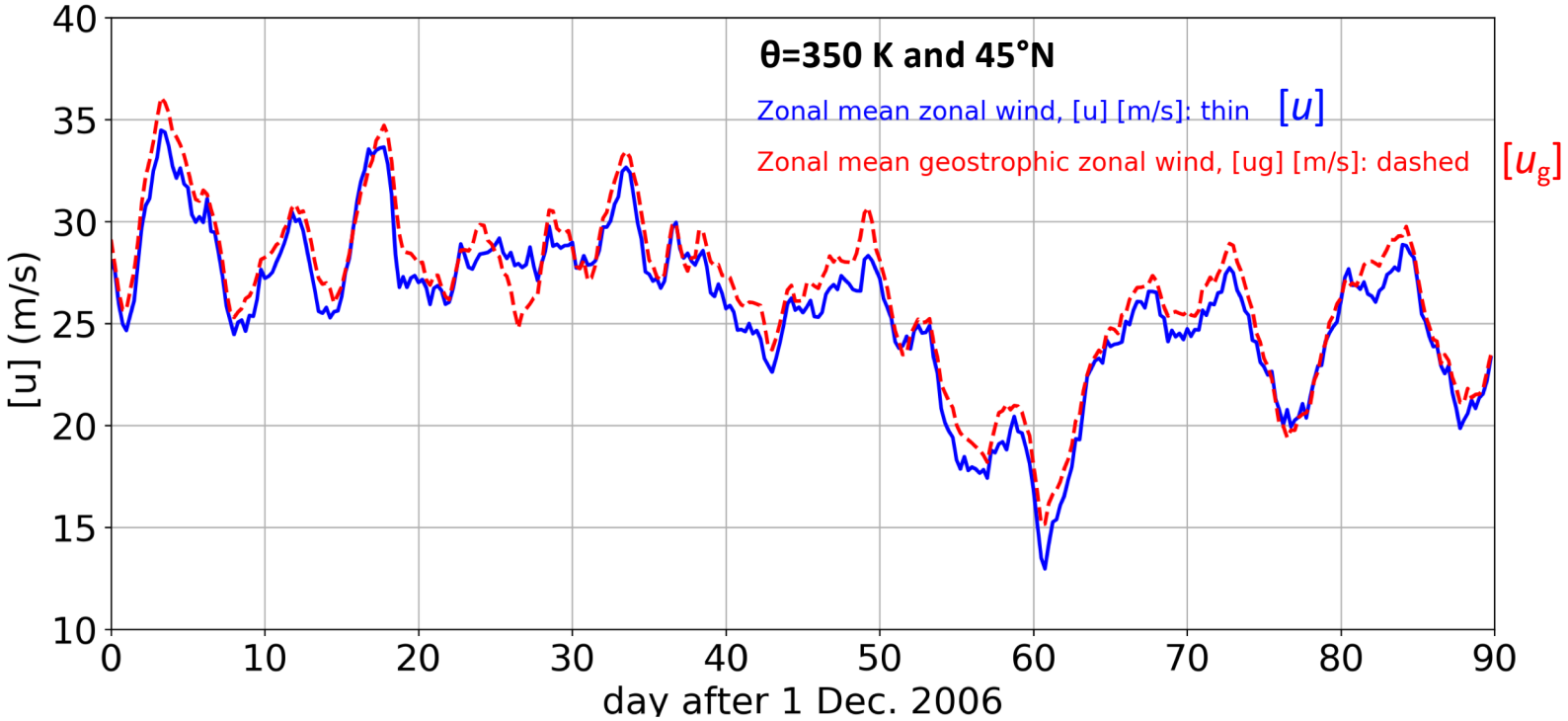


Figure 12.36

Zonal-mean zonal wind velocity (blue thin line), zonal-mean geostrophic zonal wind velocity (red dashed line) between 1 December 2006, 0 UTC and 28 February 2007, 18 UTC **at θ=350 K and 45°N**. The resolution in time of the data is 6 hours. Note: $[u]$ and $[u_g]$ are almost identical. Based on the six-hourly ERA-Interim re-analysis

Geostrophic wind balance + hydrostatic balance = thermal wind balance

Thermal wind balance

Zonal-mean Exner function

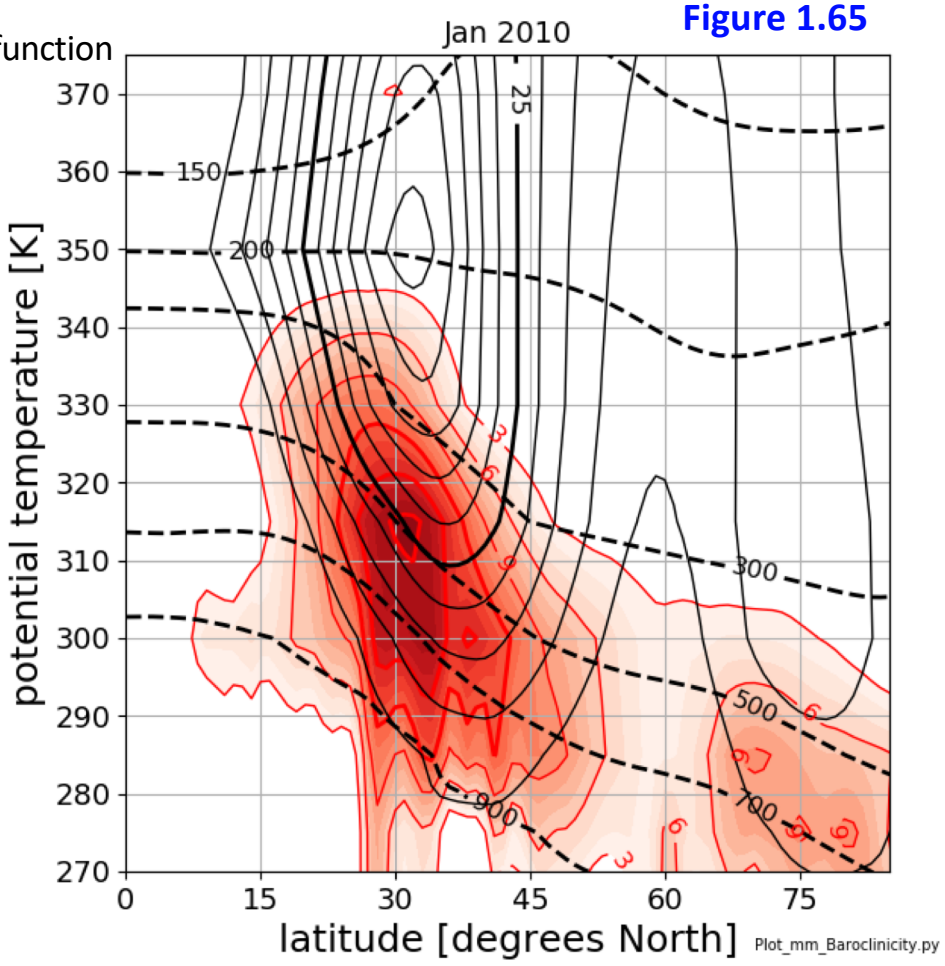
$$f_{loc}\frac{\partial[u]}{\partial\theta} = -\left(\frac{\partial[\Pi]}{\partial y}\right)_{\theta}$$

(eq..1.326)

$$f_{loc} = f + \frac{2[u]\tan\phi}{a}$$

Monthly mean, zonal mean fields of [*p*] (dashed contours, representing 900 hPa, 700 hPa, 500 hPa, 300 hPa, 200 hPa and 150 hPa), of "**baroclinicity**"=$\mathbf{|(\partial[p]/\partial y)_\theta|}$ (red shading; contour interval is 3×10^{-3} Pa m^{-1}; first contour corresponds to 3×10^{-3} Pa m^{-1}) and of **[u]** (black solid contours at 5 m/s intervals; zero contour not shown), as a function of potential temperature and latitude, for January 2010.

Figure 1.65



Potential vorticity inversion

The relation between potential vorticity and zonal-mean zonal wind velocity in a state of **thermal wind balance** is fixed by the **PV-inversion equation**

$$\frac{\partial}{\partial y}\left(\frac{\partial [u]}{\partial y}-\frac{[u]\tan\phi}{a}\right)+\frac{[Z]}{g}\frac{\partial}{\partial\theta}\left(f_{loc}\theta[\rho]\frac{\partial [u]}{\partial\theta}\right)=\frac{df}{dy}-[\sigma]\frac{\partial [Z]}{\partial y}$$

Another way of writing **thermal wind balance in isentropic coordinates**
(derivation in section 1.41 & **lecture 14 of Dynamical Meteorology**)

Given the zonal mean PV ($[Z]$), the solution of this equation yields the zonal-mean zonal velocity

(subscript, θ, has been omitted)

Section 1.41: PV-inversion on the sphere (see also **lecture 14 of Dynamical Meteorology**)

PV-inversion of the zonal mean state

PV-inversion equation for the zonal mean flow:

$$\frac{\partial}{\partial y}\left(\frac{\partial [u]}{\partial y}-\frac{[u]\tan\phi}{a}\right)+\frac{[Z]}{g}\frac{\partial}{\partial \theta}\left(f_{loc}\theta[\rho]\frac{\partial [u]}{\partial \theta}\right)=\frac{df}{dy}-[\sigma]\frac{\partial [Z]}{\partial y}$$

"**Laplace of *u***" ≈ "**forcing**"

Schematically, this equation is like the Poisson equation: $\nabla^2[u]\approx \mathrm{F}$

$$\frac{df}{dy}-[\sigma]\frac{\partial [Z]}{\partial y}$$

F = 0 if $[\sigma]\frac{\partial [Z]}{\partial y}=\frac{df}{dy}\equiv\beta$ →

Definition of the "reference state"

Previous slide: **Reference state**: $[\sigma]\frac{\partial[Z]}{\partial y}=\frac{df}{dy}\equiv\beta$ so that "**forcing"=0**

anomaly

Define $[Z]=Z_{\text{ref}}+Z'$ such that $\beta-[\sigma_{\text{ref}}]\frac{\partial[Z_{\text{ref}}]}{\partial y}=0$

with $\sigma_{\text{ref}}(\theta)=\frac{\int\sigma\cos\phi d\phi}{\int\cos\phi d\phi}$

Therefore: $Z_{\text{ref}}=\frac{f(y)}{\sigma_{\text{ref}}(\theta)}$

The “reference state” and the real state in terms of PV

“COSPAR”-International Reference Atmosphere

Zonal-mean potential vorticity:

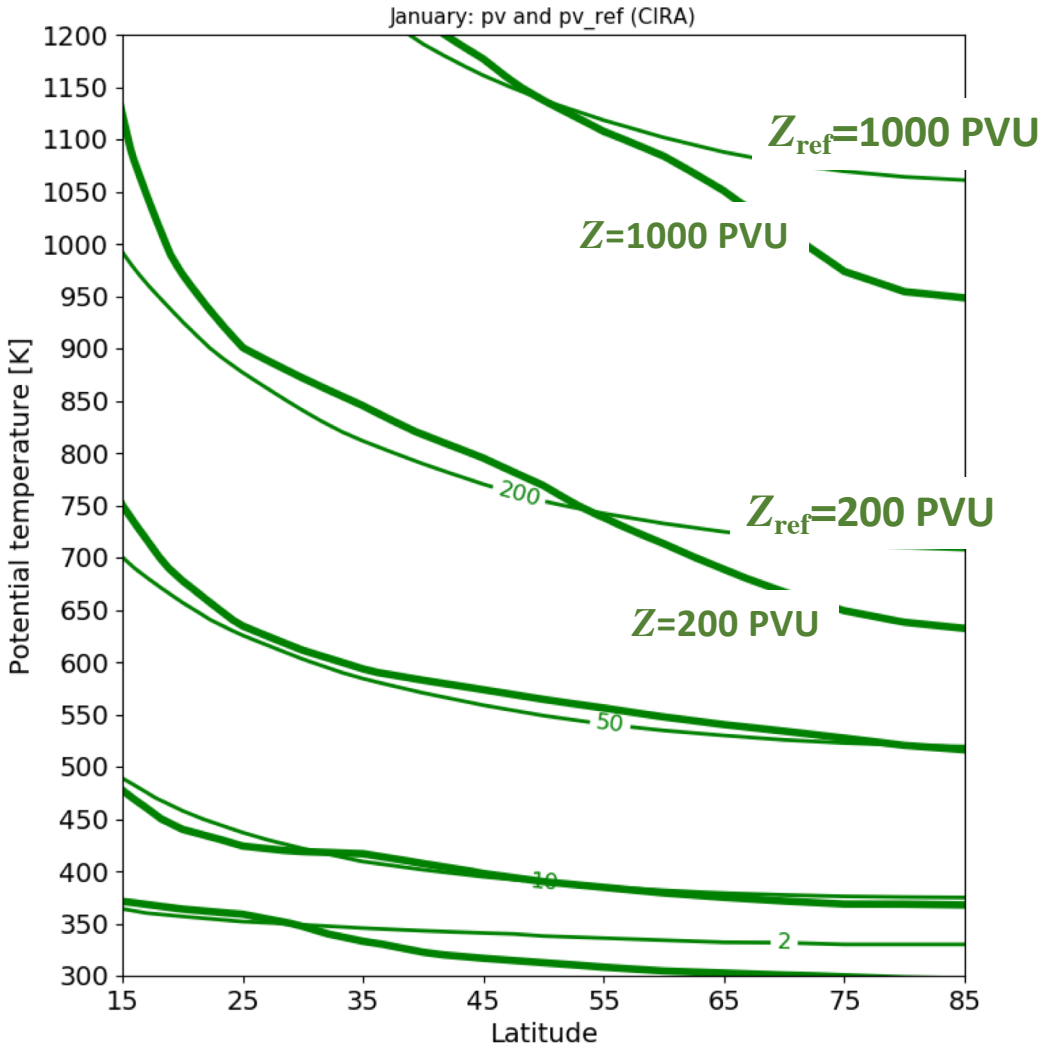
$$[Z] = \frac{[\zeta] + f}{[\sigma]}$$

Thick green contours: $[Z] = Z_{\mathrm{ref}} + Z'$

Thin green contours: $Z_{\mathrm{ref}} = \frac{f(y)}{\sigma_{\mathrm{ref}}(\theta)}$

1 PVU=10-6 K m2 $kg^{-1}s^{-1}$

January - mean



ReadPlotSol_PVinvCh7.py

Zonal-mean PV-anomalies

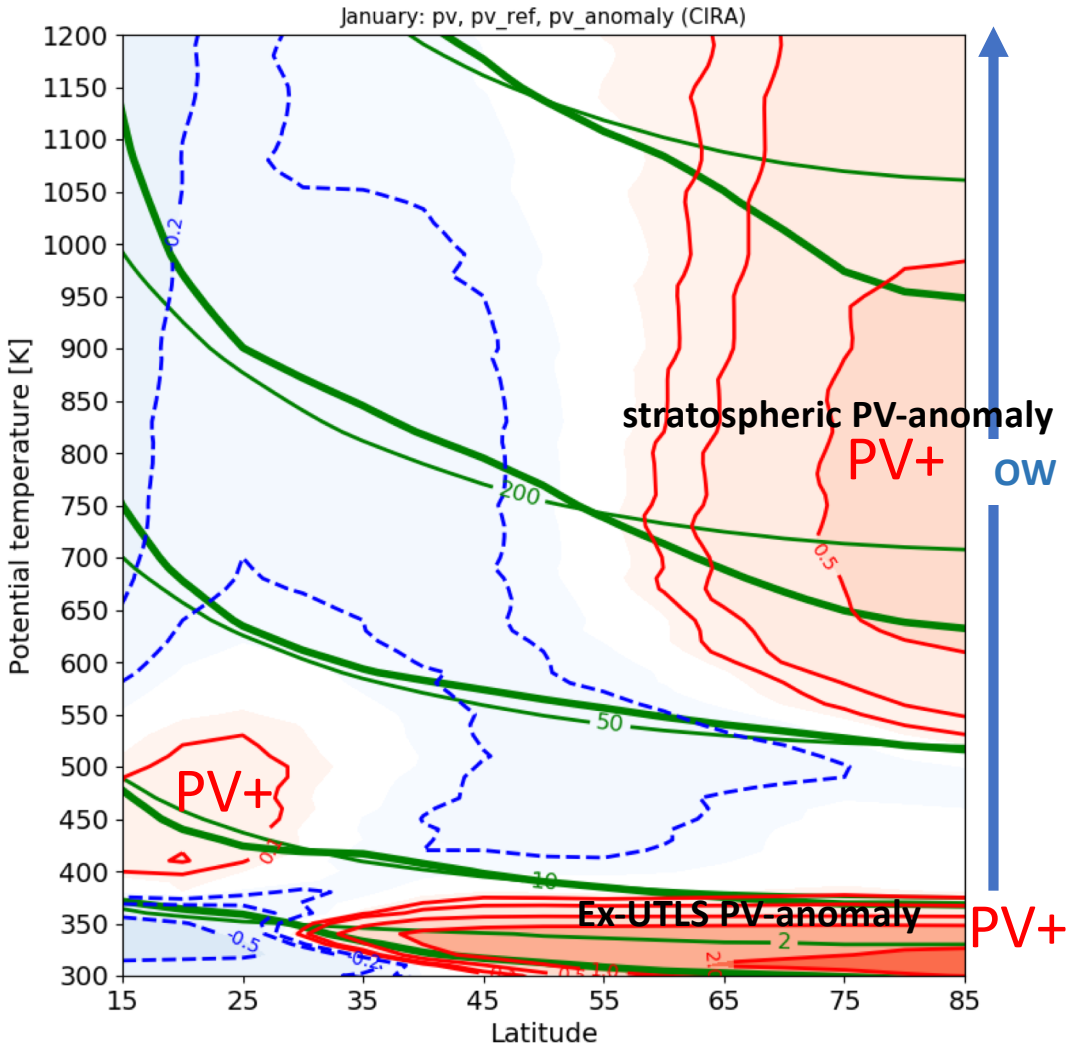
Non-dimensional anomalies:

$$Z^{+} \equiv \frac{Z'}{Z_{\mathrm{ref}}};\ \zeta^{+} \equiv \frac{\zeta'}{f};\ \sigma^{+} \equiv \frac{\sigma'}{\sigma_{\mathrm{ref}}}$$

Red & blue shading

PV+ = Positive PV-anomaly
OW = Overworld

January



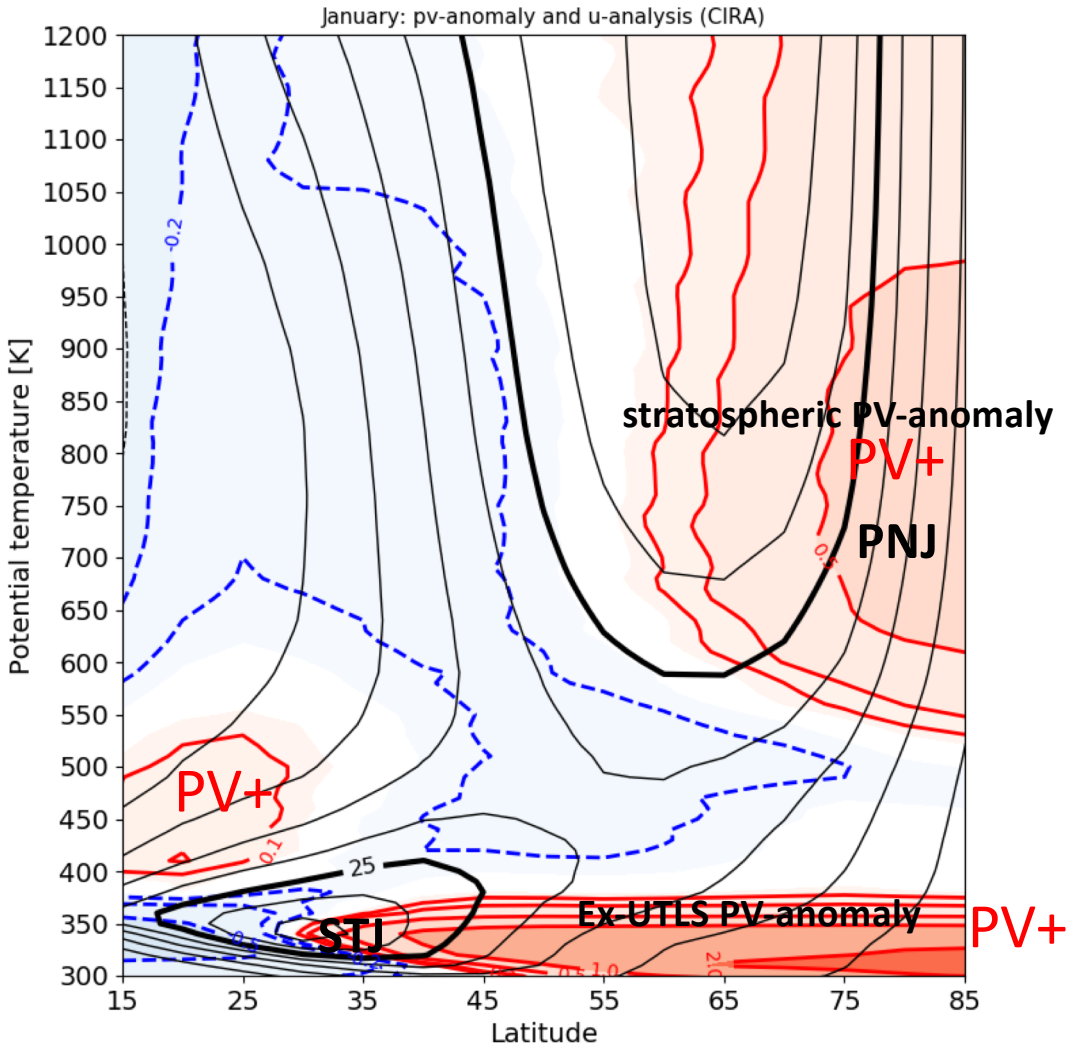
Observed [*u*]

(CIRA “re-analysis”)

Stratospheric PV-anomaly and Ex-UTLS PV-anomaly are related to the two main jets, the STJ and the PNJ

PV+ = Positive PV-anomaly
STJ = SubTropical Jet
PNJ = Polar Night Jet

January



Inverted [*u*] + [*p*]

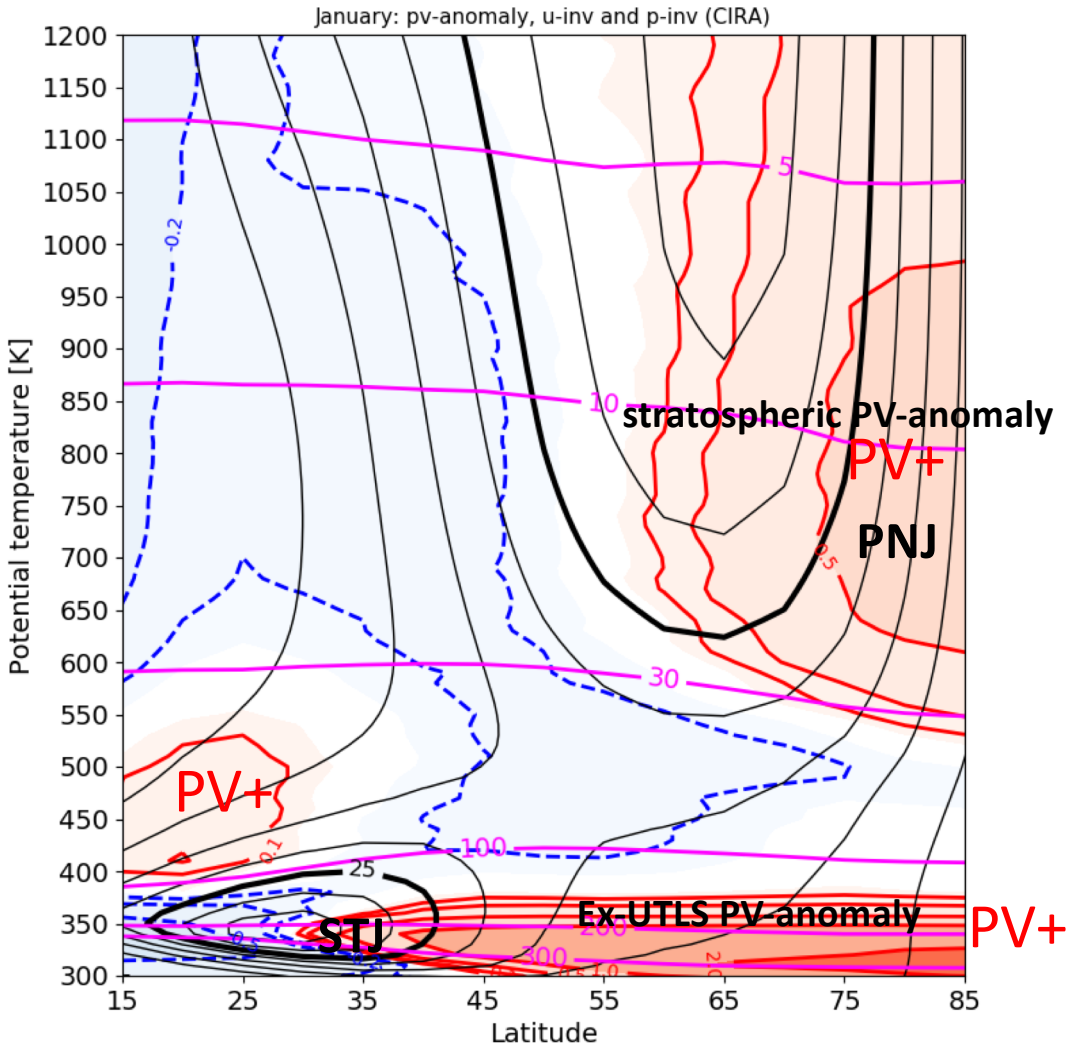
(solution PV-inversion equation*)

Stratospheric PV-anomaly and Ex-UTLS PV-anomaly are related to the two main jets, the STJ and the PNJ through the state of balance.

PV+ = Positive PV-anomaly
STJ = SubTropical Jet
PNJ = Polar Night Jet

* chapter 7

January



Kleinschmidt's principles

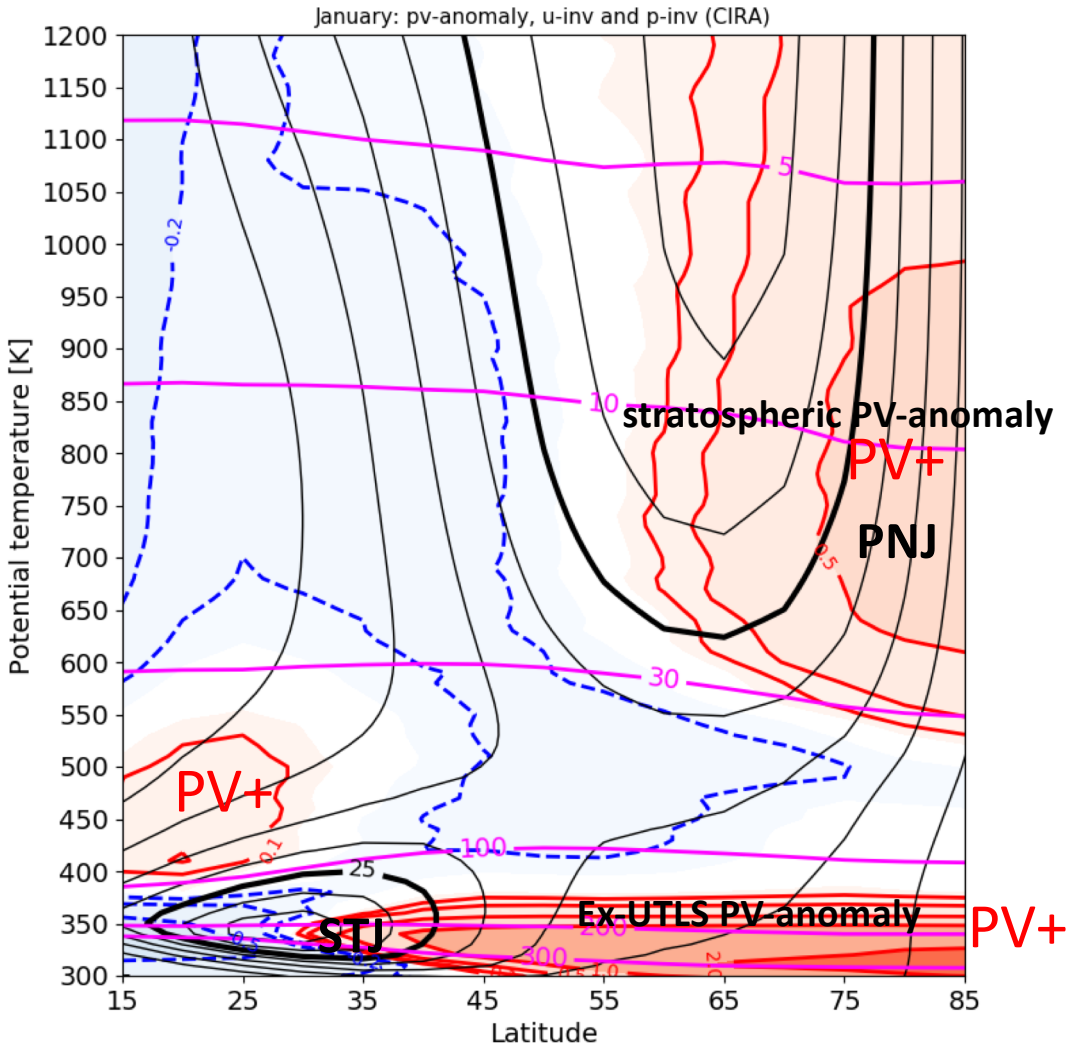
1a. **Within an isolated air mass with abnormal potential vorticity, in thermal wind balance, the absolute vorticity deviates from the normal in the *same* sense as the potential vorticity**.

1b. **Within an isolated air mass with abnormal potential vorticity, in thermal wind balance, the isentropic density deviates from the normal in the *opposite* sense as the potential vorticity**.

2. **An air mass of relatively high (low) potential vorticity gives rise to a cyclone (an anticyclone)**.

3. Isentropic surfaces are raised below an air mass of relatively high potential vorticity in thermal wind balance. Isentropic surfaces are lowered an air mass of relatively high potential vorticity in thermal wind balance. Thus, an **upper level cyclone has a cold core below and a warm core above**, while an **upper level anticyclone has a warm core below and a cold core above** the potential vorticity anomaly.

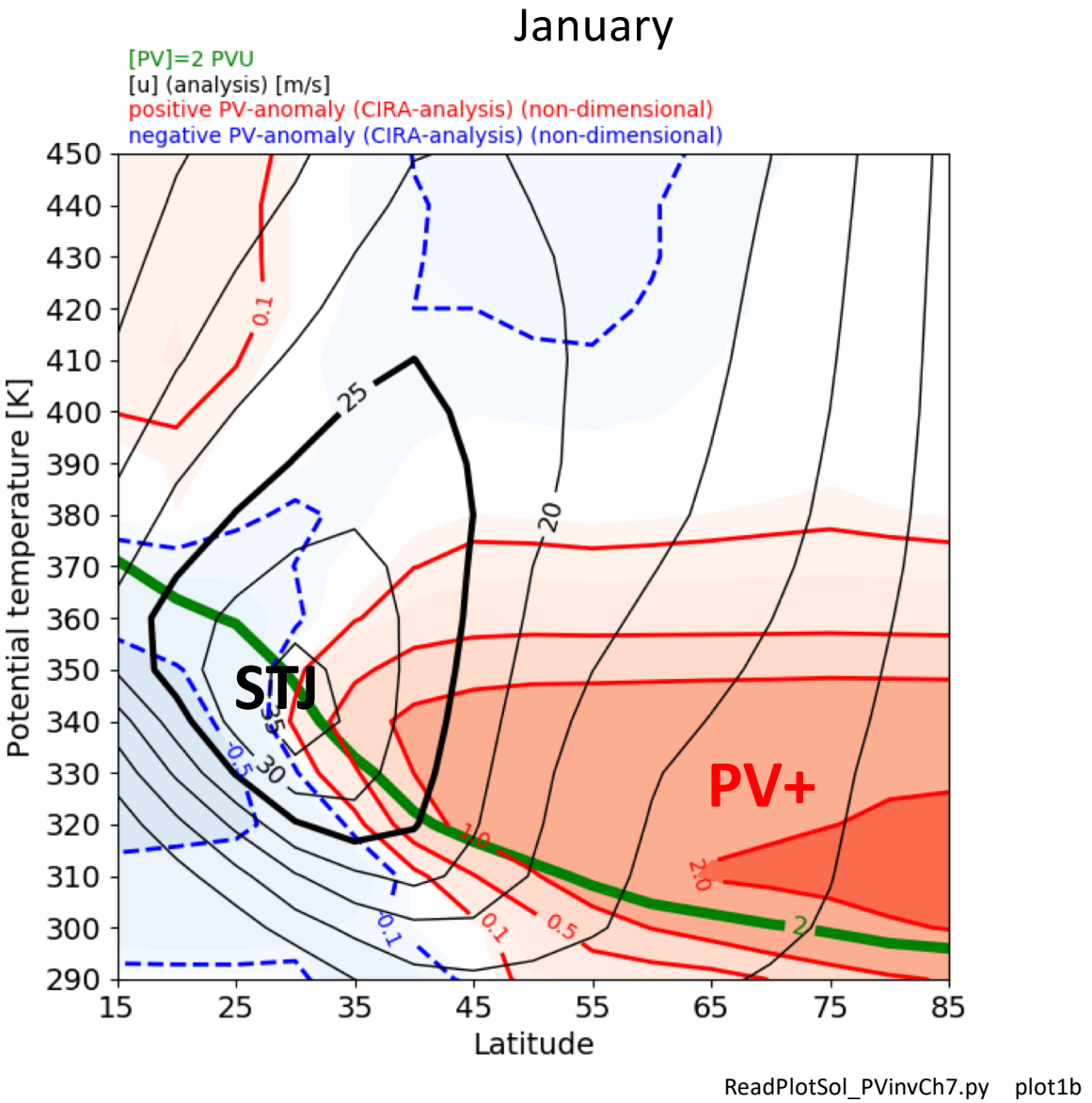
January



Close-up Middleworld

Observed [*u*]

(CIRA “re-analysis”)



ReadPlotSol_PVinvCh7.py plot1b

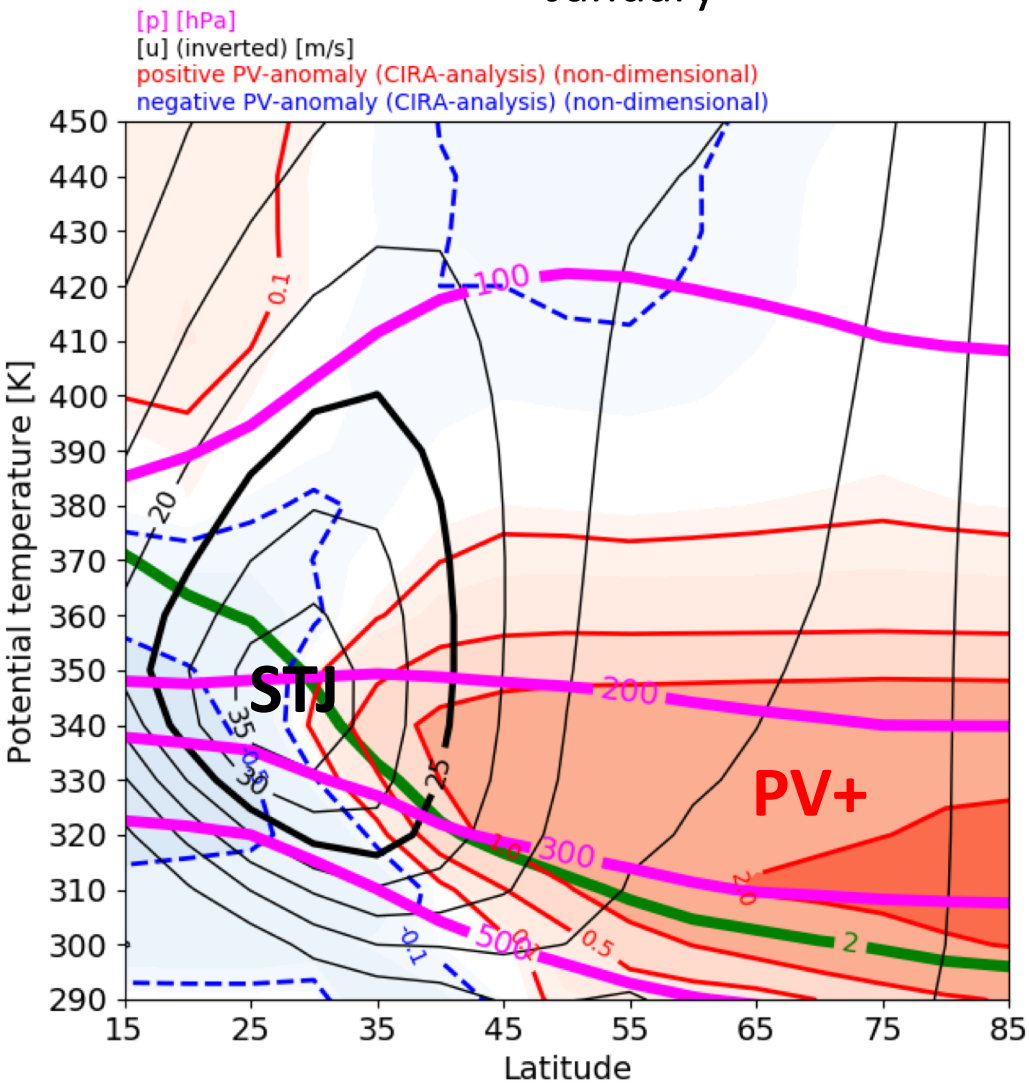
STJ = Sub-Tropical Jet
PV+ = Positive PV-anomaly
MW = Middleworld

Close-up Middleworld

Inverted [*u*] + [*p*]

(solution PV-inversion equation*)

January



PVinv_UTLS.png

STJ = Sub-Tropical Jet
PV+ = Positive PV-anomaly

* chapter 7

Reference state corresponds to state of rest

If $Z = Z_{\text{ref}}$ the PV-inversion equation for the zonal mean flow is:

$$\frac{\partial}{\partial y}\left(\frac{\partial [u]}{\partial y} - \frac{[u]\tan\phi}{a}\right) + \frac{[Z]}{g}\frac{\partial}{\partial \theta}\left(f_{loc}\theta[\rho]\frac{\partial [u]}{\partial \theta}\right) = 0$$

Remember: $Z_{\text{ref}} = \dfrac{f(y)}{\sigma_{\text{ref}}(\theta)}$

With **homogeneous boundary conditions**, solution is $[u] = 0$

The state of rest with respect to the earth's surface is associated with a *y*-dependent zonal-mean PV-distribution !

This fact invites the following **thought-experiment**:

next slide

Thought experiment (section 1.41):

Perfect mixing of the reference state by waves and eddies

materially conserving PV

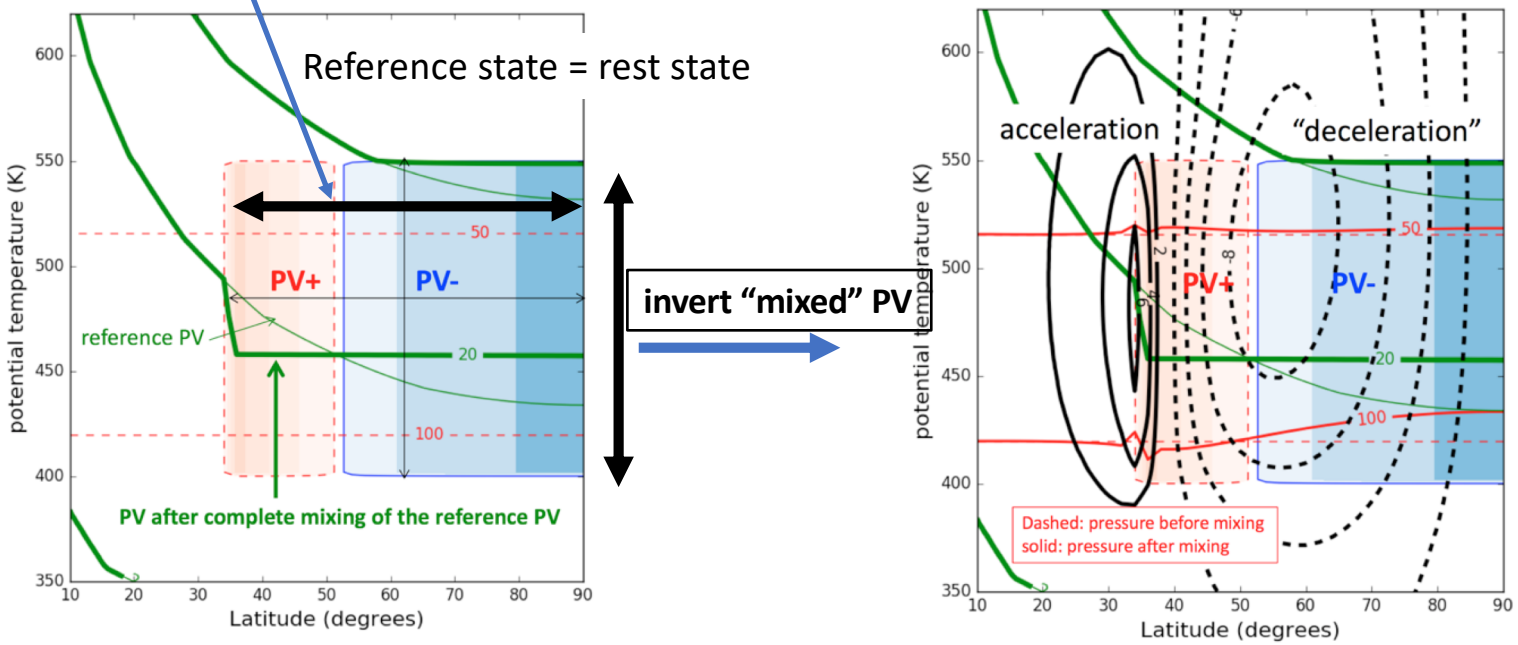
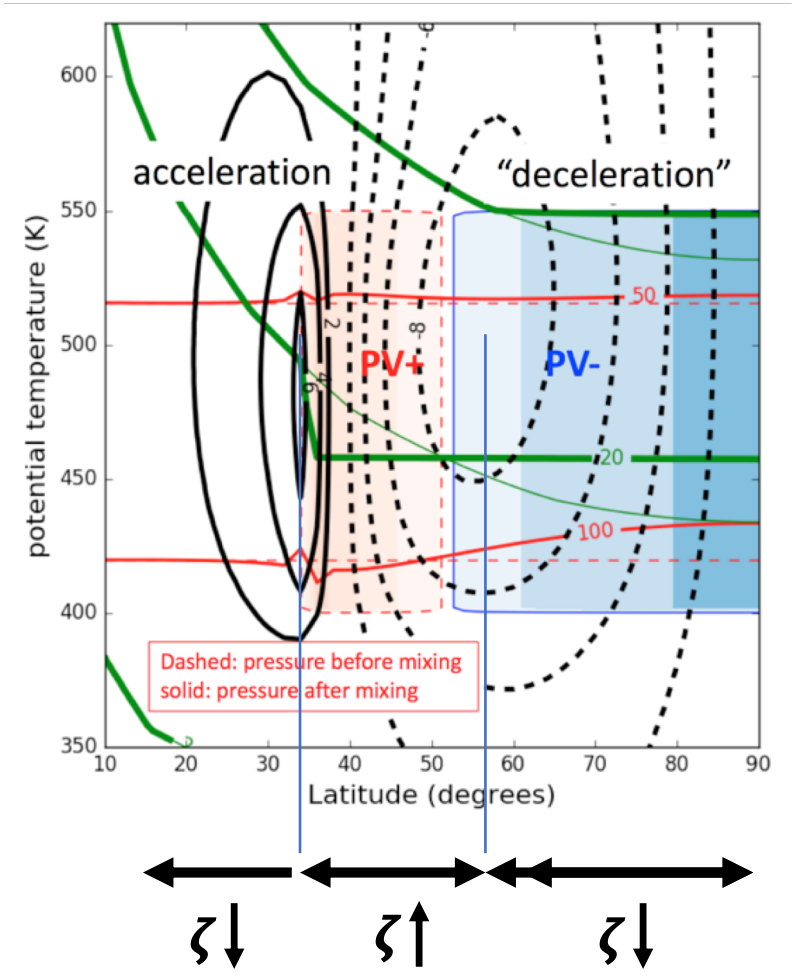


FIGURE 1.122. The latitude-dependent reference potential vorticity is "mixed completely" between 35°N and the Pole, in the layer, θ=400-550 K creating a negative PV-anomaly, approximately between 51°N and the Pole, and a positive PV-anomaly between 35°N and approximately 51°N. Thin green lines: reference potential vorticity. **Thick green lines: potential vorticity after complete mixing**. Red horizontal dashed lines represent isobars in reference state of rest, labeled in units of hPa.

FIGURE 1.123. Balanced zonal mean zonal wind velocity (**solid black contours: positive;** dashed black contours: negative), which is created by complete mixing, between the 35°N and the North Pole, of the potential vorticity associated with the state of rest (the reference state)(contour interval: 2 m s^{-1}). The solid red lines represent the isobars associated with the balanced state.

Balance requirements: "Kleinschmidt's Principles"



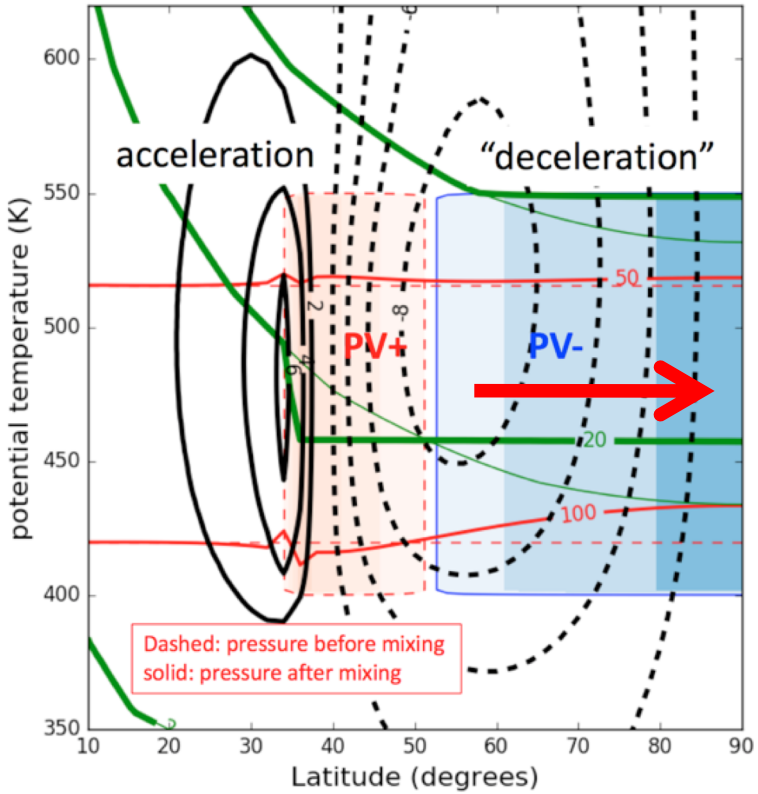
1a. **Within an isolated air mass with abnormal potential vorticity, in thermal wind balance, the absolute vorticity deviates from the normal in the *same* sense as the potential vorticity**.

1b. **Within an isolated air mass with abnormal potential vorticity, in thermal wind balance, the isentropic density deviates from the normal in the *opposite* sense as the potential vorticity**.

2. **An air mass of relatively high (low) potential vorticity gives rise to a cyclone (an anticyclone)**.

3. Isentropic surfaces are raised below an air mass of relatively high potential vorticity in thermal wind balance. Isentropic surfaces are lowered an air mass of relatively high potential vorticity in thermal wind balance. Thus, an **upper level cyclone has a cold core below and a warm core above**, while an **upper level anticyclone has a warm core below and a cold core above** the potential vorticity anomaly.

PV-mixing and balance requirement explains poleward mass flux & the Brewer-Dobson circulation



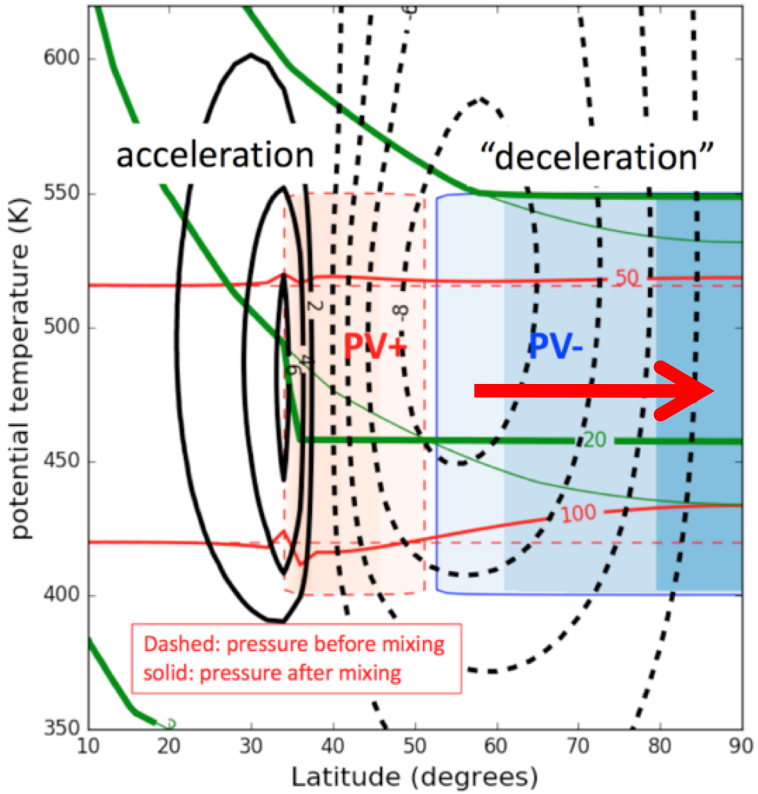
Kleinschmidt's principle 1b:

Isobars over the polar cap are squeezed in mixing region.

Arrow: *Poleward mass flux*

FIGURE 1.123. Balanced zonal mean zonal wind velocity (**solid black contours: positive;** dashed black contours: negative), which is created by complete mixing, between the 35°N and the North Pole, of the potential vorticity associated with the state of rest (the reference state)(contour interval: 2 m s^{-1}). The solid red lines represent the isobars associated with the balanced state.

PV-mixing and balance requirement explains poleward mass flux & the Brewer-Dobson circulation



In a balanced atmosphere, the *negative PV-anomaly over the pole*, which is caused by meridional PV-mixing by eddies and waves, must be associated with both a **negative vorticity anomaly** and a **positive isentropic density anomaly**.

Therefore, meridional PV-mixing and maintenance of zonal mean thermal wind balance must be associated with a net **equatorward PVS*-flux** and a **poleward mass-flux**.

The poleward mass-flux represents the upper branch of the Brewer-Dobson circulation

* PVS –absolute vorticity

Conclusions after lecture 5

A sharp meridional PV-gradient exists in the Middleworld
Due to drainage of mass into the Underworld
This creates a negative extratropical isentropic density anomaly in the Middleworld
And so also a positive PV-anomaly
Which “induces” the sub-tropical jet
Because the atmosphere is always close to zonal-mean thermal wind balance
Waves and eddies act to mix potential vorticity
This homogenises PV, but also creates sharp PV-gradients
In a balanced state, this decelerates and accelerates the zonal-mean zonal wind

Next:

Lecture by Michiel van den Broeke:

Wednesday, 16 June, 2021, online

No lecture on Friday, 18 June

Lecture 6, BLT&M-2: 25 June, 10:00-11:45, BBG 061

The eight ingredients of wave – zonal-mean flow interaction

1 The wave guide

2 wave breaking

3 PV-mixing

4 zonal mean ageostrophic response

5 mixing intensifies or creates jets

6 net meridional vorticity flux due to the secondary circulation and eddies

7 net meridional mass flux due to the secondary circulation and eddies

8 diabatic effects create the wave guide