

Climate Dynamics (lecture 3)

Absorption of radiation: Bouguer-Lambert-Beer Law.

Optical path.

Absorption cross-section.

Radiative energy budget equation: Schwarzschild's equation.

Radiative equilibrium: solution of Schwarzschild's equation.

Division of the atmosphere in **troposphere and stratosphere.**

Box 2.1, 2.2, 2.3 of the lecture notes

<http://www.staff.science.uu.nl/~delde102/climatedynamics.htm>

Temperature

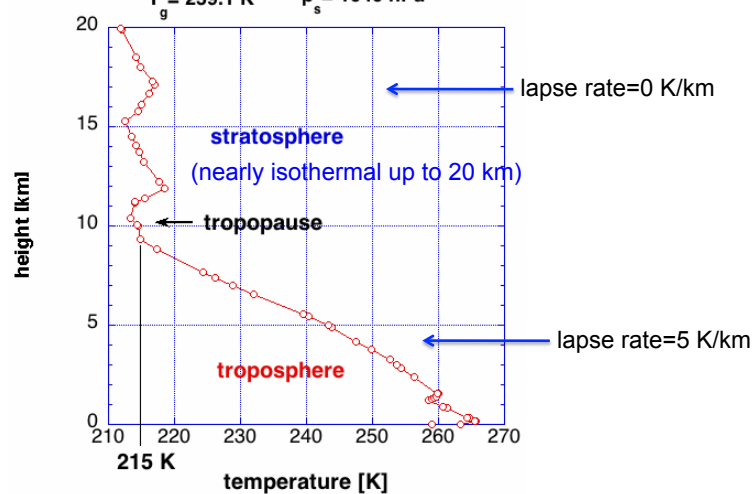
Sounding DeBilt 04/02/2012 00 UTC

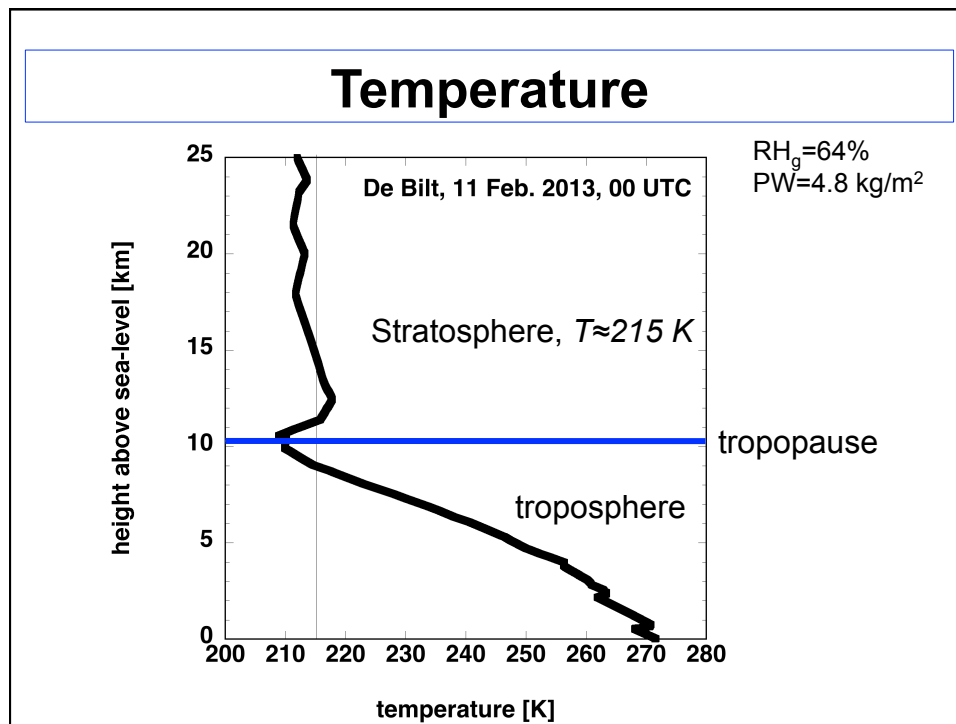
$RH_g = 93 \%$

$PW = 3.58 \text{ kg m}^{-2}$

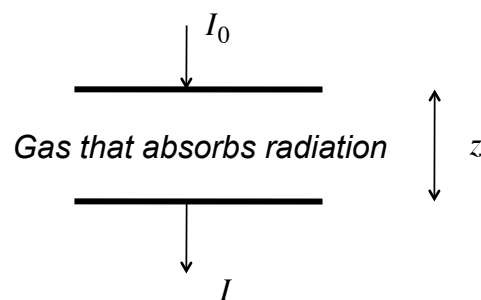
$T_g = 259.1 \text{ K}$

$p_s = 1040 \text{ hPa}$



**BOX 2.1**

Absorption of radiation



Bouguer-Lambert-Beer Law: $I = I_0 \exp(-\kappa z) \rightarrow dI = -\kappa I dz$

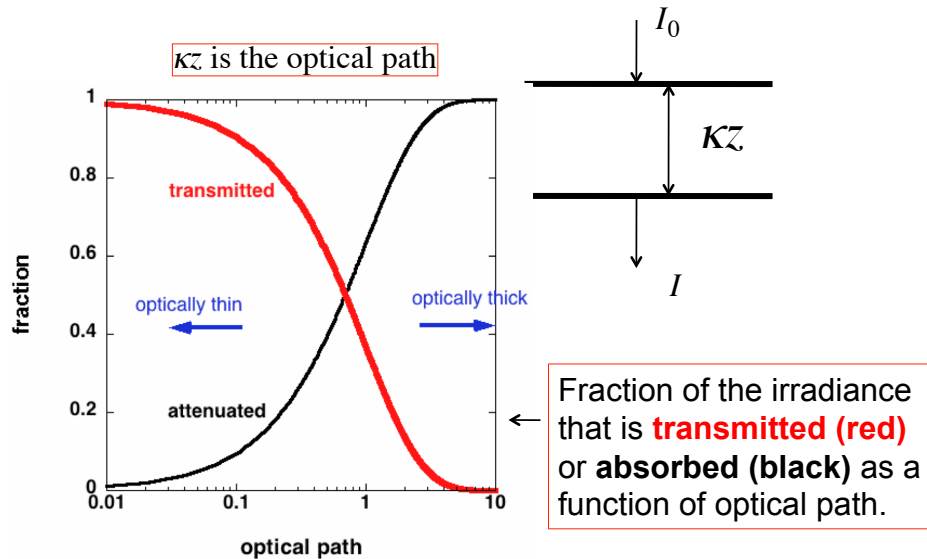
κ is the absorption coefficient

κz is the optical path

$\frac{1}{\kappa}$ is the absorption length (unit : m)

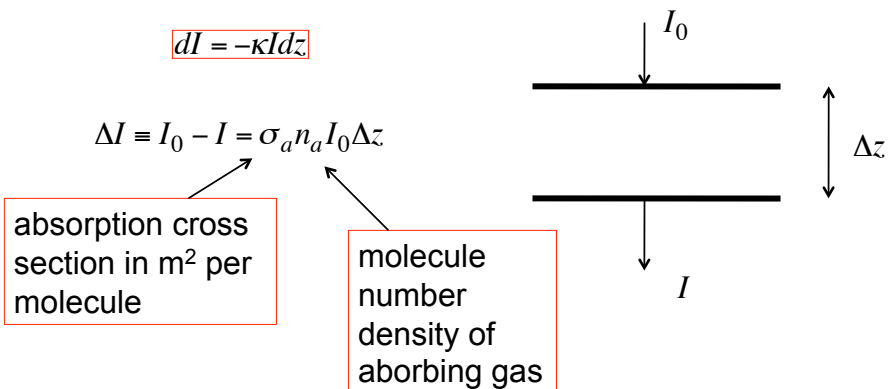
BOX 2.1

Absorption of radiation



BOX 2.2

Absorption cross section



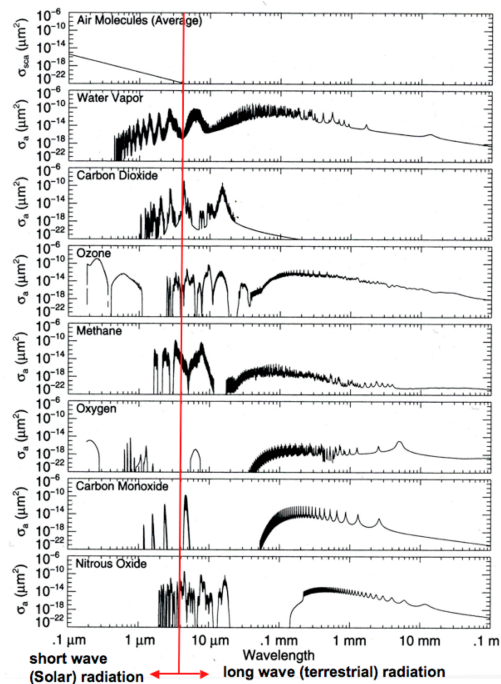
Effect of CO_2 and water vapour (H_2O):

$$\Delta I = (\sigma_a[\text{CO}_2]n[\text{CO}_2] + \sigma_a[\text{H}_2\text{O}]n[\text{H}_2\text{O}])I_0\Delta z$$

Absorption cross section

FIGURE 2.48.

Absorption cross sections of the strongest absorbing atmospheric constituents

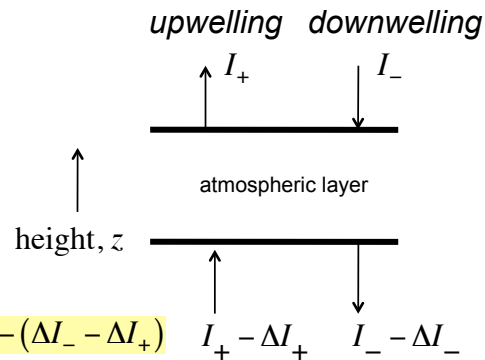


Kirchhoff's law

absorption coefficient = emission coefficient

Upwelling and downwelling radiation

net in from the top = $I_- - I_+$



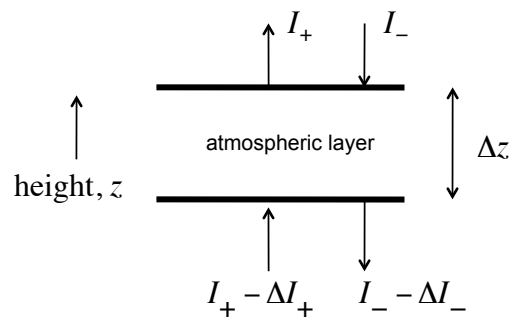
net out from the bottom = $(I_- - I_+) - (\Delta I_- - \Delta I_+)$

Net absorbed radiation is $(\Delta I_- - \Delta I_+)$

Radiation balance: $\Delta I_- = \Delta I_+$

BOX 2.3

“Planck emission”



Previous slide: Radiation balance: $\Delta I_- = \Delta I_+$

Fluxes include “Planck emission” of atmospheric itself!!!

Black body “Planck emission”: $B \equiv \sigma T^4$

Atmospheric layer itself emits: $\kappa \Delta z B$ (Kirchhoff’s law)

BOX 2.3

Schwarzschild's equations

Energy budget:upwelling radiation:

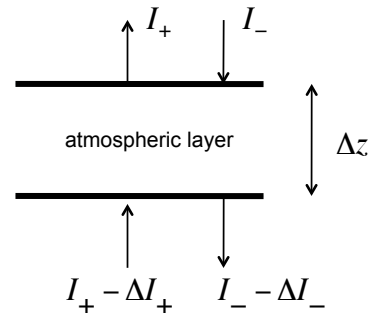
$$-\Delta I_+ = +\kappa \Delta z (I_+ - \Delta I_+) - \kappa B \Delta z$$

Thin layer: $-\kappa dz = d\delta \ll 1$!!!!!

$$-\Delta I_+ = +\kappa \Delta z I_+ - \kappa B \Delta z$$

downwelling radiation:

$$\Delta I_- = \kappa \Delta z I_- - \kappa B \Delta z$$

Previous slide: Radiation balance: $\Delta I_- = \Delta I_+$

BOX 2.3

Schwarzschild's equations

Energy budget:upwelling radiation:

$$-\Delta I_+ = +\kappa \Delta z (I_+ - \Delta I_+) - \kappa B \Delta z$$

Thin layer: $-\kappa dz = d\delta \ll 1$!!!!!

$$-\Delta I_+ = +\kappa \Delta z I_+ - \kappa B \Delta z$$

downwelling radiation:

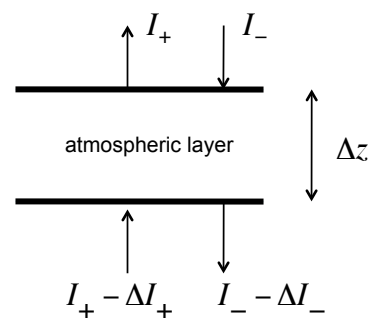
$$\Delta I_- = \kappa \Delta z I_- - \kappa B \Delta z$$

$$\downarrow \quad \Delta I_- = \Delta I_+$$

$$I_+ + I_- - 2B = 0$$

and

$$\Delta I_+ + \Delta I_- = -\kappa (I_+ - I_-) \Delta z$$



BOX 2.3

Continuous homogeneous atmosphere

Definition: $\bar{I} \equiv I_+ + I_-$ and $I \equiv I_+ - I_-$

Previous slide:

$$\begin{array}{lcl} I_+ + I_- - 2B = 0 & \longrightarrow & \bar{I} \equiv 2B \\ \Delta I_+ + \Delta I_- = -\kappa(I_+ - I_-)\Delta z & \longrightarrow & \Delta \bar{I} \equiv -\kappa I \Delta z \\ \Delta I_- = \Delta I_+ & \longrightarrow & \Delta I = 0 \end{array}$$

or in general:

$$\bar{I} \equiv 2B$$

$$\frac{d\bar{I}}{d\delta} \equiv I$$

$$\frac{dI}{d\delta} = 0$$

$$B \equiv \frac{1}{2}I\delta + B_0$$

B_0 is an integration constant
Value of B at $\delta=0$ (=TOA)

BOX 2.3

Schwarzschild's equation: solution

$$\bar{I} \equiv 2B$$

$$\frac{d\bar{I}}{d\delta} \equiv I$$

$$\frac{dI}{d\delta} = 0$$

$$B \equiv \frac{1}{2}I\delta + B_0$$

B_0 is an integration constant
Value of B at $\delta=0$ (TOA)

At **TOA**:

$$I_+ = \frac{1}{4}S_0(1 - \alpha_p) \equiv Q$$

and

$$I \equiv I_+ - I_- = I_+ = Q$$

$$B \equiv \frac{1}{2}(Q\delta + 2B_0) \quad \text{for all } \delta$$

$$\frac{dI}{d\delta} = 0$$

holds for all optical paths

BOX 2.3

Schwarzschild's equation: solution

$$\bar{I} \equiv 2B \longrightarrow \bar{I} = I + 2I_- \equiv 2B$$

$$\frac{d\bar{I}}{d\delta} \equiv I$$

$$\frac{dI}{d\delta} = 0$$

at **TOA** this is: $I = 2B$ or $Q = 2B_0$

$$B \equiv \sigma T^4 = \frac{1}{2} Q(\delta + 1)$$

$$B \equiv \frac{1}{2} (Q\delta + 2B_0) \quad \text{for all } \delta$$

$$I = Q$$

BOX 2.3

Skin temperature

General solution:

$$B \equiv \sigma T^4 = \frac{1}{2} Q(\delta + 1) \longrightarrow T = \sqrt[4]{\frac{Q(\delta + 1)}{2\sigma}}$$

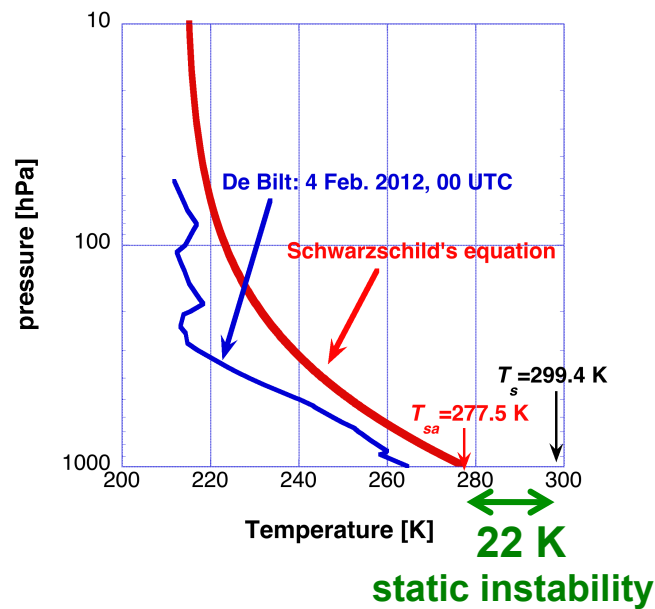
$$\text{If } \delta \ll 1 \text{ then } T = \sqrt[4]{\frac{Q}{2\sigma}} \equiv T_{skin}$$

“skin temperature”

T_{skin} is temperature of the **optically thin stratosphere**

BOX 2.3

Radiative equilibrium

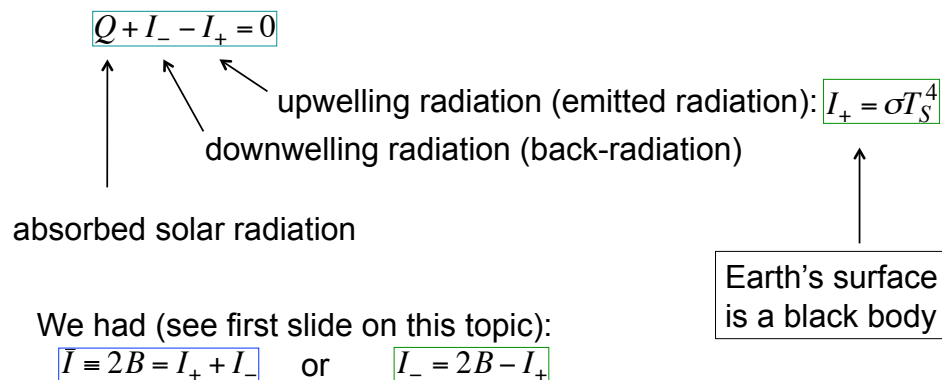


$$T = \sqrt[4]{\frac{Q(\delta+1)}{2\sigma}}$$

δ is related to mass of the absorber above level of interest

BOX 2.3

Radiative equilibrium at surface



BOX 2.3

Radiative equilibrium at surface

So we have:

$$Q + I_- - I_+ = 0$$

$$I_+ = \sigma T_S^4$$

$$I_- = 2B - I_+$$

and

optical path at earth's surface

$$B \equiv \sigma T^4 = \frac{1}{2} Q(\delta + 1) = \frac{1}{2} Q(\delta_s + 1)$$

at earth's surface

Surface temperature:

$$T_S = \left(\frac{Q(\delta_S + 2)}{2\sigma} \right)^{1/4}$$

Atmospheric temperature, just above surface:

$$T_{Sa} = \left(\frac{Q(\delta_S + 1)}{2\sigma} \right)^{1/4}$$

BOX 2.3

Radiative equilibrium at surface

Surface temperature:

$$T_S = \left(\frac{Q(\delta_S + 2)}{2\sigma} \right)^{1/4}$$

Atmospheric temperature, just above surface:

$$T_{Sa} = \left(\frac{Q(\delta_S + 1)}{2\sigma} \right)^{1/4}$$

$$T_S > T_{Sa}$$

Surface temperature
higher than atmospheric
temperature near surface

BOX 2.3

Radiative equilibrium: full solution

Atmospheric temperature:

$$T = \sqrt[4]{\frac{Q(\delta+1)}{2\sigma}} = T_{skin}(\delta+1)^{1/4}$$

Surface temperature:

$$T_S = \left(\frac{Q(\delta_S + 2)}{2\sigma} \right)^{1/4} = T_{skin}(\delta_S + 2)^{1/4}$$

Atmospheric temperature, just above surface:

$$T_{Sa} = \left(\frac{Q(\delta_S + 1)}{2\sigma} \right)^{1/4} = T_{skin}(\delta_S + 1)^{1/4}$$

BOX 2.3

Radiative equilibrium: full solution

Atmospheric temperature:

$$T = T_{skin}(\delta+1)^{1/4}$$

Surface temperature:

$$T_S = T_{skin}(\delta_S + 2)^{1/4}$$

Atmospheric temperature, just above surface:

$$T_{Sa} = T_{skin}(\delta_S + 1)^{1/4}$$

δ is related to height or pressure:

$$-\kappa dz = (\sigma_a q_a / g) dp = d(\sigma_a q_a p / g) \equiv d\delta$$

q is the specific concentration by mass of the absorber

$$\delta = \sigma_a q_a p / g \approx \sigma_a r_a p / g$$

$$\delta_s \approx \sigma_a r_a p_s / g$$

r_a is the mixing ratio by mass of the absorber

BOX 2.3

Radiative equilibrium: full solution

Atmospheric temperature:

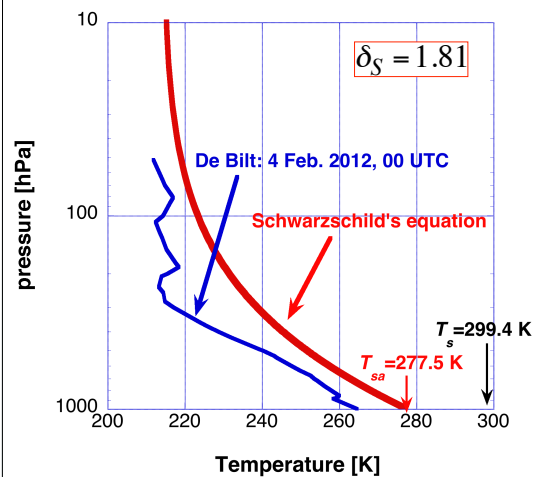
$$T = T_{skin}(\delta + 1)^{1/4}$$

Surface temperature:

$$T_S = T_{skin}(\delta_S + 2)^{1/4}$$

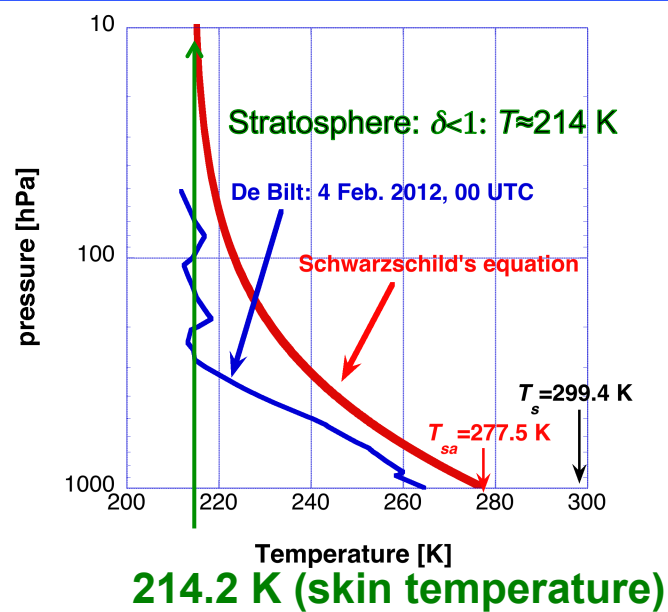
$$\sigma_a = 0.3 \text{ m}^2 \text{ kg}^{-1}; r_a = 390 \text{ ppm}$$

$$\alpha_p = 0.3; S_0 = 1366 \text{ W m}^{-2}; p_s = 1000 \text{ hPa}$$



BOX 2.3

Radiative equilibrium: full solution



Stratosphere is the optically thin upper part of the atmosphere

BOX 2.3

Radiative equilibrium: full solution

Assumption:

$$\sigma_a = 0.3 \text{ m}^2 \text{ kg}^{-1}$$

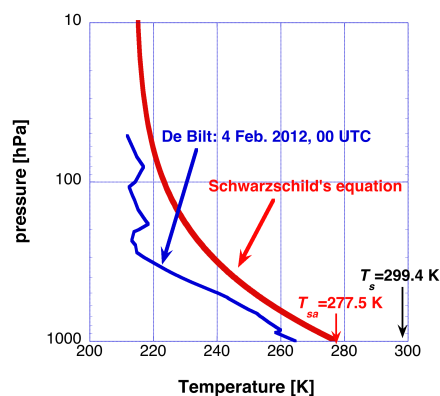
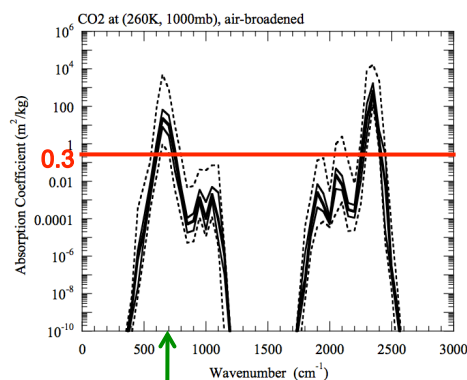


FIGURE 2.52

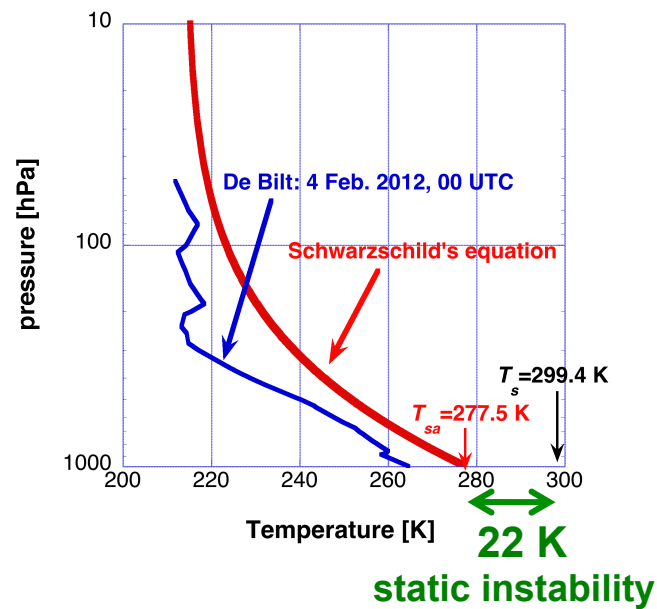
Actually:
Absorption crosssection CO_2
depends on wavelength



peak long wave radiation spectrum

BOX 2.3

Radiative equilibrium: full solution



Next lecture

Wednesday 13/2, 2013, 9:00-10:45

What do with the static instability at the surface?

Potential temperature

Parametrization of the sensible heat flux

The radiative-convective model (RCM)

Radiatively determined state