

Climate Dynamics (lecture 4)

What to do with the static instability at the surface?

Potential temperature

Parametrization of the sensible heat flux

The radiative-convective model (RCM)

Radiatively determined state

Sections 2.7, 2.8, 2.9

<http://www.staff.science.uu.nl/~delde102/climatedynamics.htm>

schedule

Week	Day	Date	Type*	Topic, sections or problems	
6	Mon	04/2	HC	Introduction/ Atmospheric Composition, parts of sec. 1.8-1.12 (Aarnout)	
6	Wed	06/2	HC	Greenhouse Effect/ Radiation Balance, sections 2.1-2.5 (Aarnout)	
7	Mon	11/2	HC	Radiative-Convective Model, sections 2.7-2.9 (Aarnout)	← today!
7	Wed	13/2	HC	Energy fluxes associated with the water cycle, sec. 2.10-2.15 (Aarnout)	
8	Mon	18/2	WC	problems 2.2, 2.4, 2.6 (Wim)	
8	Wed	20/2	WC	Excel problem (computers required) (hand in answers: 27/2) (Wim) ⁺	
9	Mon	25/2	HC	Climate sensitivity & radiative forcing, sec. 2.16-2.20 (Aarnout)	←
9	Wed	27/2	WC	problems 2.9, 2.11, (Wim) deadline Excel problem (5%)	
10	Mon	04/3	HC	Ice-albedo feedback, sec. 2.20-2.22 (Aarnout)	
10	Wed	06/3	HC	Energy- and CO ₂ -budget of Oceans (Aarnout)	
Choose Theme for essay					
11	Mon	11/3	no classes		
11	Wed	13/3	no classes		
12	Mon	18/3	WC	problem 2.15 (computers required) (hand in answers: 27/3) (Niels) ⁺	
12	Wed	20/3	HC	History of Earth's climate (Roderik)	
13	Mon	25/3	WC	problems 2.16, 2.17 (Wim)	
13	Wed	27/3	WC	deadline problem 2.15 (5%)	← HC
Choose theme for presentations					
14	Mon	01/4	Easter		
14	Wed	03/4	HC	Paleoclimatological observations 1 (Roderik)	
15	Mon	08/4	WC	Paleo1 (Wim) deadline essay (15%; obligatory).	
15	Wed	10/4	no classes		
16	Mon	15/4	Test 1 (09:00-12:00) (25%; minimum grade: 5) (Wim)		
16	Wed	17/4	no classes		

Optical path

Optical path: $\delta = \frac{\kappa p}{\rho g}$

Absorption coefficient: $\kappa = \sigma_a \rho_a$

density of the absorber [kg m⁻³]

absorption cross-section [m² kg⁻¹]

Therefore $\delta = \frac{\sigma_a \rho_a p}{\rho g}$ and $\delta_s = \frac{\sigma_a \rho_a p_s}{\rho g}$

(optical path at the surface)

BOX 2.3

Radiative equilibrium

Atmospheric temperature:

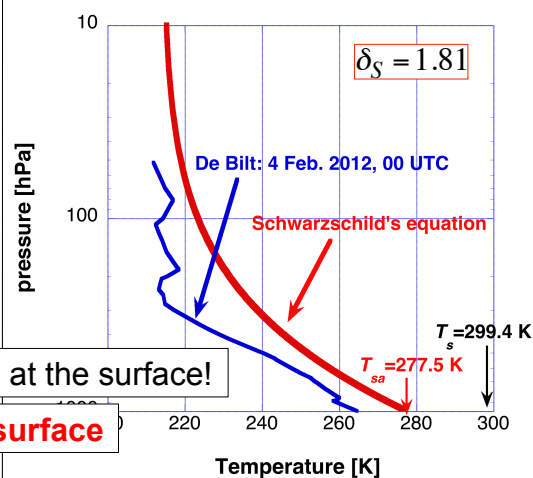
$$T = T_{skin}(\delta + 1)^{1/4}$$

Surface temperature:

$$T_s = T_{skin}(\delta_s + 2)^{1/4}$$

logarithmic scale →

$\sigma_a = 0.3 \text{ m}^2 \text{ kg}^{-1}$; $r_a = 390 \text{ ppm}$
 $\alpha_p = 0.3$; $S_0 = 1366 \text{ W m}^{-2}$; $p_s = 1000 \text{ hPa}$



Discontinuity in temperature at the surface!

>> **Static instability at the surface**

Without the temperature discontinuity at the surface there would be “radiative imbalance” at the surface, i.e. a positive downward net radiation flux.

In the simplified model this net radiation leads to an increase of the temperature of the surface

In reality non-radiative processes offset the imbalance. The radiative energy is “used” to evaporate water and to reduce the *static instability* by “sensible heat” transfer

Energy balance at Earth's surface

Transport of energy from the Earth's surface to the atmosphere is not only realized by radiation. **Sensible and latent heat (evaporation)** are important components of the energy balance!

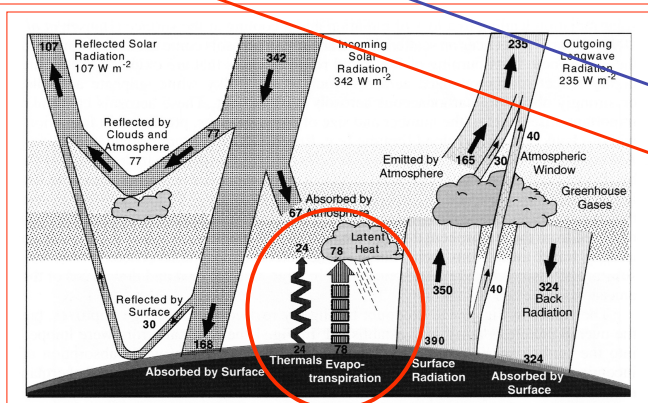
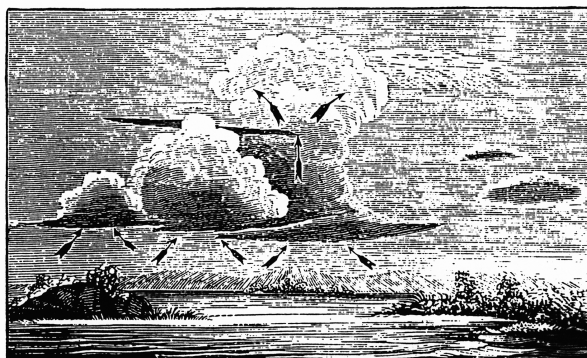


Figure 1. The earth's radiation balance. The net incoming solar radiation of 342 W m^{-2} is partially reflected by clouds and the atmosphere or at the surface, but 49% is absorbed by the surface. Some of that heat is returned to the atmosphere as sensible heating and most as evapotranspiration that is realized as latent heat in precipitation. The rest is radiated as thermal infrared radiation and most of that is absorbed by the atmosphere and re-emitted both upwards and downwards, producing a greenhouse effect, as the radiation lost to space comes from cloud tops and parts of the atmosphere much colder than the surface. From Kiehl and Trenberth (1997).

about 78 W m^{-2}

about 24 W m^{-2}

Convection



Arrows: convection currents

Upward transport of heat (sensible and latent) from the earth's surface by convection makes temperature profile "neutral". This process is called "**convective adjustment**".

The role of water is rather complex.

First we only consider the role of sensible heat transport.

Static instability will lead to *convection* in the surface layer, which will mix heat and reduce the temperature gradient.

To *include this process in the radiation model* is not a trivial matter, mathematically.

We must resort to *numerical methods*.

We'll construct a *layer model of the atmosphere*.

The *one layer model* is a special case of this model

Recapitulation of second lecture:

One layer radiation model

Emission temperature

$$T_e = \sqrt[4]{\frac{S_0 / 4 (1 - \alpha_p)}{\sigma}} = 255 \text{ K}$$

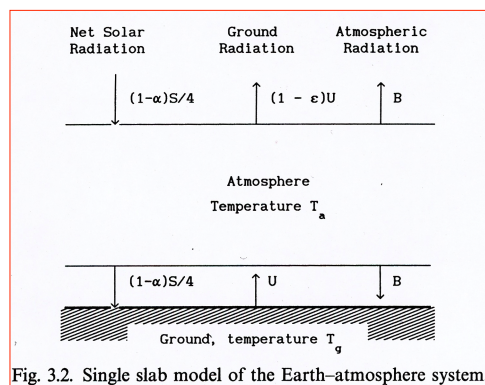
Ground emits energy at rate U of which a fraction, $(1-\epsilon)$, escapes to space.

Energy “balance”:

$$C \frac{dT_g}{dt} = (1 - \alpha) \frac{S_0}{4} + B - U = 0$$

$$\frac{c_p P_s}{g} \frac{dT_a}{dt} = \epsilon U - 2B = 0$$

This leads to: $T_g = \left\{ \frac{2}{2 - \epsilon} \right\}^{1/4} T_e$ and $T_a = \left\{ \frac{\epsilon}{2 - \epsilon} \right\}^{1/4} T_e$



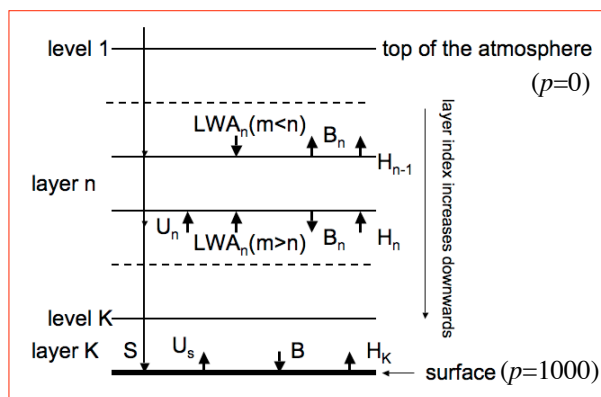
C is heat capacity of ground

Extension:

K-layer radiation model

Divide the atmosphere into K layers of equal mass. Write down the radiation budget for each layer and integrate these equations in time numerically until a steady state is reached.

LWA_n: longwave radiation flux from atmosphere reaching layer n
 B_n: longwave radiation flux emitted by layer n
 U_n: longwave radiation flux emitted by Earth's surface
 H: convective heat flux
 S: solar radiation reaching Earth's surface



Model consists of equations of the type: $C \frac{dT_g}{dt} = (1 - \alpha) \frac{S_0}{4} + B - U$ (slide 2)

Numerical integration

To solve a **system of first order differential equations** of the type

$$\frac{dX}{dt} = F(X, Y); \quad \frac{dY}{dt} = G(X, Y)$$

We choose a **time increment Δt** , and then find $X(t + \Delta t)$ by the following possible **recipes**:

Matsuno scheme

$$\begin{aligned} X_0 &= X(t), \\ X_1 &= X_0 + F(X_0, Y_0)\Delta t, \\ X(t + \Delta t) &= X_0 + F(X_1, Y_1)\Delta t, \end{aligned}$$

Gives excellent results if timestep is small enough →

Runge-Kutta scheme

$$\begin{aligned} X_0 &= X(t), \\ X_1 &= X_0 + F(X_0, Y_0)\Delta t / 2, \\ X_2 &= X_0 + F(X_1, Y_1)\Delta t / 2, \\ X_3 &= X_0 + F(X_2, Y_2)\Delta t, \\ X_4 &= X_0 - F(X_3, Y_3)\Delta t / 2, \\ X(t + \Delta t) &= (X_1 + 2X_2 + X_3 - X_4) / 3, \end{aligned}$$

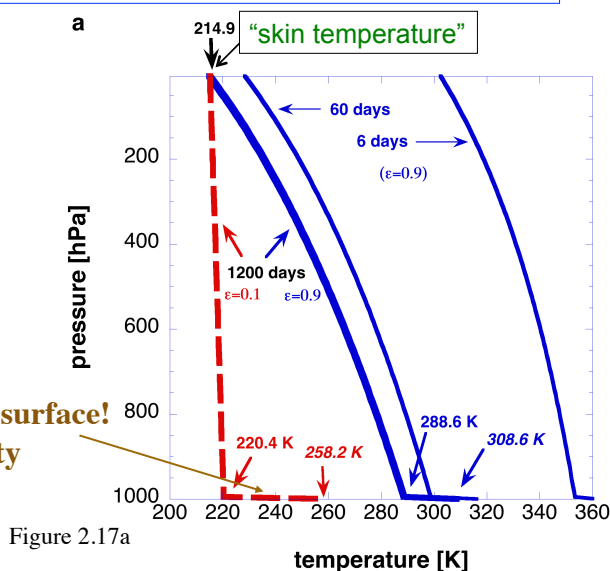
Homogeneous atmosphere, transparent to Solar radiation

Adjustment to radiative equilibrium

Initially temperature is 360 K everywhere ($K=100$)

Discontinuity at Earth's surface!

→ “convective” instability



Source code of model: <http://www.staff.science.uu.nl/~delde102/RCM.htm>

Homogeneous atmosphere, transparent to Solar radiation

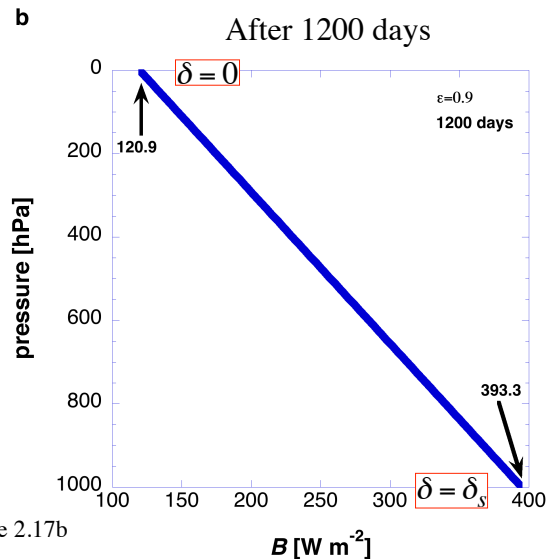
Radiative equilibrium for $\varepsilon=0.9$

Solution of
Schwarzschild's equation:

$$B \equiv \sigma T^4 = \frac{1}{2} Q(\delta + 1)$$

Relation between ε and δ ?

$$\varepsilon = 1 - \exp(-\delta_s)$$



Source code of model: <http://www.staff.science.uu.nl/~delde102/RCM.htm>

Homogeneous atmosphere, transparent to Solar radiation

Radiative equilibrium for $\varepsilon=0.9$

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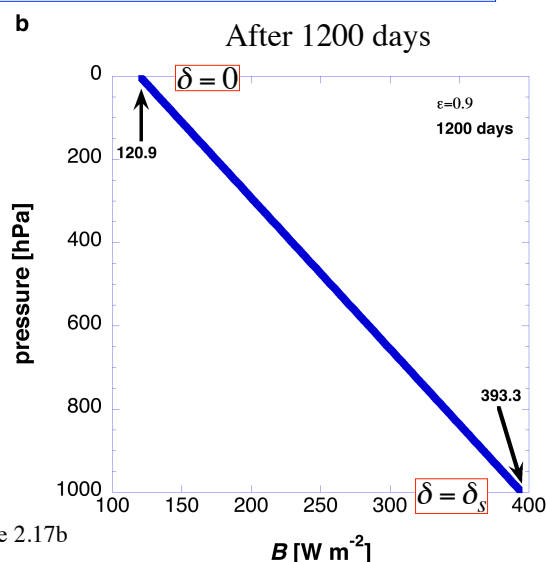
$$B \equiv \sigma T^4 = \frac{1}{2} Q(\delta + 1)$$

Relation between ε and δ ?

$$\varepsilon = 1 - \exp(-\delta_s)$$

At the surface:

$$B \equiv \sigma T^4 = \frac{1}{2} Q(\delta + 2)$$



Source code of model: <http://www.staff.science.uu.nl/~delde102/RCM.htm>

Homogeneous atmosphere, transparent to Solar radiation

Radiative equilibrium for $\varepsilon=0.9$

Solution of
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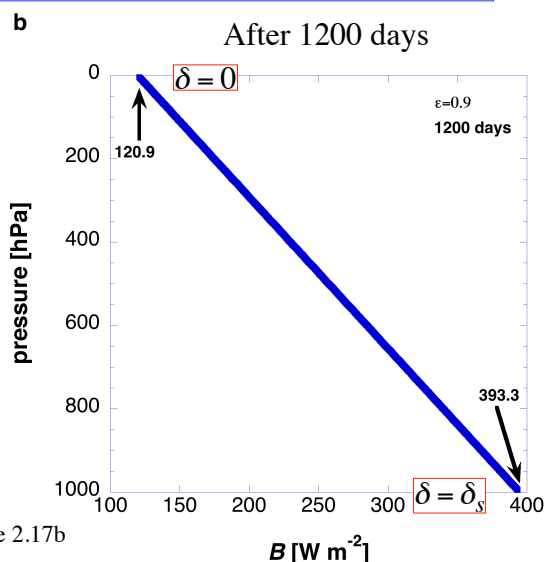
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At the surface:

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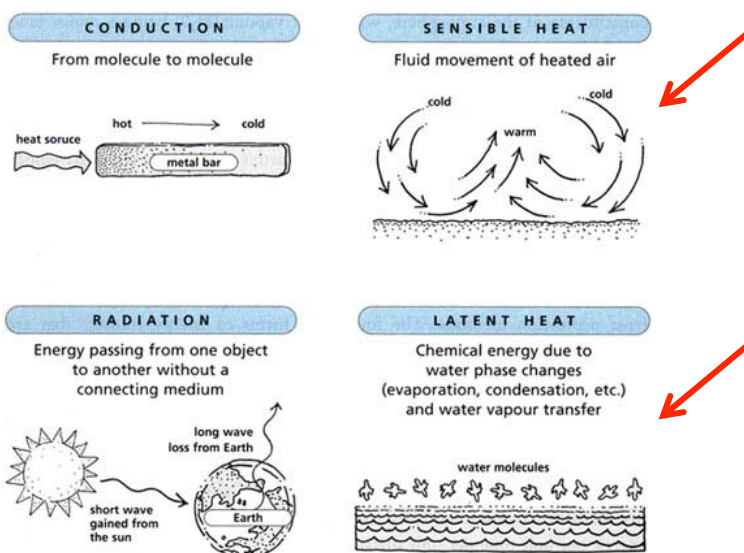
Static instability!!

Figure 2.17b



Source code of model: <http://www.staff.science.uu.nl/~delde102/RCM.htm>

Energy balance at Earth's surface



Potential temperature

Potential temperature of an air parcel: $\theta \equiv T \left(\frac{p_{ref}}{p} \right)^{R/c_p}$

From first law of thermodynamics and equation of state for an ideal gas:

$$\frac{d\theta}{dt} = \frac{\theta}{c_p T} J$$

J is the heating per unit mass, per unit time (**section 1.13**)

Static instability

If potential temperature decreases with increasing height, the state of hydrostatic balance is unstable (**section 1.15**). This is called “static instability” and leads to convection currents which are the cause of the formation cumulus clouds.

Static instability if $\frac{\partial \theta}{\partial z} < 0$

Brunt-Vaisala frequency: $N^2 \equiv \left(\frac{g}{\theta} \frac{d\theta}{dz} \right) < 0$

Homogeneous atmosphere, transparent to Solar radiation

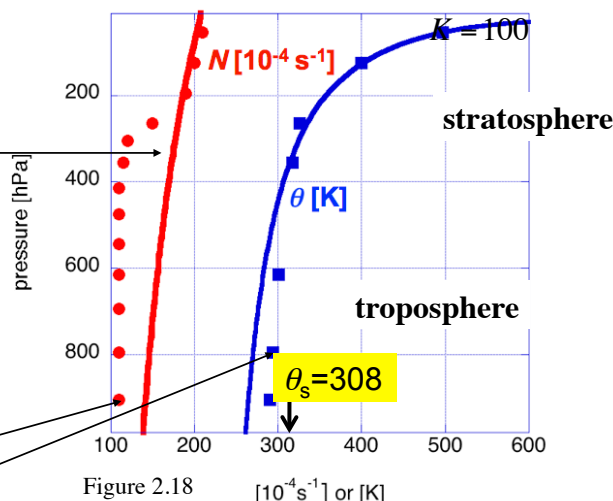
Radiative equilibrium solution

Abrupt transition troposphere to stratosphere is not reproduced by model

$$\theta \equiv T \left(\frac{p_{ref}}{p} \right)^{R/c_p}$$

$$N \equiv \left(\frac{g}{\theta} \frac{d\theta}{dz} \right)^{1/2}$$

standard atmosphere
(table 2.1 lecture notes)



N is the Brunt-Väisälä frequency (see section 1.15)

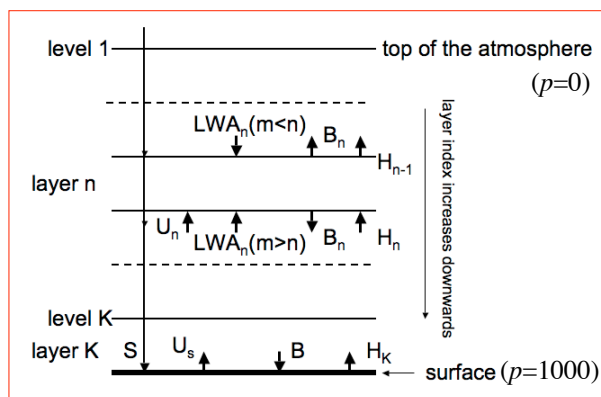
θ is potential temperature: conserved by air parcels under adiabatic conditions

How to take account of this effect in a model that does not explicitly contain dynamics??

K-layer radiation model

Divide the atmosphere into K equally spaced vertical intervals (in terms of pressure). Write down the radiation budget for each layer and integrate these equations in time numerically until a steady state is reached.

LWA_n : longwave radiation flux from atmosphere reaching layer n
 B_n : longwave radiation flux emitted by layer n
 U_n : longwave radiation flux emitted by Earth's surface
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Parametrization of convective adjustment

The convective heat flux from model layer $n+1$ to model layer n , H_n , in the one-dimensional model (see figure below) is “prescribed” as follows

$$H_n = k_H (\theta_{n+1} - \theta_n) \text{ if } \theta_{n+1} > \theta_n$$

$$H_n = 0 \text{ if } \theta_{n+1} \leq \theta_n$$

Where, k_H , is a transfer coefficient, much like a molecular diffusivity.

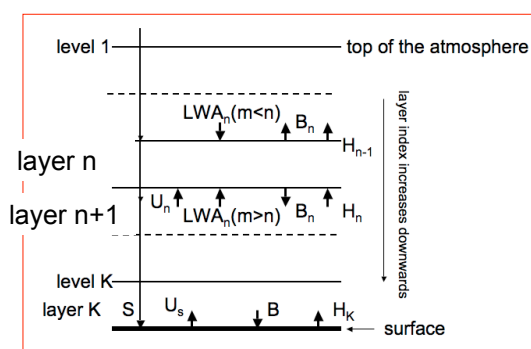
(see section 2.8)

Potential temperature:

$$\theta \equiv T \left(\frac{p_{ref}}{p} \right)^{R/c_p}$$

p_{ref} = reference pressure

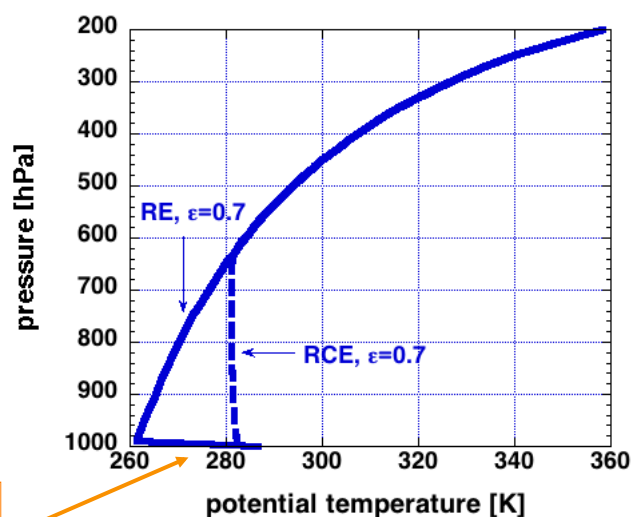
= 1000 hPa



Radiative-equilibrium

Figure 2.19 lecture notes

The state of *radiative-equilibrium* (RE) and the state of *radiative-convective equilibrium* (RCE) for a homogeneous atmosphere with $\epsilon=0.7$, computed by integrating the one-dimensional radiation model (section 2.7) until $ASR=OLR-TOA$.

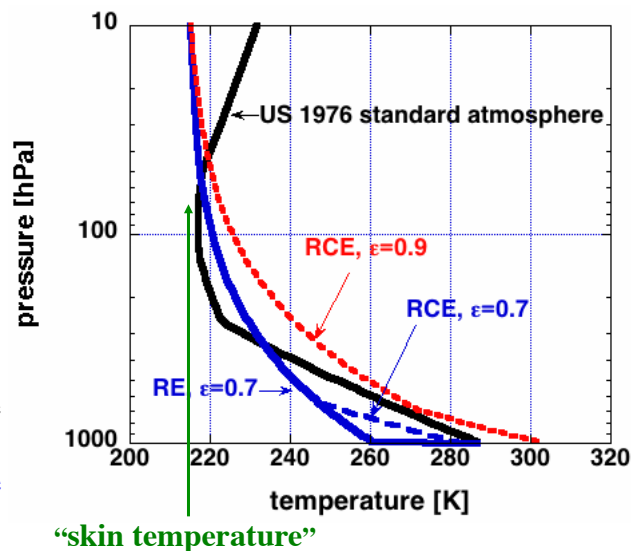


Static instability!
--> convection

Radiative-convective equilibrium

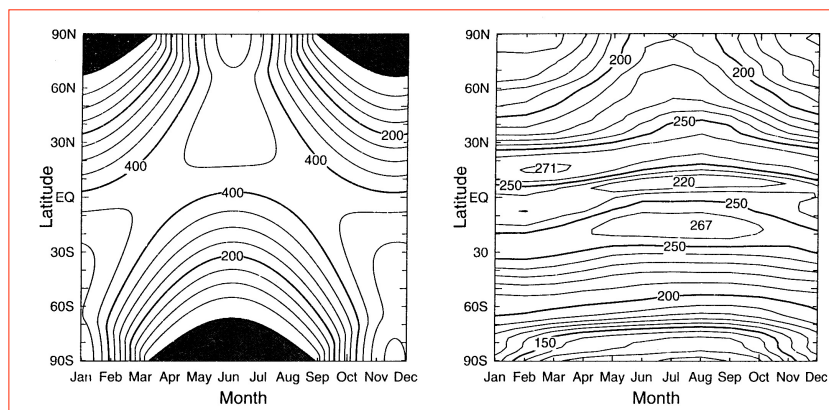
Figure 2.19 lecture notes

The state of **radiative-convective equilibrium (RCE)** for a **homogeneous atmosphere** computed by integrating the one-dimensional radiation model (section 2.7) including convective adjustment until $ASR=OLR-TOA$. The US-standard atmosphere is also shown (black solid line). **RE=radiative equilibrium**.



Radiatively determined state is not the same as radiative equilibrium

Insolation is a function of latitude and day



Daily mean **insolation (left)** and **OLR-TOA (right)** ($W m^{-2}$) as a function of latitude and month (Figure 2.8 lecture notes)

Insolation at specific point changes with seasons

Radiatively determined state at 45°lat.

The radiative-convective determined state at a latitude of 45° as a function Julian day, in two cases: (case 1) denoted by 'c' for which $C=10^6 \text{ J K}^{-1}\text{m}^{-2}$, and (case 2) denoted by 'm' for which $C=10^8 \text{ J K}^{-1}\text{m}^{-2}$, computed by integrating the one-dimensional radiation model over three model years with insolation representative 45°N on Earth, showing only the result for the last (third) year.

C is heat capacity of ground

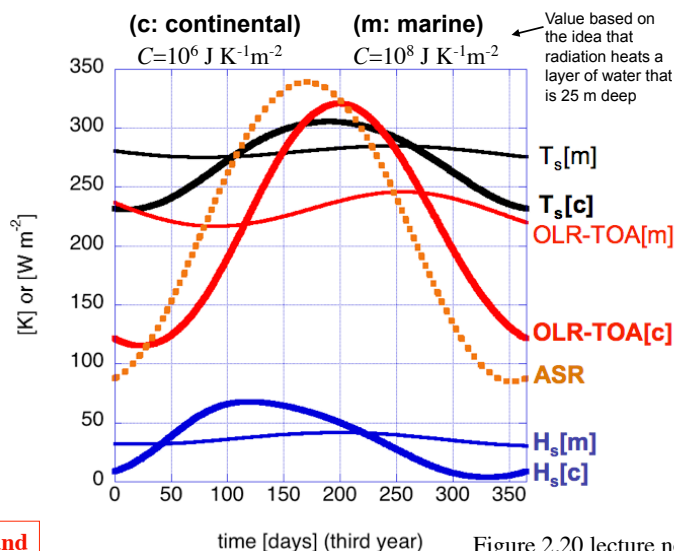


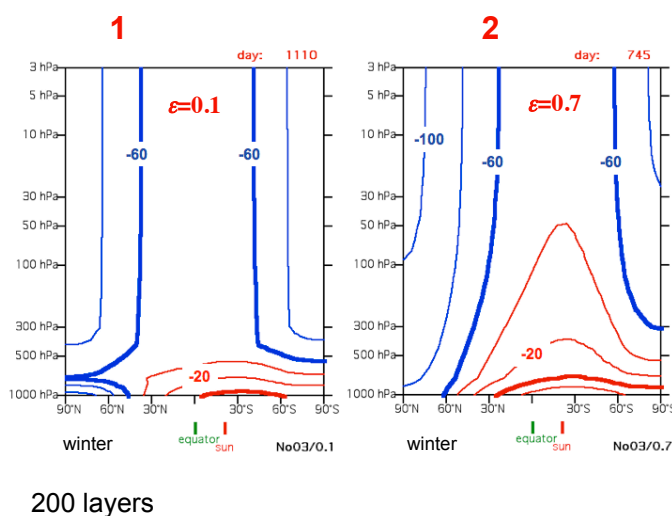
Figure 2.20 lecture no

Insolation changes with seasons

Radiatively determined state on 16 Jan.

Figure 2.21 lecture notes

The radiative-convective determined temperature at a latitude on 16 January as a function latitude and height, in two cases: (case 1) $\epsilon=0.1$, and (case 2) $\epsilon=0.7$, computed by integrating the one-dimensional radiation model over 1110 days (case 1) or 745 days (case 2) for different latitudes with insolation representative of the particular latitude. ($C=10^7 \text{ J K}^{-1}\text{m}^{-2}$)



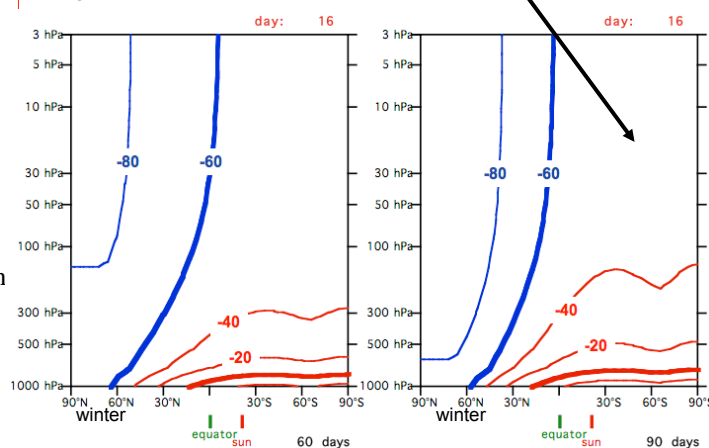
Insolation changes with seasons

Radiative “equilibrium” on 16 Jan.

Radiative-convective “equilibrium” temperature on 16 January as a function latitude and height if $\epsilon=0.7$, computed by integrating the one-dimensional radiation model over 60/90 days (initial state: isothermal) for different latitudes with constant insolation representative of 16 January at each latitude. ($C=10^7 \text{ J K}^{-1}\text{m}^{-2}$)

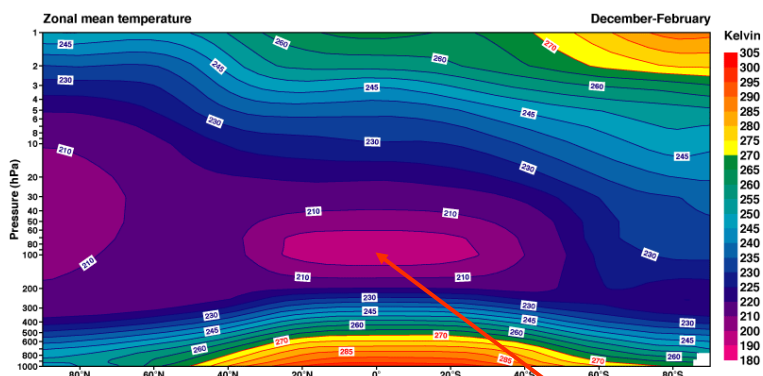
Summer hemisphere much warmer than in radiatively determined state. This is effect of thermal inertia

Figure 2.23 lecture notes



Observations (ERA-40)

Several aspects of the temperature distribution are not reproduced



ERA: ECMWF Reanalysis project

Reanalysis of all observations 1979-2002

ECMWF: European Centre for Medium Range Weather Forecasts

Practical sessions

Monday 18/2, 11:00-10:45 and Wednesday 20/2, 09:00-10:45

Wednesday 27/2, 09:00-10:45, problems 2.6, 2.9 and 2.11

With Wim Ridderinkhof

Excel problem (Energy balance model)
Hand in answer before or on Wed. 27/2
(5% of the final grade)

<http://www.staff.science.uu.nl/~delde102/climatedynamics.htm>

About the essay

Writing a short essay of 750-1000 words on a ("hot") topic in climate research is part of the course (15% of the final grade). Inspiration can be obtained by going to the electronic library and browsing through scientific journals, such as *Nature*, *Nature Geoscience*, *Science*, *PNAS*, *Journal of Climate* and many more. You should give me the title or subject of the essay on 6 March, or earlier.

See

<http://www.staff.science.uu.nl/~delde102/climatedynamics.htm>
for some examples of typical essays of 750-1000 words in Dutch and in English

Next lecture

Monday 4/3, 2013, 11:00-12:45

Water cycle and energy balance (sections 2.10-2.13)

Cooling of Earth's surface due to evaporation

Latent heat release due to condensation and freezing

Planetary albedo is determined by cloud cover fraction!

Clouds absorb long wave radiation

<http://www.staff.science.uu.nl/~delde102/climatedynamics.htm>