

Ice albedo feedback

sections 2.21 and 2.22

The “icon/toy” climate model: Energy Balance Model of Budyko and Sellers

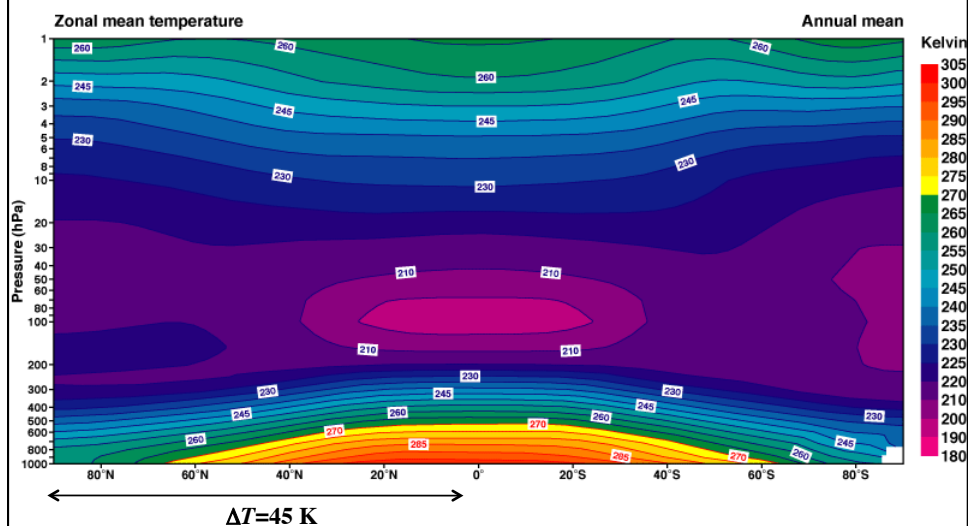
Messages of this toy-model:

1. “climate system is **non-linear**”
2. “taking account of **meridional transport of energy** is crucial to explain **equator-pole temperature difference**”

Aarnout van Delden, IMAU, Utrecht University

<http://www.staff.science.uu.nl/~delde102/climatedynamics.htm>

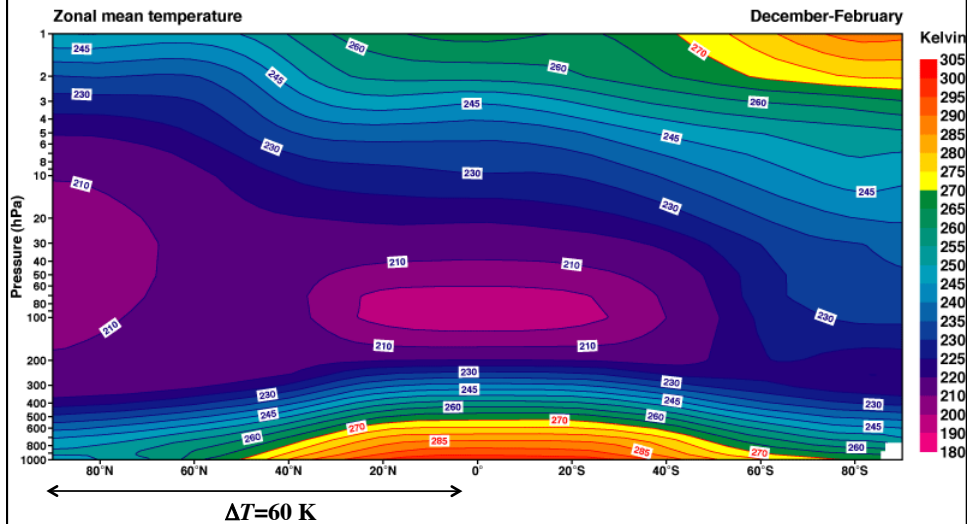
Zonal mean* temperature



ERA-40 reanalysis (http://www.ecmwf.int/research/era/ERA-40_Atlas/docs/index.html)

*Zonal mean: mean with respect to longitude

Zonal mean* temperature



ERA-40 reanalysis (http://www.ecmwf.int/research/era/ERA-40_Atlas/docs/index.html)

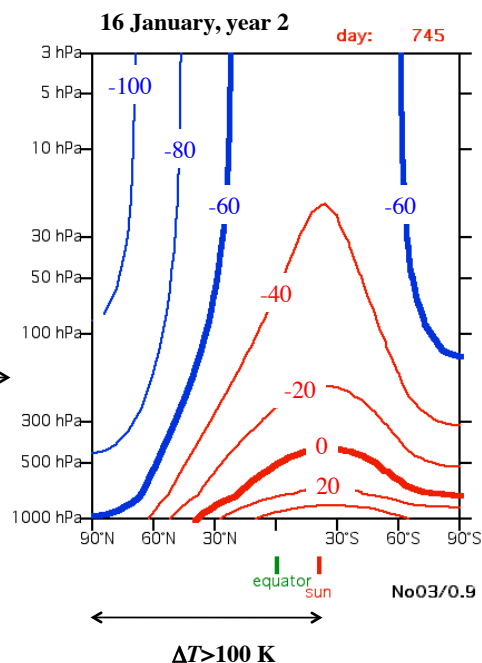
*Zonal mean: mean with respect to longitude

Zonal mean radiatively determined temperature

Temperature distribution by integrating the RCM for each latitude separately and making the composite for 16 January in the second year of the integration.

Temperature contrast pole-equator very large!

Need to take into account the meridional energy transport by motions in the atmosphere and ocean



Section 2.9, figure 2.21

We are going to construct a **simplified model** of the meridional surface temperature distribution, with which we can also study the ice-albedo feedback and possible causes of ice ages

Energy balance as a function of latitude in terms of surface temperature

-Formulation of the model: Parametrization of outgoing long-wave radiation and of meridional transport of energy in terms of surface temperature

-Solutions of this model exhibit Multiple equilibria and hysteresis

Energy balance model due to Budyko and Sellers (1969)

Predicts **global average surface equilibrium temperature as a function of latitude** taking into account very crudely the meridional transport of energy from the equator to the poles.

Focus on the estimation of the effect on the surface temperature of **changes in the planetary albedo** (due to changes in snow/ice cover) as a function of latitude, ϕ .

One-dimensional energy balance model

For **mean annual conditions**, the equation for the **zonally averaged heat balance** of the earth-atmosphere system is:

$$Q(1 - \alpha) = I + A$$

$Q(\phi)$ =Solar radiation at TOA (top of the atmosphere)

$\alpha(\phi, T)$ =albedo;

$I(\phi, T)$ =outgoing long-wave radiation at TOA

$A(\phi, T)$ =loss (if $A > 0$) of energy from a particular latitude belt as a result of the atmosphere and hydrosphere circulation, including heat redistribution of phase water transformations.

For the Earth-atmosphere system as a whole we have, $A=0$, giving the “**zero-dimensional EBM**”.

$$Q(1 - \alpha) = I + A$$

“Toy” models

“It may be that we **seek enlightenment, rather than imitation**. We might then look for the **essence of the physical system**, disregarding all processes that we think are not absolutely essential. This set of essential processes defines our model, though perhaps we should call it a toy, rather than a model. The object is to reproduce the consequences of what we think are the essentials, and the only thing that is to prevent us is our inaccurate specification of the physics. What we can **hope to get right is the feedback between processes**”

John Green , 1999: **Atmospheric Dynamics**, Cambridge University press (p. 160)

$$Q(1 - \alpha) = I + A$$

Latitudinal dependence of insolation

For obliquity angle of 23.5° :

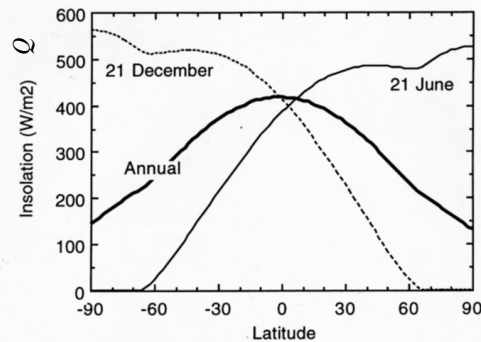


Fig. 2.7 Annual-mean and solstice insolation as functions of latitude.

How does the “Annual” curve change with increasing obliquity angle?

Hartmann, 1994, fig.2.7

$$Q(1 - \alpha) = I + A$$

Energy imbalance at each latitude

Annual mean radiation

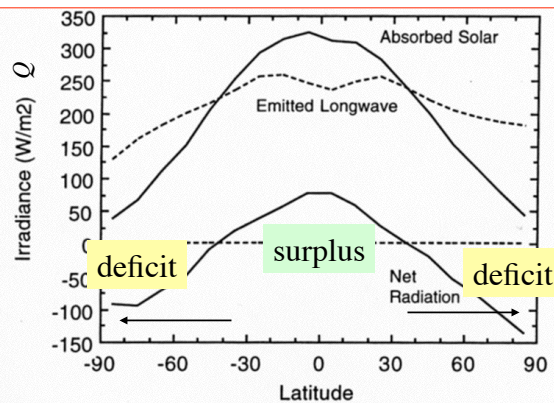


Fig. 2.12 Graphs of annual-mean absorbed solar radiation, OLR, and net radiation averaged around latitude circles.

Why is the emitted radiation less at poleward latitudes?

Hartmann, 1994, fig.2.12

Parametrization in Budyko-Sellers energy balance model

$$Q(1 - \alpha) = I + A$$

$Q(\phi)$ =Solar radiation at TOA (top of the atmosphere)

$\alpha(\phi, T)$ =albedo;

$I(\phi, T)$ =outgoing long-wave radiation at TOA

$A(\phi, T)$ =loss (if $A > 0$) of energy from a particular latitude belt as a result of the atmosphere and hydrosphere circulation, including heat redistribution of phase water transformations.

Can we express I , A and α in terms of T_g ?

Dependence of I on temperature of Earth's surface, T_g

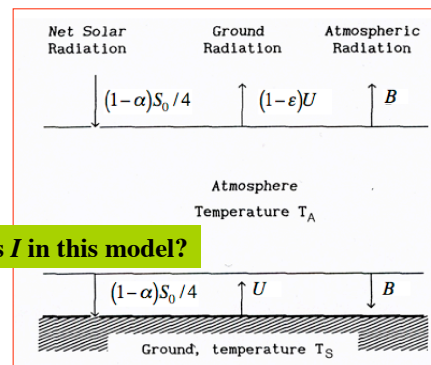
Parametrization of outgoing radiation

This depends on the radiation absorption-emission characteristics of the atmosphere.

An **empirical formula** has been suggested by **Budyko** (1969) and others:

$$I = I_0 + b(T - h\Gamma)$$

What is I in this model?



Single-layer model of the atmosphere. (fig 2.5 lecture notes) .

T is **sea-level temperature in °C(!)**, Γ is the lapse rate (6°C km^{-1}) and h the zonal mean height of Earth's surface

See also problem 2.16

Parametrization of outgoing radiation continued

The **linear relation** $I = I_0 + b(T - h\Gamma)$

due to Budyko can be obtained also by linearization of the black body emission. Assuming that $h=0$, we can write:

$$\sigma(273.15 + T)^4 \approx \sigma(273.15)^4 + 4\sigma(273.15)^3 T \quad * \quad (T \text{ in } ^\circ\text{C}!)$$

Now, comparing the above formulae we obtain:

$$I_0 = \sigma(273.15)^4 = 320.64 \text{ W m}^{-2}$$

$$b = 4\sigma(273.15)^3 = 4.69 \text{ W m}^{-2} \text{ } ^\circ\text{C}^{-1}$$

Budyko obtains $I_0=205 \text{ W m}^{-2}$ & $b=2.23 \text{ W [m}^2 \text{ } ^\circ\text{C}]^{-1}$

(but this depends on mean relative humidity and concentration of greenhouse gasses, i.e. the emission coefficient(s) of the atmosphere)

Explain the difference in the value of the coefficients

Estimate the emission coefficient of the Earth-atmosphere system

$$\sigma = 5.76 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \quad * \text{using the Binomial theorem: see appendix to this lecture}$$

A parametrization is empirical

$$I = I_0 + b(T - h\Gamma)$$

Estimate the value of the parameter, b , from both graphs

R.T. Pierrehumbert (Nature, 2002, vol. 432, p. 677)

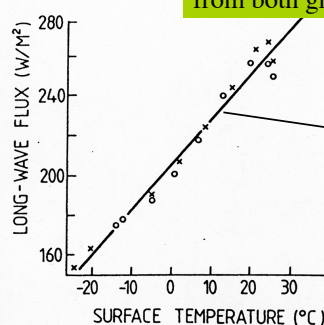
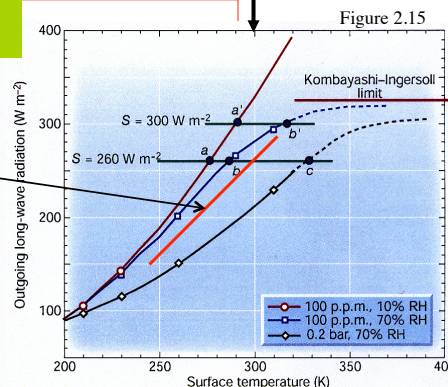


Figure 1.3. A plot of surface temperature vs net outgoing long-wave radiation. Each symbol represents a 10° wide latitude belt. Crosses refer to the southern hemisphere, circles to the northern hemisphere.

Figure due to Oerlemans & v.d. Dool (J.Atmos.Sci.,1978), based on first satellite measurements of OLR-TOA.



Is the parameter, b , smaller or larger for an atmosphere with more greenhouse gases?

Intermezzo

Climate Sensitivity

For the whole planet (atmosphere-earth system) we have the equilibrium

$$Q(1 - \alpha) - I_0 - bT = 0$$

(assuming for simplicity that $h=0$). Differentiating this equation with respect to Q * we obtain

$$\frac{dT}{dQ} = \frac{(1 - \alpha)}{b}$$

where we have assumed that the albedo and the other empirical parameters are independent of Q . The above relation implies that the **sensitivity of the climate to changes in the incoming solar radiation depends on the albedo and the parameter b .**

$$*Q = S_0/4$$

Climate sensitivity is frequently defined as the **global temperature change for a 1% change in solar constant**, i.e. using the parameter

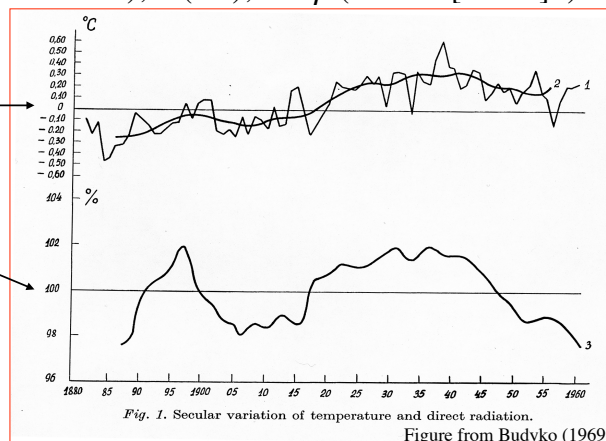
$$\mu = \frac{Q}{100} \frac{dT}{dQ} = Q \frac{(1 - \alpha)}{100b}$$

With the values of Q (342 Wm^{-2}), α (0.3), and β ($2.23 \text{ W [m}^2 \text{ }^\circ\text{C]}^{-1}$) we obtain $\mu = 1.07^\circ\text{C}$.

Global mean temperature deviation

Absorbed solar radiation (%)

Is theory in accord with observation?



More up to date version of this graph: see also figure 2.14 lecture notes

$$Q(1-\alpha) - I = A$$

meridional energy flux divergence

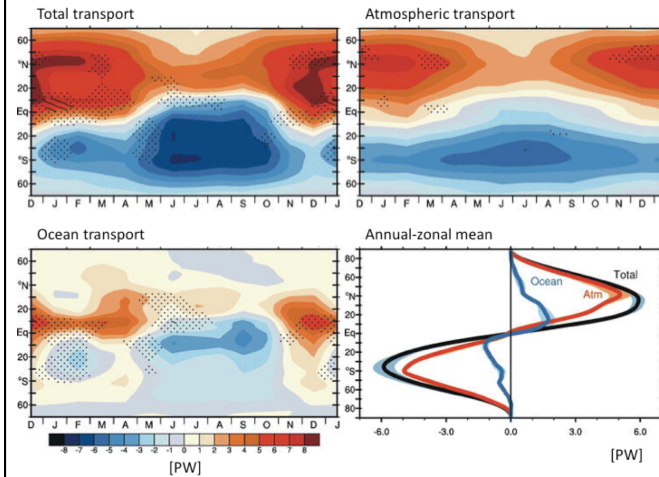


Figure 1 of box 2.11. Meridional energy transport in Peta-Watts (PW=10¹⁵ Watts). *Upper left*: total; *upper right*: atmospheric transport; *lower left*: ocean transport; *lower right*: annual-zonal mean energy transport by ocean, atmosphere and total (Fasullo, John T., Kevin E. Trenberth, 2008: *J. Climate*, **21**, 2313–2325)

Dependence of A on temperature of Earth's surface, T

Parametrization of meridional flux divergence

$$Q(1-\alpha) - I = A$$

To facilitate the solution of the above equation, the energy flux divergence within the atmosphere and oceans is **prescribed** as follows*.

$$A = \beta(T - T_p) \quad \text{with} \quad T_p = \frac{1}{2} \int_{-1}^1 T dx \quad x \equiv \sin \phi$$

T_p is the planetary mean sea-level temperature;

β is a **relaxation coefficient**.

*after M.I.Budyko (Tellus, 21 (1969), 610-619)

Dependence of A on temperature of Earth's surface, T

Parametrization of meridional flux divergence

$$Q(1 - \alpha) - I = A$$

Another possibility is to describe the energy flux divergence by a **diffusive term** as follows**.

$$A = \frac{\partial}{\partial x} \left(1 - x^2 \right) K_H \frac{\partial T}{\partial x}$$

K_H is diffusivity [$\text{W m}^{-2} \text{K}^{-1}$]

**after W.D. Sellers (J.Appl.Meteor., 8 (1969), 392-400)

$$Q(1 - \alpha) - I = A$$

Model with Budyko's parametrization

$$Q(1 - \alpha) = I_0 + bT + \beta(T - T_p) = a^* + b^*T$$

$$\text{with } a^* = I_0 - \beta T_p; \quad b^* = b + \beta$$

Introduce a **temperature dependent albedo**:

$$\begin{aligned} \alpha &= \alpha_0 \text{ if } T \leq T_0 \\ \alpha &= \alpha_0 + \frac{T - T_0}{T_1 - T_0} (\alpha_1 - \alpha_0) \text{ if } T_1 \geq T > T_0 \\ \alpha &= \alpha_1 \text{ if } T > T_1 \end{aligned}$$

For each latitude we have this equilibrium:

$$Q(1 - \alpha) = a^* + b^* T$$

The **absorbed solar radiation (ASR)** now depends on temperature as shown by the solid line in the figure (l.h.s. of eq. above).

The broken line in the figure represents the **energy loss** (r.h.s. of eq. above)

Equilibrium states are represented by the **intersections** of solid and broken lines

(OLR)+meridional flux divergence

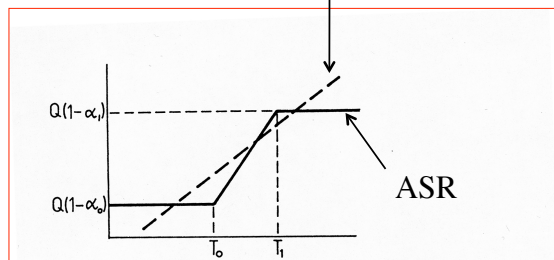


Figure 2.1. Absorbed solar radiation (solid line) and energy loss by outgoing terrestrial radiation and dynamical energy fluxes (broken line).

For each latitude we have this equilibrium:

$$Q(1 - \alpha) = a^* + b^* T$$

Slope of the solid line (ASR) between $T=T_0$ and $T=T_1$ is $\mu = \frac{Q(\alpha_0 - \alpha_1)}{T_1 - T_0}$

If $\mu < b^*$ there is **one equilibrium**

If $\mu > b^*$ there are possibly **three equilibria** depending on the value of a^*

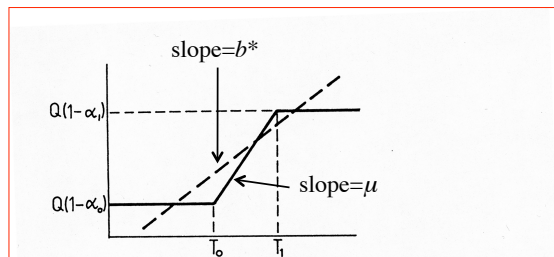
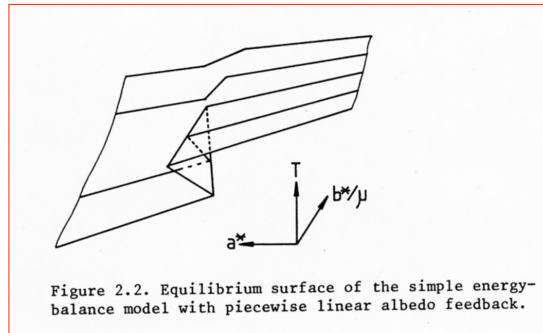


Figure 2.1. Absorbed solar radiation (solid line) and energy loss by outgoing terrestrial radiation and dynamical energy fluxes (broken line).

Multiple equilibria

The relevant **control parameters** are b^*/μ and a^* . The figure below shows the equilibrium surface. Factors favouring the occurrence of three equilibria are **strong albedo feedback** (large μ), **weak “damping”** due to the combined effect of outgoing radiation and horizontal energy flux divergence (small b^*).



See also problem 2.17

Stability characteristics

The stability of these equilibria can be investigated by **linearizing** the equation

$$C \frac{dT}{dt} = Q(1 - \alpha(T)) - a^* - b^* T$$

C is the heat capacity of the system per unit surface area.

This is a nonlinear first order differential equation. The standard method of investigating the stability of the equilibrium solution is to add a perturbation:

$$T(t) = T_{eq} + T'(t)$$

The albedo may be written as:

$$\alpha(T) \cong \alpha(T_{eq}) + \frac{d\alpha}{dT} T'$$

Inserting the above two equations into the top equation we obtain

See also **problem 2.17**

$$\frac{dT'}{dt} = -\lambda T' \quad \text{with} \quad \lambda = \frac{b^* + Q \frac{d\alpha}{dT}}{C}$$

The solution of this differential equation is $T' = T'(0) \exp(-\lambda t)$

Therefore there is decay back to equilibrium if $\lambda > 0$ and **exponential growth of the amplitude of the perturbation (instability) if $\lambda < 0$** .

The sign of λ is determined by the sign of $d\alpha/dT$.

For $T > T_1$ or $T < T_0$ $d\alpha/dT = 0$. Therefore, the solution for these equilibria are stable (because $b^* > 0$).

For $T_0 < T < T_1$ $d\alpha/dT < 0$. Therefore, the solution for this intermediate equilibrium state is unstable if

$$\left| \frac{d\alpha}{dT} \right| < \frac{b^*}{Q}$$

Parametrization of solar radiation

$$Q = \frac{S_0}{4} s(x) \quad \text{where} \quad x = \sin \phi$$

The distribution function $s(x)$ is defined as the **annual mean insolation at a particular latitude** divided by the global average insolation.

The global average of $s(x)$ is 1.

The annual mean insolation distribution function for current conditions can be approximated as

$$s(x) = 1.0 - 0.477 P_2(x)$$

$$\text{where} \quad P_2(x) = \frac{1}{2}(3x^2 - 1)$$

Model with meridional flux divergence

Equilibrium: $Q(1-\alpha) - I = A$

With: $Q = \frac{S_0}{4} s(x)$
 $I = I_0 + b(T - h\Gamma)$
 $A = \beta(T - T_p)$

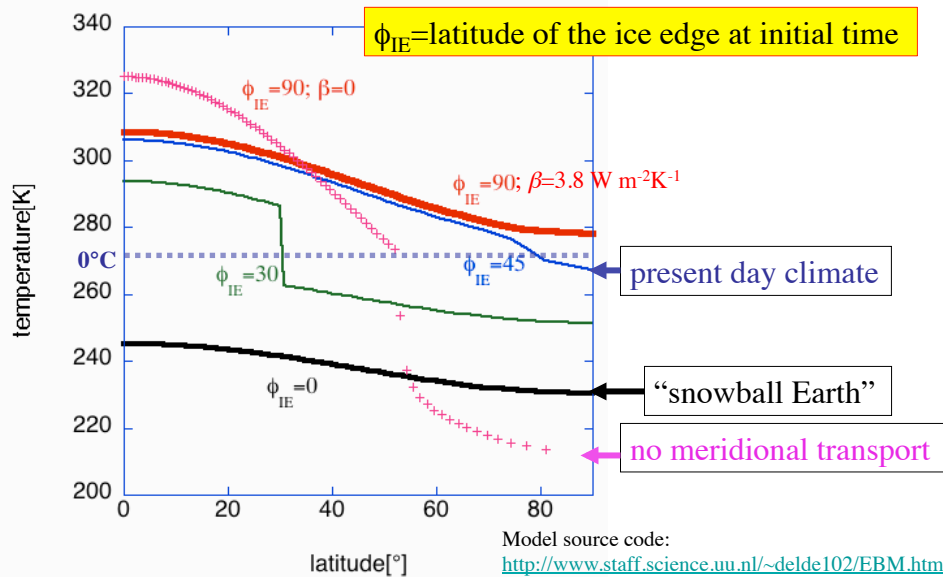
gives

$$\beta(T - T_p) - bT = -\frac{S_0}{4} s(x)(1-\alpha) + I_0 - bh\Gamma$$

This equation is solved as an initial-value problem by adding a term $C \frac{dT}{dt}$ and integrating the equation until a steady state is reached.

Solution depends on initial condition

Initial condition: $T = -40^\circ\text{C}$ over ice and $T = +40^\circ\text{C}$ if no ice



750-650 million years BP: first “ice ages”

“Snowball Earth”

Gabrielle Walker:

Picture a world entirely sheathed in ice, its oceans smothered by a freezing white blanket nearly a mile thick. Vast glaciers creep across the continents. Nothing else moves. There are no clouds, save perhaps a handful of high wispy streaks made from frozen crystals of carbon dioxide. With the temperature a chilling 40 degrees below zero, only a few living things survive. Algae cling to the meager warmth of volcanic springs, and bacteria eke out a living around hot-water vents deep in the ocean. For millions of years, nothing changes. This is no far-off planet or alien moon. It's a view of Earth just a few hundred million years ago.

See also:

http://en.wikipedia.org/wiki/Snowball_earth#cite_note-Crowley2001-38

This “ice ages”: glacials and interglacials

Extent of ice cover

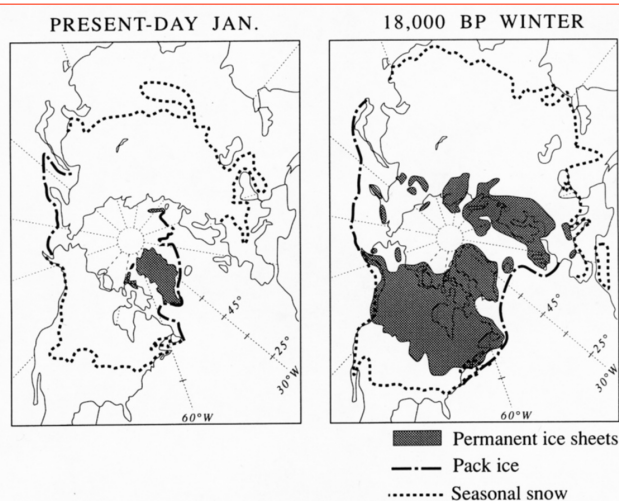


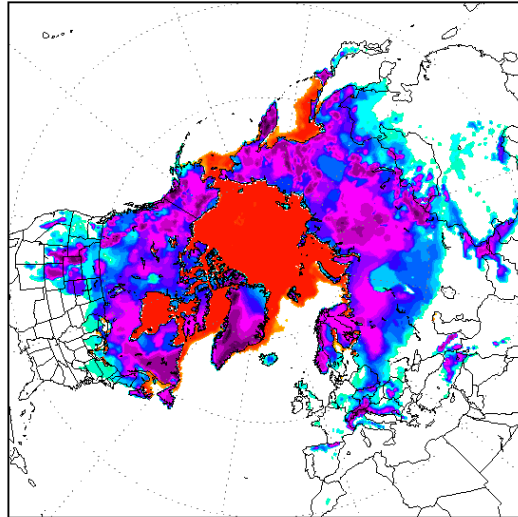
Fig. 9.4 Extent of permanent ice sheets (shaded), seasonal snowcover (dotted line), and pack ice (perennial sea ice, dash-dot line) at present and estimated for the last glacial maximum. [From Robinson, unpublished, as published in Kukla (1979).]

End of northern hemisphere winter: 2004

Init : Tue,02MAR2004 00Z

Valid: Tue,02MAR2004 00Z

Eisbedeckung (rot) und Schneehoehe in cm



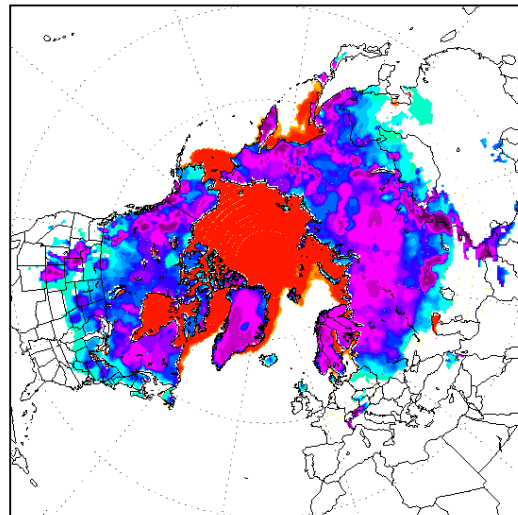
Daten: Eis und Schneehoehenanalyse des NCEP
Wetterzentrale Karlsruhe
Top Karten : <http://www.wetterzentrale.de/topkarten/>

End of northern hemisphere winter: 2010

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Eisbedeckung (rot) und Schneehoehe in cm



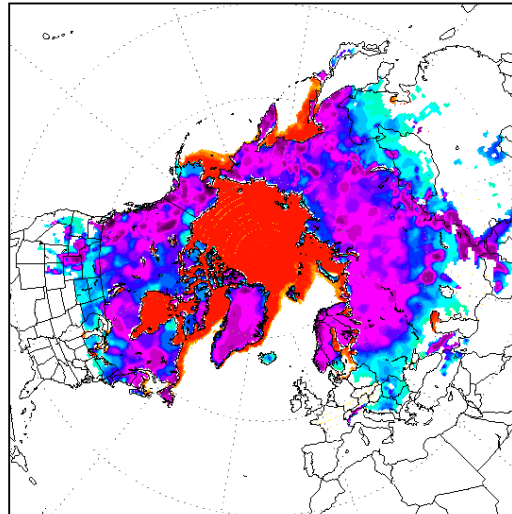
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(C) Wetterzentrale
www.wetterzentrale.de

End of northern hemisphere winter: 2011

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Eisbedeckung (rot) und Schneehoehe in cm



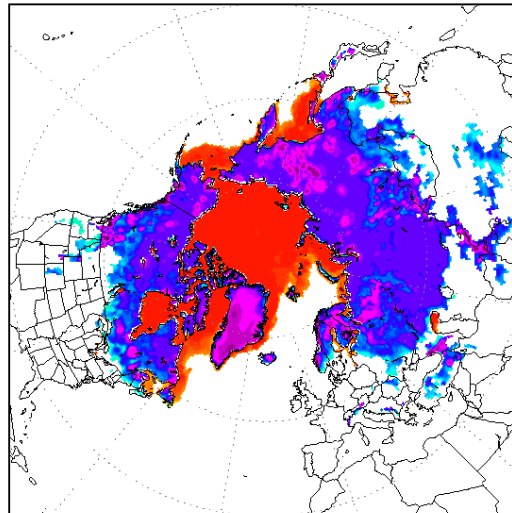
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End of northern hemisphere winter: 2012

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Eisbedeckung (rot) und Schneehoehe in cm



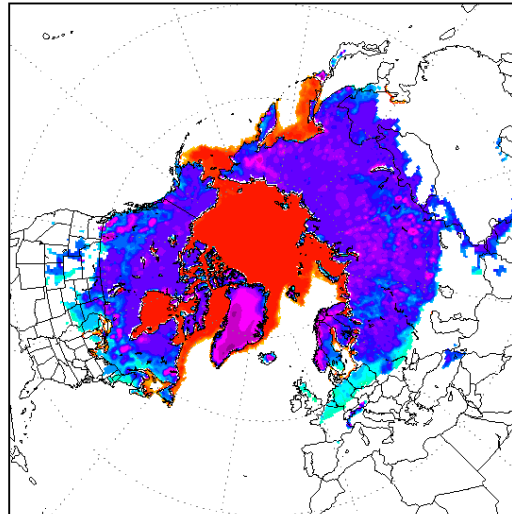
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End of northern hemisphere winter: 2013

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Valid: Tue,12MAR2013 00Z

Eisbedeckung (rot) und Schneehöhe in cm



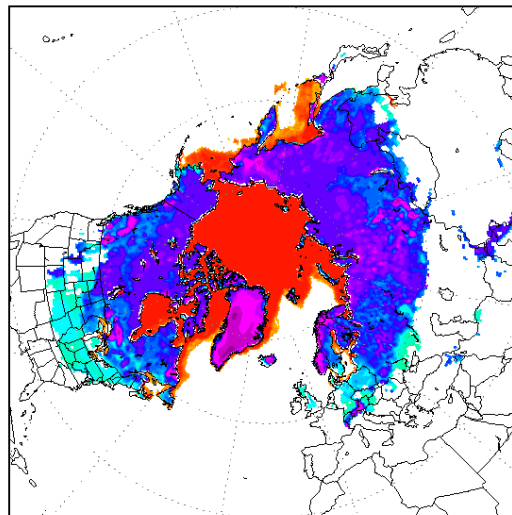
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End of northern hemisphere winter: 2013

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Valid: Mon,25MAR2013 00Z

Eisbedeckung (rot) und Schneehöhe in cm



Daten: Eis und Schneehöhenanalyse des NCEP
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Radiative forcing and albedo feedback from the Northern Hemisphere cryosphere between 1979 and 2008

M. G. Flanner^{1*}, K. M. Shell², M. Barlage³, D. K. Perovich⁴ and M. A. Tschudi⁵

The extent of snow cover¹ and sea ice² in the Northern Hemisphere has declined since 1979, coincident with hemispheric warming and indicative of a positive feedback of surface reflectivity on climate. This albedo feedback of snow on land has been quantified from observations at seasonal timescales^{3–6}, and century-scale feedback has been assessed using climate models^{7–10}. However, the total impact of the cryosphere on radiative forcing and albedo feedback has yet to be determined from measurements. Here we assess the influence of the Northern Hemisphere cryosphere on Earth's radiation budget at the top of the atmosphere—termed cryosphere radiative forcing—by synthesizing a variety of remote sensing and field measurements. We estimate mean Northern Hemisphere forcing at -4.6 to -2.2 W m^{-2} , with a peak in May of $-9.0 \pm 2.7 \text{ W m}^{-2}$. We find that cryospheric cooling declined by 0.45 W m^{-2} from 1979 to 2008, with nearly equal contributions from changes in land snow cover and sea ice. On the basis of these observations, we conclude that the albedo feedback from the Northern Hemisphere cryosphere falls between 0.3 and $1.1 \text{ W m}^{-2} \text{ K}^{-1}$, substantially larger than comparable estimates obtained from 18 climate models.

Model with *diffusive* meridional flux divergence

Equilibrium: $Q(1 - \alpha) - I = A$

With: $Q = \frac{S_0}{4} s(x)$
 $I = I_0 + b(T - h\Gamma)$

gives $A = \frac{\partial}{\partial x} (1 - x^2) K_H \frac{\partial T}{\partial x}$

$$\frac{\partial}{\partial x} (1 - x^2) K_H \frac{\partial T}{\partial x} - bT = -\frac{S_0}{4} s(x)(1 - \alpha) + I_0 - bh\Gamma$$

Because the coefficients before the two terms on the l.h.s. of this equation are positive, this is an **elliptic differential equation** and therefore can be solved numerically by **iterative methods** (e.g. by successive relaxation).

Specification of albedo

The albedo must be specified as a function of x by letting it depend on whether the particular point on earth is covered with snow (0.61), ice (0.56), vegetation without ice or snow (0.31) or ocean (a complex function of the zenith angle). The fraction of land around a latitude is also accounted for (see later).

The potential **snow cover**, i.e. the snow cover that would exist if open water did not melt the falling snow, can be derived from observations. The figure gives an impression of the temperature dependence of snow cover.

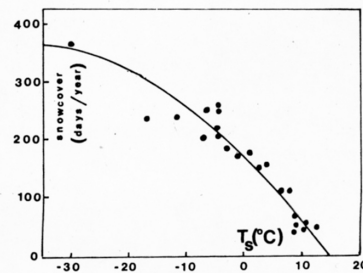


FIG. 3. The number of days with snow cover as a function of the mean annual temperature (data from Lamb, 1972). The curve gives the parameterization used in the model.

The snow cover is represented in the model by

$$\text{snow cover} \begin{cases} = 1 & \text{if } T \leq -40^\circ\text{C}; \\ = 1 - 0.00033(T + 40)^2 & \text{if } -40^\circ\text{C} < T \leq +15^\circ\text{C} \\ = 0 & \text{if } T > +15^\circ\text{C} \end{cases}$$

(the solid line in graph on previous slide)

The graph below shows the computed albedo corresponding to the present temperature distribution together with observed values.

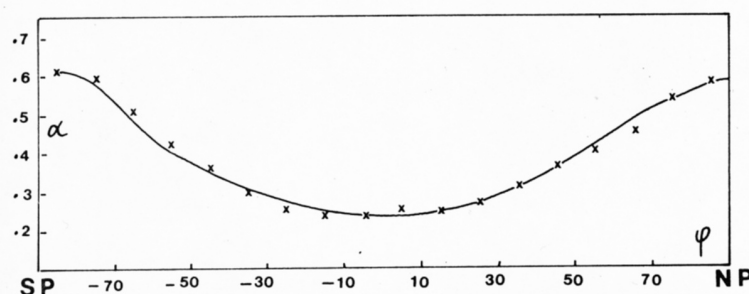


FIG. 4. Computed albedo (solid line) compared with observed values given by Ellis and Vonder Haar (1976).

$$I = I_0 + b(T - h\Gamma)$$

Inclusion of effect of orography on surface temperature

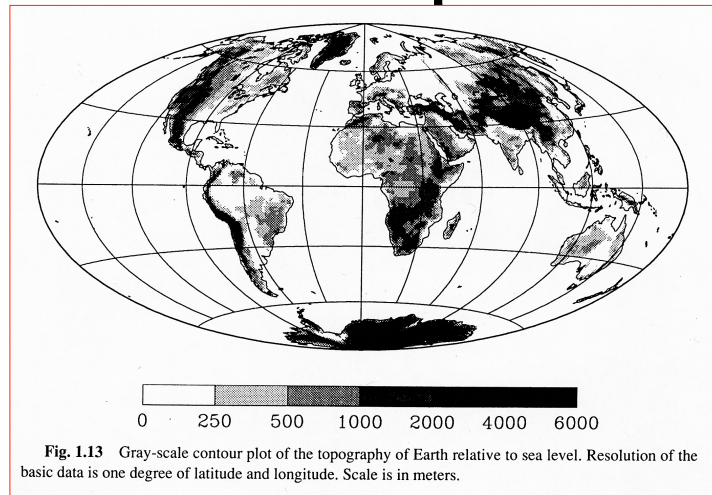


Fig. 1.13 Gray-scale contour plot of the topography of Earth relative to sea level. Resolution of the basic data is one degree of latitude and longitude. Scale is in meters.

Hartmann, 1994, fig.1.13

Land fraction as a function of latitude

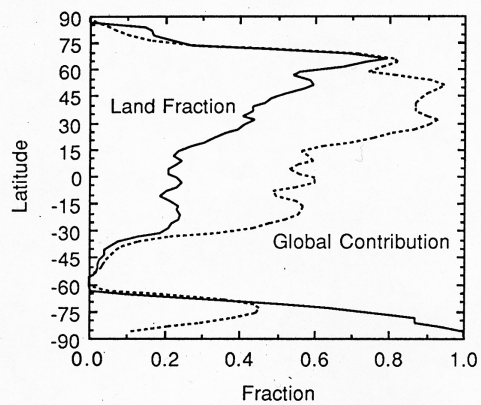


Fig. 1.12 Fraction of surface area covered by land as a function of latitude (solid line) and contribution of each latitude belt to the global land surface area (dashed line).

Hartmann, 1994, fig.1.12

We repeat the equation governing the latitude dependent energy balance:

$$\frac{\partial}{\partial x} (1 - x^2) K_H \frac{\partial T}{\partial x} - bT = -\frac{S_0}{4} s(x)(1 - \alpha) + I_0 - bh\Gamma$$

Oerlemans and van den Dool (1978)* have solved this equation for the albedo parametrization just discussed and realistic distribution land-sea ratio as a function of latitude. Because of these facts the equation is **non-linear**. It therefore has more than one solution for the same parameter values.

Problem. Write a computer program to solve the above equation by **successive relaxation** using the albedo-parametrization introduced on slide nr. 13. Divide domain into 2.5°-latitude intervals and approximate the equation by finite differences. The solution will depend on the first guess.

*J. Oerlemans and H.M. Van den Dool, 1978: J.Atmos.Sci., 35, 371-381.

Solution

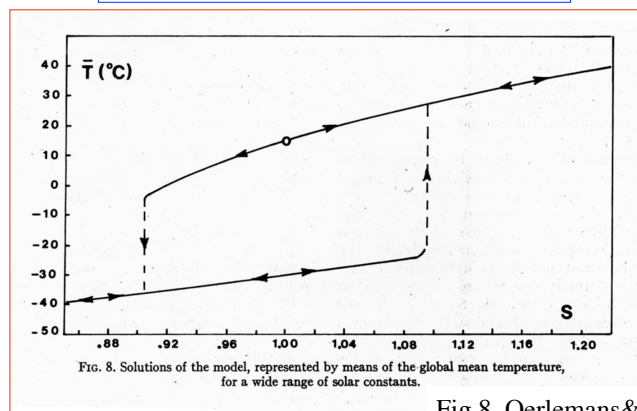


Fig.8, Oerlemans&vdDool, 1978

For a wide range of values of the solar constant around the present value the model possesses two steady state solutions: a partially ice-free earth, resembling the **present climate** and a completely ice-covered earth, referred to as “**snowball earth**”.

Hysteresis

This is an example of hysteresis.

Presently we are in “warm solution”. A negative perturbation of 10% of the present value of the solar “constant” is required to bring the climate into the glaciated state. However, a much larger positive perturbation of the solar “constant” is required to bring us back into the warm climate.

Climate sensitivity depends on its history. Although the real climate system is much more complex (more equilibria or “attractors”), it is thought that the basic concept of “discontinuous steps” remains valid.



“Tipping point”

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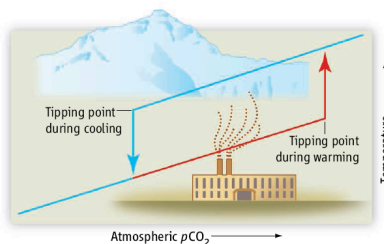
ATMOSPHERIC SCIENCE

Tipping Pointedly Colder

Lee R. Kump

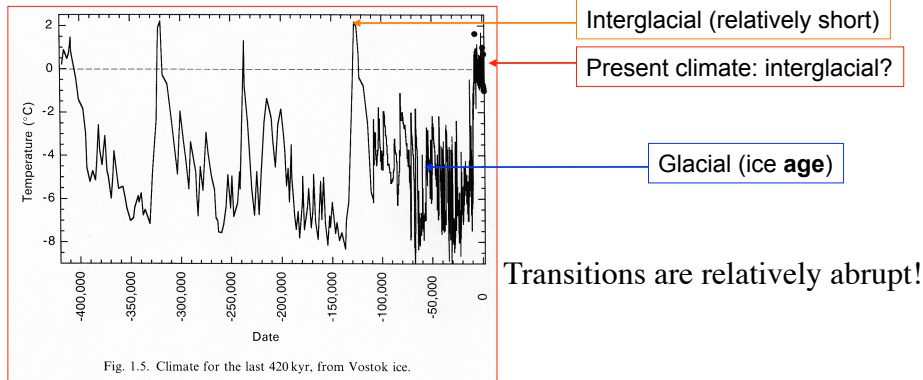
Analysis of the geologic record shows that Earth's temperature has skyrocketed suddenly, and plunged precipitously, many times in its history. These abrupt climate transitions presumably occurred because tipping points in the climate system were exceeded (1). What are these tipping points, and are we pushing the climate system to another with fossil-fuel burning? On page 1187 of this issue, Liu *et al.* (2) bring new tools to bear on these questions. Using data from multiple ocean basins, they study the relation between temperature and ice-sheet initiation during an ancient tipping point: a transition from greenhouse to icehouse.

For much of Earth history, the cli-



How to get into and out of the icehouse. As the atmospheric partial pressure of CO₂ ($p\text{CO}_2$) declines during a cooling phase of an ice-free greenhouse interval (red path), perhaps driven by mountain building and enhanced continental weathering, the climate system approaches a tipping point beyond which it shifts abruptly from greenhouse to icehouse. The escape from the icehouse (blue path) requires $p\text{CO}_2$ to rise above the level that triggered the ice age because of hysteresis in ice-sheet dynamics. Eventually the melting occurs, and when it does, it does so abruptly.

Ice ages and interglacials



Two quasi-equilibrium states, like in EBM??

Conclusion

Budyko-Sellers model is an “icon” for the interpretation of the solutions of more realistic but complex General Circulation models.

Isaac Held (2005):

“The health of climate/modeling in the coming decades is threatened by a growing gap between high-end simulations and idealized theoretical work”

“We typically gain some understanding of a complex system by relating its behaviour to that of other, especially simpler, systems”. The Budyko-Sellers model is such a simplified system.

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