

Exercise 3:

Numerical Simulation of wave propagation

Solve the linear one-layer shallow water equations numerically:

$$\frac{\partial u}{\partial t} = -g \frac{\partial h}{\partial x}; \quad \frac{\partial h}{\partial t} = -H \frac{\partial u}{\partial x}$$

on a **periodic spatial domain** with length, $L=10^4$ km, and with $H=1000$ m and $g=10 \text{ m s}^{-2}$, employing an unstaggered spatial grid. The fluid is at rest initially and a perturbation in h is inserted in the middle of the domain at $x=0$ according to

$$h = H + \eta_0 \exp\left(-\left(\frac{x}{a_0}\right)^2\right) \quad (-5000 \leq x \leq 5000 \text{ km}, \quad \eta_0 = 50 \text{ m and } a = 200 \text{ km})$$

Perform integrations for different values of the grid distance, Δx .

The philosophy of these experiments is similar to that of the Molenkamp test. You should observe *two* perturbations in the free surface that propagate in both directions with constant phase velocity without loss of form (no dispersion). If the boundary conditions are periodic, the waves will meet after a certain time and the state at this time, in terms of the height of the free surface, h , should be exactly as at $t=0$.

Exercise 3: continued

- (a) What is the phase speed of the gravity waves on the free surface?
- (b) At what time will the waves be at the same position as at initial time?
- (c) Perform numerical integrations, lasting this time (see part b), for different spatial grid resolutions by varying the total number of grid points between 100 and 1000. Do this for a fixed time step, $\Delta t = 50$ s. Plot the result at initial time ($t=0$) and at the end of the integration for each integration and interpret the result in the light of “the overview of errors in the numerical simulation of gravity waves” on slide 91.
- (d) Can you improve on the result by reducing Δt ?
- (e) Repeat exercise (c) with a staggered grid in space with the same resolution. In which way does this improve the numerical solution?

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Deadline hand-in exercises of Part 1

Exercise 1: Thursday 16 September 2021

Exercises 2 and 3: Thursday 7 October 2021